HighApproach

NOTICE OF APPRECIATION

We, at HighApproach, extend our warmest gratitude to **Mr. Mandeep Thapa**, **Mr. Sijan Parajuli**, and **Ms. Nisha Rimal** for their generous contribution of invaluable educational resources. Your efforts have significantly bolstered our mission to deliver high-quality education and support to our learners and educators across the globe.

10th March, 2024

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Mid Term Examination, 2024

Full Marks: 100
Time: 3 hours
Set B

BIM/ First Semester/ MTH 204: Basic Mathematics

Candidates are required to give their answers in their own words as far as practicable.

Group A

Brief Answer Questions: [10X2=20]

1. Solve the differential eqn $\frac{dy}{dx} = e^{ax+by}$.
2. Write down the order and degree of differential eqn $\frac{d^3y}{dx^2} + 5\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 6 = 0$
3. Integrate $\int 3^x dx$
4. Find the point of inflection when $y = 3x^3 - 4x^2 + 8x - 9$
5. Find the area bounded by the curve $y^2 = 4ax$, x-axis and two ordinates at x = 0 and x = 2.
6. Evaluate $\lim_{x \to \infty} \left(\frac{3x^3+4x^2-8}{4x^3-5x^2+7}\right)$
7. What are the complex cube roots of unity?
8. Write the inequality $-5 \le x \le -2$ using absolute value sign.
9. If A = [- 1, 3) and B = [2, 4], find A − B and B − A.
10. If $n(A - B) = 24, n(B - A) = 14, n(A \cap B) = 11$, find $n(A \cup B)$.

Group B

Short Answer Questions (Attempt any SIX questions): [6X5=30]

11. a. Find the square root of 3 − 4i.
    b. Express $2 + 2\sqrt{3}i$ in polar form.
12. Evaluate $\lim_{x \to a} \frac{\sqrt{3a-x}-\sqrt{x+a}}{4(x-a)}$
13. A function y = f(x) is defined by $\begin{cases} Ax^2 + 5x - 9 & ,x < 1 \\ B & ,x = 1 \\ (3-x)(A-2x) & ,x > 1 \end{cases}$

    If it is continuous for all x, find 'A' & 'B'
14. Find $\frac{dy}{dx}$ when
    a. $y = (3x^2 + 4x - 5)^{\frac{1}{2}}$
    b. $x^2 + y^2 = 4$
15. Integrate the following
    a. $\int \frac{dx}{x(1+\log x)}$
    b. $\int x^n \log x dx$
16. Solve the differential rqn $\frac{dy}{dx} + ay = e^{mx}$
17. If the marginal revenue function is $MR = \frac{ab}{(x+b)^2} - c$, find the demand law (or function) p(x)

Group C

Long Answer Questions (Attempt any THREE questions): **[3X10=30]**

18. There are three brands of fertilizers X, Y and Z. 'X' contains 1 unit of nitre, 2 units of potash and 3 units of phosphate. Y contains 3 units of nitre, 1 unit of potash and 2 units of phosphate. Z contains 2 units of nitre, 3 units of potash and 1 unit of phosphate. If 11 units of nitre, 10 units of potash and 9 units of phosphate are necessary for a field, how much each type of fertilizers required for it? Solve by Cramer's rule or inverse matrix method.
19. In a group of 200 students, 100 are interested in music, 70 are interested in photography, 40 like swimming, 40 are interested in music and photography, 30 are interested in music and swimming, 20 are interested in photography and swimming and 10 are interested in all three activities. Find the number if students that are interested in
    a. exactly two activities.
    b. at least one activities.
    c. None of the activities.
20. If the marginal revenue and the marginal cost for an output x of a commodity are given by $MR = 5 - 4x + 3x^2$ and $MC = 3 + 2x$ and if the fixed cost is zero, find
    a. the total revenue function.
    b. the cost function.
    c. profit function.
    d. demand function.
21. The demand and supply functions for a commodity are $P_d = 23 - x^2$ and $P_s = 2x^2 - 4$ respectively. Find the consumer's surplus (CS), producer's Surplus (PS) and the total surplus (TS) at the market equilibrium price.

Group D

Comprehensive Answer Question: **[1X20=20]**

22. Let the cost function of a firm be given by $c(x) = 300x - 10x^2 + \frac{x^3}{3}$ where x is an output. Calculate:
    a. the minimum marginal cost.
    b. the minimum average cost.
    c. the output at which average cost is equal to marginal cost.
    d. show that the marginal cost and average cost are equal at the minimum average cost. (6+6+6+2)

Group A

Q. No. 1

Solution,

The given differential equation is $\frac{dy}{dx} = e^{ax+by}$

or, $\frac{dy}{dx} = e^{ax+by}$

or, $\frac{dy}{dx} = e^{ax} \cdot e^{by}$

or, $\frac{dy}{e^{by}} = e^{ax}\, dx$

or, $e^{-by}\, dy = e^{ax}\, dx$

Integrating on both sides,

$\int e^{-by}\, dy = \int e^{ax}\, dx$

or, $\frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + c$

or, $-\frac{e^{-by}}{b} = \frac{e^{ax}}{a} + c$

or, $\frac{e^{ax}}{a} + \frac{e^{-by}}{b} = c$ is the required solution.

2

Q.NO.2

Solution,

The given differential equation is $\frac{d^3y}{dx^2} + 5\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 6 = 0$

$$\frac{d^3y}{dx^2} + 5\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 6 = 0$$

Order (Highest order derivative) = 3

Degree (power of Highest order derivative) = 1

2

Q.No.3

Solution,

$$\int 3^x \, dx$$

$$= \frac{3^x}{\ln 3} \qquad \left[\because \int a^x \, dx = \frac{a^x}{\ln a}\right]$$

2

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Q.No.4

Solution,

$$y = 3x^3 - 4x^2 + 8x - 9$$

Let, $y = f(x) = 3x^3 - 4x^2 + 8x - 9$

Differentiating both sides with respect to 'x':

$$f'(x) = \frac{d(3x^3 - 4x^2 + 8x - 9)}{dx}$$

$$= \frac{d(3x^3)}{dx} - \frac{d(4x^2)}{dx} + \frac{d(8x)}{dx} - \frac{d(9)}{dx}$$

$$= 3 \cdot 3x^2 - 4 \cdot 2x + 8 - 0$$

$$\therefore f'(x) = 9x^2 - 8x + 8$$

Again,

Differentiating both sides with respect to 'x'

$$f''(x) = \frac{d(9x^2 - 8x + 8)}{dx}$$

$$= \cancel{9 \cdot 2} \cdot \frac{d(9x^2)}{dx} - \frac{d(8x)}{dx} + \frac{d(8)}{dx}$$

$$= 9 \cdot 2x - 8 + 0$$

$$\therefore f''(x) = 18x - 8$$

To find point of inflection.

$$f''(x) = 0$$

or, $18x - 8 = 0$

or, $18x = 8$

or, $x = \frac{8}{18}$

$\therefore x = 4/9$

2

$\therefore$ The point of inflection is at $x = 4/9$.

Q.No.5

solution,

Given,

$$y^2 = 4ax$$

we know that,

$$\text{Required Area} = \int_a^b y\,dx \quad \text{---(i)}$$

To get the value of y

$$y^2 = 4ax$$

or $y = \pm\sqrt{4ax}$

or $y = \pm 2\sqrt{a}\sqrt{x}$

$\therefore y = 2\sqrt{a}\sqrt{x}$

Eq^n(i) becomes

$$\text{Required Area} = \int_0^2 2\sqrt{a}\sqrt{x}\,dx$$

2

$$= 2\sqrt{a}\int_0^2 x^{1/2}\,dx$$

$$= 2\sqrt{a}\left[\frac{x^{1/2+1}}{1/2+1}\right]_0^2$$

$$= 2\sqrt{a}\left[\frac{2x^{3/2}}{3}\right]_0^2$$

$$= 2\sqrt{a}\left[\frac{2\times 2^{3/2}}{3} - \frac{2\times 0^{3/2}}{3}\right]$$

$$= 2\sqrt{a}\cdot\frac{2}{3}\cdot 2^{3/2} \text{ sq. units}$$

Q. No. 6

Solution,

Given,

$$\lim_{x\to\infty}\left(\frac{3x^3+4x^2-8}{4x^3-5x^2+7}\right)$$

Dividing Numerator and Denominator by 'x^3'.

$$\cancel{\lim_{x\to\infty}}$$

$$= \lim_{x\to\infty}\frac{\left(\frac{3x^3+4x^2-8}{x^3}\right)}{\left(\frac{4x^3-5x^2+7}{x^3}\right)}$$

$$= \lim_{x\to\infty}\frac{\left(\frac{3x^3}{x^3}+\frac{4x^2}{x^3}-\frac{8}{x^3}\right)}{\left(\frac{4x^3}{x^3}-\frac{5x^2}{x^3}+\frac{7}{x^3}\right)}$$

2

$$= \lim_{x\to\infty} \frac{\left(3 + \frac{4}{x} - \frac{8}{x^3}\right)}{\left(4 - \frac{5}{x} + \frac{7}{x^3}\right)}$$

$$= \frac{3 + \frac{4}{\infty} - \frac{8}{(\infty)^3}}{4 - \frac{5}{\infty} + \frac{7}{(\infty)^3}}$$

$$= \frac{3+0+0}{4-0+0} \quad \left[\because \frac{\text{Finite value}}{\infty} = 0\right]$$

$$= \frac{3}{4} //$$

Q. No. 7

Solution

Let $z^3 = 1$

or, $z^3 - 1 = 0$

or, $z^3 - 1^3 = 0$

or, $(z-1)(z^2+z+1) = 0$ $[\because (a^3-b^3) = (a-b)(a^2+a$

or, $(z-1)(z^2+z+1) = 0$

Either,

$z - 1 = 0$

$z = 1$

Or, $z^2 + z + 1 = 0$

Comparing with $ax^2 + bx + c = 0$

$\therefore a = 1, \; b = 1, \; c = 1$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$= \frac{-1 \pm \sqrt{1 - 4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm \sqrt{-1 \times 3}}{2}$$

$$= \frac{-1 \pm \sqrt{i^2 \times 3}}{2} \qquad [\because i^2 = -1]$$

$$= \frac{-1 \pm i\sqrt{3}}{2}$$

Taking +ve sign

$\therefore z = \dfrac{-1 + i\sqrt{3}}{2}$

Taking -ve sign

$\therefore z = \dfrac{-1 - i\sqrt{3}}{2}$

$\therefore$ The complex cube roots of unity are,

$1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$.

Let 'ω' $= \frac{-1+i\sqrt{3}}{2}$ Then 'ω^2' $= \frac{-1-i\sqrt{3}}{2}$.

Q.No.8

Solution,

Given,

$$-5 \leq x \leq -2$$

Now,

Adding $7/2$ on all sides

or, $-5+\frac{7}{2} \leq x+\frac{7}{2} \leq -2+\frac{7}{2}$

or, $\frac{-10+7}{2} \leq \frac{2x+7}{2} \leq \frac{-4+7}{2}$

or, $\frac{-3}{2} \leq \frac{2x+7}{2} \leq \frac{3}{2}$

or, $-3 \leq 2x+7 \leq 3$

$|2x+7| \leq 3$ [$\because -a \leq x \leq a \Rightarrow |x| \leq a$]

Rough

$a = -5$ $b = -$

$= -\left(\frac{a+b}{2}\right)$

$= -\left(\frac{-5-2}{2}\right)$

$= -\left(-\frac{7}{2}\right)$

$= \frac{7}{2}$

2

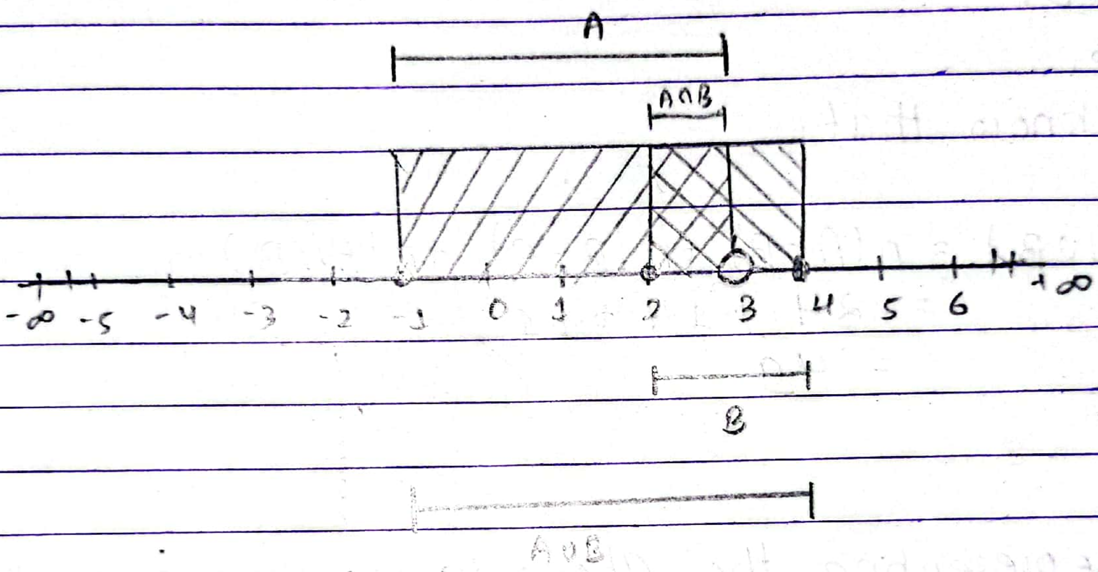
Q. No. 9

Soln,

Given,

$A = [-1, 3)$

$B = [2, 4]$



$A - B = [-1, 3) - [2, 4]$
$= [-1, 2)$

$B - A = [2, 4] - [-1, 3)$
$= [3, 4]$

Q. No. 10

Solution,

Given.

$n(A-B) = 24$

$n(B-A) = 14$

$n(A\cap B) = 11$

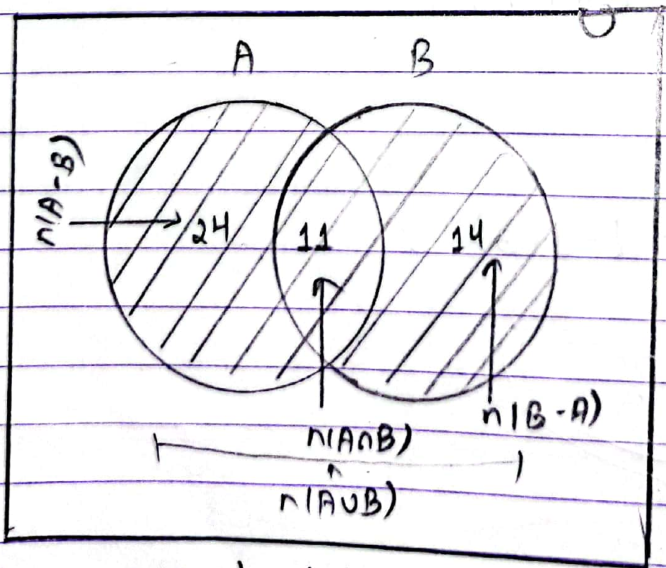
$n(A\cup B) = ?$

Now,

we know that,

$$n(A\cup B) = n(A-B) + n(B-A) + n(A\cap B)$$
$$= 24 + 14 + 11$$
$$= 49$$

Representing the above information in venn-diagram



n(A∪B) = shaded region.

Group 'B'

Q.No. ~~11 a)~~ '12'

Solution,

Given,

$$\lim_{x \to a} \frac{\sqrt{3a-x} - \sqrt{x+a}}{4(x-a)}$$

$$= \lim_{x \to a} \frac{\sqrt{3a-x} - \sqrt{x+a}}{4(x-a)} \times \frac{\sqrt{3a-x} + \sqrt{x+a}}{\sqrt{3a-x} + \sqrt{x+a}}$$

$$= \lim_{x \to a} \frac{(\sqrt{3a-x} - \sqrt{x+a})(\sqrt{3a-x} + \sqrt{x+a})}{4(x-a)(\sqrt{3a-x} + \sqrt{x+a})}$$

$$= \lim_{x \to a} \frac{(\sqrt{3a-x})^2 - (\sqrt{x+a})^2}{4(x-a)(\sqrt{3a-x} + \sqrt{x+a})}$$

$$= \lim_{x \to a} \frac{(3a-x) - (x+a)}{4(x-a)(\sqrt{3a-x} + \sqrt{x+a})}$$

$$= \lim_{x \to a} \frac{3a-x-x-a}{4(x-a)(\sqrt{3a-x} + \sqrt{x+a})}$$

$$= \lim_{x \to a} \frac{2a-2x}{4(x-a)(\sqrt{3a-x} + \sqrt{x+a})}$$

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$$= \lim_{x \to a} \frac{-2x + 2a}{4(x - a)(\sqrt{3a - x} + \sqrt{x + a})}$$

$$= \lim_{x \to a} \frac{-2(x - a)}{4(x - a)(\sqrt{3a - x} + \sqrt{x + a})}$$

$$= \lim_{x \to a} \frac{-1}{2(\sqrt{3a - x} + \sqrt{x + a})}$$

$$= \frac{-1}{2(\sqrt{3a - a} + \sqrt{a + a})}$$

$$= \frac{-1}{2(\sqrt{2a} + \sqrt{2a})}$$

$$= \frac{-1}{2(2\sqrt{2a})}$$

$$= \frac{-1}{4\sqrt{2a}} \quad //$$

Q.No.13

Solution,
Given,

$$y = f(x) = \begin{cases} Ax^2 + 5x - 9, & x < 1 \\ B & x = 1 \\ (3-x)(A-2x), & x > 1 \end{cases}$$

Since the Given function is continuous for all x.

Left Hand limit = Right Hand limit = Functional v

or, $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

or, $\lim_{x \to 1^-} (Ax^2 + 5x - 9) = \lim_{x \to 1^+} (3-x)(A-2x) = B$

or, $A \times 1^2 + 5 \times 1 - 9 = (3-1)(A - 2 \times 1) = B$
or, $A + 5 - 9 = 2(A-2) = B$
or, $A - 4 = 2A - 4 = B$
Taking 1st and 2nd

$A - 4 = 2A - 4$
or, $-4 + 4 = 2A - A$
or, $A = 0$

Taking 1st and 3rd
$A - 4 = B$
or, $0 - 4 = B$
or, $B = -4$

5

$\therefore$ The value of A is 0 and the value of B is -4.

Q.No. 14 (a)

a. $y = (3x^2 + 4x - 5)^{1/2}$

Differentiating both sides with respect to 'x'.

$$\frac{dy}{dx} = \frac{d(3x^2+4x-5)^{1/2}}{dx}$$

Using chain rule.

$$= \frac{d(3x^2+4x-5)^{1/2}}{d(3x^2+4x-5)} \cdot \frac{d(3x^2+4x-5)}{dx}$$

$$= \frac{1}{2}(3x^2+4x-5)^{1/2-1} \cdot \left[\frac{d(3x^2)}{dx} + \frac{d(4x)}{dx} - \frac{d(5)}{dx}\right]$$

$$= \frac{1}{2}(3x^2+4x-5)^{-1/2} \left[3 \cdot 2x + 4 - 0\right]$$

$$= \frac{1}{2(3x^2+4x-5)^{1/2}} (6x+4)$$

$$= \frac{(6x+4)}{2\sqrt{3x^2+4x-5}}$$

$$= \frac{2(3x+2)}{2\sqrt{3x^2+4x-5}}$$

$$= \frac{(3x+2)}{\sqrt{3x^2+4x-5}}$$ //

2.5

Q.No. 14(b)

b. $x^2 + y^2 = 4$

Differentiating both sides with respect to 'x'!

$$\frac{d(x^2+y^2)}{dx} = \frac{d(4)}{dx}$$

or $$\frac{d(x^2)}{dx} + \frac{d(y^2)}{dx} = 0 \qquad \left[\because \frac{d(\text{constant})}{dx} = 0\right]$$

or $$2x + \frac{dy^2}{dy} \cdot \frac{dy}{dx} = 0$$

or $$2x + 2y \cdot \frac{dy}{dx} = 0 \qquad \left[\because \frac{dx^n}{dx} = nx^{n-1}\right]$$

or $$2x = -2y\frac{dy}{dx}$$

or $$\frac{2x}{-2y} = \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{-x}{y} \,//$$

2

Q. No. 15 (a)

a. $\int \frac{dx}{x(1+\log x)}$

Solution,

put $(1+\log x) = t$ --- (i)

Differentiating both sides with respect to 'x'

$\frac{d(1+\log x)}{dx} = \frac{dt}{dx}$

or, $\frac{d(1)}{dx} + \frac{d(\log x)}{dx} = \frac{dt}{dx}$ $\left[\because \frac{d(\log x)}{dx} = \frac{1}{x}\right]$

or, $0 + \frac{1}{x} = \frac{dt}{dx}$

or, $\frac{dx}{x} = dt$ --- (ii)

Substituting the value of $\frac{dx}{x}$.

$\int \frac{dx}{x(1+\log x)} = \int \frac{dx}{x} \cdot \frac{1}{(1+\log x)}$

$= \int dt \cdot \frac{1}{t}$ (From eq^n (i) & (ii))

2.5 $= \int \frac{dt}{t}$ $\left[\because \int \frac{dx}{x} = \log x + c\right]$

$= \log t + k$ (From eq^n (i))

$= \log(1+\log x) + k$ //

Q. No. 15 (b)

$\int x^n \log x \, dx$

$= \int \log x \, x^n \, dx$

Integrating by parts

$= \log x \int x^n dx - \int \left(\frac{d}{dx} \log x \int x^n dx \right) dx \quad \left[\because \int uv\,dx = u\int v - \int \left(\frac{du}{dx} \int v\right) \right]$

$= \log x \cdot \frac{x^{n+1}}{n+1} - \int \left(\frac{1}{x} \cdot \frac{x^{n+1}}{n+1} \right) dx$

$= \frac{x^{n+1}}{n+1} \cdot \log x - \frac{1}{n+1} \int \frac{x^{n+1}}{x^1} dx$

$= \frac{x^{n+1}}{n+1} \cdot \log x - \frac{1}{n+1} \int x^{n+1-1} dx$

$= \frac{x^{n+1}}{n+1} \log x - \frac{1}{n+1} \int x^n dx$

$= \frac{x^{n+1}}{n+1} \log x - \frac{1}{n+1} \cdot \frac{x^{n+1}}{n+1} + c$

2

$= \frac{x^{n+1}}{n+1} \cdot \log x - \frac{x^{n+1}}{(n+1)^2} + c$

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Q.No.16

$$\frac{dy}{dx} + ay = e^{mx}$$

comparing with $\frac{dy}{dx} + Py = Q$.

$\therefore P = a$. $\therefore Q = e^{mx}$

Integrating Factor (I.F) $= e^{\int P dx}$
$= e^{\int a dx}$
$= e^{a \int dx}$
$= e^{ax}$

Now,

$y \times I.F = \int (Q \times I.F) dx + c$

or, $y \times e^{ax} = \int (e^{mx} \times e^{ax}) dx + c$

or, $y e^{ax} = \int e^{mx + ax} dx + c$

or, $y e^{ax} = \int e^{x(m+a)} dx$

or, $y e^{ax} = \frac{e^{x(m+a)}}{(m+a)} + c$ $\left[\because \int e^{ax} dx = \frac{e^{ax}}{a} + c\right]$

$\therefore y e^{ax} = \frac{e^{x(m+a)}}{(m+a)} + c$ //

5

Q.No.17

Solution,

Given,

Marginal Revenue $MR(x) = \frac{ab}{(x+b)^2} - c$

Total Revenue $TR(x) = \int MR(x)\,dx$

$$= \int\left(\frac{ab}{(x+b)^2} - c\right)dx$$

$$= ab\int \frac{1}{(x+b)^2}\,dx - \int c\,dx$$

$$= ab\int (x+b)^{-2}\,dx - cx + k$$

$$= ab \cdot \frac{(x+b)^{-2+1}}{1(-2+1)} - cx + k$$

$$= ab\,\frac{(x+b)^{-1}}{-1} - cx + k$$

$$\therefore TR(x) = \frac{-ab}{(x+b)} - cx + k \quad \text{---(i)}$$

When $x = 0$

$$TR(0) = \frac{-ab}{(0+b)} - c\times 0 + k$$

or, $0 = \frac{-ab}{b} - 0 + k$

$\therefore k = a$

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Eqⁿ(i) becomes,

$$TR(x) = \frac{-ab}{(x+b)} - cx + a$$

$$\text{Demand Law} = \frac{TR(x)}{x}$$

$$= \frac{\frac{-ab}{(x+b)} - cx + a}{x}$$

$$= \frac{-ab}{x(x+b)} + \frac{a(x+b)}{x(x+b)} - c$$

$$= \frac{-ab + a(x+b)}{x(x+b)} - c$$

$$= \frac{-ab + ax + ab}{x(x+b)} - c$$

$$= \frac{ax}{x(x+b)} - c$$

$$= \frac{a}{(x+b)} - c \,//$$

Group 'C'

Q.No.18

Solution,

	Nitre	Potash	Phosphate
X	~~Potash~~ 1	2	3
Y	3	1	2
Z	2	3	1
Total	11	10	9

Let the type of fertilizer 'X' be x, 'Y' be y and 'Z' be z. Then.

$$x + 3y + 2z = 11 \quad ---- (i)$$
$$2x + y + 3z = 10 \quad --- (ii)$$
$$3x + 2y + z = 9 \quad --- (iii)$$

Using Cramer's rule

Coefficient of x	Coefficient of y	Coefficient of z	constant terms
1	3	2	11
2	1	3	10
3	2	1	9

5

$$D = \begin{vmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{vmatrix}$$

Expanding 'D' along 'R_1'

$= 1\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$

$= 1(1\times1 - 3\times2) - 3(2\times1 - 3\times3) + 2(2\times2 - 3\times1)$

$= 1(1-6) - 3(2-9) + 2(4-3)$

$= 1(-5) - 3(-7) + 2(1)$

$= -5 + 21 + 2$

$= 18$

$$D_1 = \begin{vmatrix} 11 & 3 & 2 \\ 10 & 1 & 3 \\ 9 & 2 & 1 \end{vmatrix}$$

Expanding 'D_1' along 'R_1'

$= 11\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 10 & 3 \\ 9 & 1 \end{vmatrix} + 2\begin{vmatrix} 10 & 1 \\ 9 & 2 \end{vmatrix}$

$= 11(1\times1 - 2\times3) - 3(10\times1 - 9\times3) + 2(10\times2 - 9\times1)$

$= 11(1-6) - 3(10-27) + 2(20-9)$

$= 11(-5) - 3(-17) + 2(11)$

$= -55 + 51 + 22$

$= 18$

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$$D_2 = \begin{vmatrix} 1 & 11 & 2 \\ 2 & 10 & 3 \\ 3 & 9 & 1 \end{vmatrix}$$

Expanding 'D_2' along 'R_1'

$$= 1\begin{vmatrix} 10 & 3 \\ 9 & 1 \end{vmatrix} - 11\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 10 \\ 3 & 9 \end{vmatrix}$$

$= 1(10\times1 - 9\times3) - 11(2\times1 - 3\times3) + 2(9\times2 - 10\times3)$

$= 1(10-27) - 11(2-9) + 2(18-30)$

$= 1(-17) - 11(-7) + 2(-12)$

$= -17 + 77 - 24$

$= 36$

$$D_3 = \begin{vmatrix} 1 & 3 & 11 \\ 2 & 1 & 10 \\ 3 & 2 & 9 \end{vmatrix}$$

Expanding 'D_3' along 'R_1'

$$= 1\begin{vmatrix} 1 & 10 \\ 2 & 9 \end{vmatrix} - 3\begin{vmatrix} 2 & 10 \\ 3 & 9 \end{vmatrix} + 11\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$$

$= 1(1\times9 - 2\times10) - 3(9\times2 - 3\times10) + 11(2\times2 - 3\times1)$

$= 1(9-20) - 3(18-30) + 11(4-3)$

$= 1(-11) - 3(-12) + 11(1)$

$= -11 + 36 + 11$

$= 36$

Using Cramer's rule

$x = \frac{D_1}{D} = \frac{18}{18} = 1$

$y = \frac{D_2}{D} = \frac{36}{18} = 2$

$z = \frac{D_3}{D} = \frac{36}{18} = 2$

∴ 1 unit of fertilizer 'x', 2 unit of fertilizer 'y' and '2' unit of fertilizer 'z' is required for it.

Q. No. 20

solution,

Given,

marginal Revenue $MR(x) = 5 - 4x + 3x^2$

marginal cost $MC(x) = 3 + 2x$

Fixed cost $= 0$

a) Total Revenue Function:

Total Revenue $TR(x) = \int MR(x)\,dx$

$= \int (5 - 4x + 3x^2)\,dx$

$= \int 5\,dx - \int 4x\,dx + \int 3x^2\,dx$

$= 5x - \frac{4x^2}{2} + \frac{3x^3}{3} + k$

$\therefore TR(x) = 5x - 2x^2 + x^3 + k$ --- (i)

when $x = 0$

$TR(0) = 5\times0 - 2\times0^2 + 0^3 + k$

$\therefore k = 0$

Eqⁿ (i) becomes

$TR(x) = 5x - 2x^2 + x^3$

$\therefore$ The total revenue function is $5x - 2x^2 + x^3$.

---

b. The cost function.

$\therefore$ Total cost $TC(x) = \int MC(x)\,dx$

$= \int (3 + 2x)\,dx$

$= \int 3\,dx + \int 2x\,dx$

$= 3x + \frac{2x^2}{2} + k$

$\therefore TC(x) = 3x + x^2 + k$ --- (i)

when $x = 0$

$TC(0) = 3\times0 + 0^2 + k$.

or $0 = k$

eqⁿ (i) becomes

$\therefore TC(x) = 3x + x^2$

$\therefore$ The Total cost function is $3x + x^2$.

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c. Profit. Function:

$$\pi(x) = TR(x) - TC(x)$$
$$= 5x - 2x^2 + x^3 - 3x - x^2$$
$$= 2x - 3x^2 + x^3$$
$$= x^3 - 3x^2 + 2x$$

∴ The profit function is $x^3 - 3x^2 + 2x$.

d. Demand Function:

$$\text{Demand} = \frac{TR(x)}{x} \qquad \left(\frac{\text{Total Revenue}}{\text{Quantity}}\right)$$

$$= \frac{5x - 2x^2 + x^3}{x}$$

$$= 5 - 2x + x^2$$

∴ The demand function is $5 - 2x + x^2$.

Q.No.21

Solution,

Given,

Demand $(P_d) = 23 - x^2$

Supply $(P_s) = 2x^2 - 4$

At market equilibrium

$P_d = P_s$

or, $23 - x^2 = 2x^2 - 4$

or, $23 + 4 = 2x^2 + x^2$

or, $27 = 3x^2$

or, $x^2 = 9$

or, $x = \pm\sqrt{9}$

or, $x = \pm 3$

$\therefore x = 3$ (-3 is rejected)

when $x = 3$

Price $= 2(3)^2 - 4$

$= 2 \times 9 - 4$

$= 18 - 4$

$= 14$

Consumer's surplus $C.S(x) = \int_0^x P_d(x)\,dx - P \times x$

$= \int_0^3 (23 - x^2)\,dx - 14 \times 3$

$= \left[23x - \frac{x^3}{3}\right]_0^3 - 42$

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$$= \left[\left(23 \times 3 - \frac{3^3}{3}\right) - \left(23 \times 0 - \frac{0^3}{3}\right)\right] - 42$$

$$= (69 - 9) - 42$$

$$= 60 - 42$$

$$= 18$$

Producer's surplus $P.S(x) = p \times x - \int_0^x Ps(x)\,dx$

$$= 14 \times 3 - \int_0^3 (2x^2 - 4)\,dx$$

$$= 42 - \left[\frac{2x^3}{3} - 4x\right]_0^3$$

$$= 42 - \left[\left(\frac{2 \times 3^3}{3} - 4 \times 3\right) - \left(\frac{2 \times 0}{3} - 4 \times 0\right)\right]$$

$$= 42 - (18 - 12) - 0$$
$$= 42 - 6$$
$$= 36$$

Total surplus (ts) = Consumer surplus + Producer surplus
$$= 18 + 36$$
$$= 54$$

$\therefore$ The consumer's surplus is 18, producer surplus is 36 and total surplus is 54.

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Group 'D'

Q. No. 22

Solution

Given,

$$C(x) = 300x - 10x^2 + \frac{x^3}{3}$$

a. Minimum marginal cost

Soln,

For minimum marginal cost

~~$c'(x) = d$~~

Marginal cost $MC(x) = \frac{d\,C(x)}{dx}$

$$= \frac{d}{dx}\left(300x - 10x^2 + \frac{x^3}{3}\right)$$

$$= \frac{d(300x)}{dx} - \frac{d(10x^2)}{dx} + \frac{d}{dx}\left(\frac{x^3}{3}\right)$$

$$= 300\frac{dx}{dx} - 10\frac{d(x^2)}{dx} + \frac{1}{3}\frac{d(x^3)}{dx}$$

$$= 300 - 20x + \frac{1}{3}\cdot 3x^2$$

$$\therefore MC(x) = 300 - 20x + x^2$$

$MC''(x) = \frac{d}{dx}(300 - 20x + x^2)$

$= \frac{d(300)}{dx} - \frac{d(20x)}{dx} + \frac{d(x^2)}{dx}$

$= 0 - 20 + 2x$

$\therefore MC'(x) = -20 + 2x$ ---- (i)

At critical point,

$MC'(x) = 0$

or, $-20 + 2x = 0$

or, $2x = 20$

$\therefore x = 10$

Again

Differentiating eqn (i) with respect to x.

$\frac{d(MC'(x))}{dx} = \frac{d(-20+2x)}{dx}$

$= \frac{d(-20)}{dx} + \frac{d(2x)}{dx}$

$\therefore MC''(x) = 0 + 2$

when $x = 10$

$MC''(10) = 2 > 0$ (minimum)

$\therefore$ The marginal cost is minimum at $x = 10$.

marginal cost $= 300 - 20 \times 10 + 10^2$

$= 300 - 200 + 100$

$= 200$

b. Minimum Average cost

Average cost = $\frac{\text{Cost function}}{\text{Quantity}}$

$$= \frac{C(x)}{x}$$

$$= \frac{300x - 10x^2 + \frac{x^3}{3}}{x}$$

$$\therefore A(x) = 300 - 10x + \frac{x^2}{3} \quad \text{---- (i)}$$

For minimum $AC(x)$

Differentiating eq^n (i) with respect to 'x'.

$$\frac{d\,A(x)}{dx} = \frac{d}{dx}\left(300 - 10x + \frac{x^2}{3}\right)$$

$$= \frac{d(300)}{dx} - \frac{d(10x)}{dx} + \frac{d}{dx}\left(\frac{x^2}{3}\right)$$

$$= 0 - 10.(1) + \frac{1}{3}\frac{d}{dx}(x^2)$$

$$= -10 + \frac{1}{3}.2x$$

$$\therefore AC'(x) = -10 + \frac{2}{3}x \quad \text{---- (ii)}$$

At critical point, $AC'(x) = 0$

or, $-10 + \frac{2}{3}x = 0$

or, $\frac{2}{3}x = 10$

or, $2x = 30$

or, $x = 15$

Again,
Differentiating eq^n (ii) with respect to 'x'.

$$\frac{d}{dx} AC'(x) = \frac{d}{dx}\left(-10 + \frac{2}{3}x\right)$$

$$= \frac{d(-10)}{dx} + \frac{d}{dx}\left(\frac{2}{3}x\right)$$

$$= 0 + \frac{2}{3} \cdot \frac{dx}{dx}$$

$$\therefore AC''(x) = \frac{2}{3}$$

when $x = 15$

$AC''(15) = \frac{2}{3} > 0$ (minimum)

$\therefore$ The average cost is minimum at $x = 15$.

Minimum Average cost $= 300 - 10 \times 15 + \frac{15^2}{3}$

$= 300 - 150 + 75$

$= 225$

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c) output at which average cost is equal to marginal cost.

Solution,

According to question,

Average cost = Marginal cost

$A(x) = M(x)$

or, $300 - 10x + \frac{x^2}{3} = 300 - 20x + x^2$

or, $0 = 300 - 300 - 20x + 10x + x^2 - \frac{x^2}{3}$

or, $-10x + \frac{2}{3}x^2 = 0$

or, $\frac{2}{3}x^2 - 10x = 0$

or, $x\left(\frac{2}{3}x - 10\right) = 0$

Either,

$x = 0$

or

$\frac{2}{3}x = 10$

$x = \frac{30}{2}$

$\therefore x = 15$

$\therefore$ The output at which average cost is equal to marginal cost is at $x = 0$ or 15.

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d) Show that the marginal cost and average cost are equal at the minimum average cost.

Solution,
Here,
Minimum average cost = 15

Marginal cost at minimum average cost

$= 300 - 20x + x^2 \quad [x = 15]$
$= 300 - 20 \times 15 + (15)^2$
$= 300 - 300 + 225$
$= 225$

Average cost at minimum

$= 300 - 10x + \frac{x^2}{3} \quad [x = 15]$

$= 300 - 10 \times 15 + \frac{15^2}{3}$

$= 300 - 150 + \frac{225}{3}$

$= 150 + 75$
$= 225$

$\therefore$ The marginal cost and average cost are equal at the minimum average cost.