Exercise 8(B)

1. Evaluate the following determinants.

a) $\begin{vmatrix} 20 & 21 \\ 22 & 23 \end{vmatrix}$

Solution

Let $|A| = \begin{vmatrix} 20 & 21 \\ 22 & 23 \end{vmatrix}$

$= 20 \times 23 - 22 \times 21$

$= 460 - 462$

$\therefore |A| = -2$

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b) $\begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix}$ [evaluate by all 7 process]

Solution

Let $|A| = \begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix}$

Expanding $|A|$ using R_1

$= +1 \begin{vmatrix} 5 & 8 \\ 6 & 9 \end{vmatrix} - 4 \begin{vmatrix} 2 & 8 \\ 3 & 9 \end{vmatrix} + 7 \begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix}$

$= 1(5\times9 - 6\times8) - 4(2\times9 - 3\times8) + 7(2\times6 - 3\times5)$

$= 1(45 - 48) - 4(18 - 24) + 7(12 - 15)$

$= -3 + 24 - 21$

$= -24 + 24$

$= 0$

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Expanding $|A|$ using R_2

$$= -2\begin{vmatrix} 5 & 8 \\ 6 & 9 \end{vmatrix} + 5\begin{vmatrix} 2 & 8 \\ 3 & 9 \end{vmatrix} - 8\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix}$$

$= -2(5\times9 - 6\times8) + 5(2\times9 - 3\times8) - 8(2\times6 - 3\times5)$
$= -2(45 - 48) + 5(18 - 24) - 8(12 - 15)$
$= 6 - 30 + 24$
$= -30 + 30$
$= 0$

Expanding $|A|$ using R_3

$$= +3\begin{vmatrix} 4 & 7 \\ 5 & 8 \end{vmatrix} - 6\begin{vmatrix} 1 & 7 \\ 2 & 8 \end{vmatrix} + 9\begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix}$$

$= 3(4\times8 - 5\times7) - 6(1\times8 - 2\times7) + 9(1\times5 - 2\times4)$
$= 3(32 - 35) - 6(8 - 14) + 9(5 - 8)$
$= -9 + 36 - 27$
$= -36 + 36$
$= 0$

Expanding $|A|$ using C_1

$$= +1\begin{vmatrix} 5 & 8 \\ 6 & 9 \end{vmatrix} - 2\begin{vmatrix} 4 & 7 \\ 6 & 9 \end{vmatrix} + 3\begin{vmatrix} 4 & 7 \\ 5 & 8 \end{vmatrix}$$

$= 1(5\times9 - 6\times8) - 2(4\times9 - 6\times7) + 3(4\times8 - 5\times7)$
$= 1(45 - 48) - 2(36 - 42) + 3(32 - 35)$
$= -3 + 12 - 9$
$= -12 + 12$
$= 0$

Expanding $|A|$ using C_2

$$= -4\begin{vmatrix} 2 & 8 \\ 3 & 9 \end{vmatrix} + 5\begin{vmatrix} 1 & 7 \\ 3 & 9 \end{vmatrix} - 6\begin{vmatrix} 1 & 7 \\ 2 & 8 \end{vmatrix}$$

$$= -4(2\times9 - 3\times8) + 5(1\times9 - 3\times7) - 6(1\times8 - 2\times7)$$
$$= -4(18-24) + 5(9-21) - 6(8-14)$$
$$= 24 - 60 + 36$$
$$= 60 - 60$$
$$= 0$$

Expanding $|A|$ using C_3

$$= +7\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} - 8\begin{vmatrix} 1 & 4 \\ 3 & 6 \end{vmatrix} + 9\begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix}$$

$$= 7(2\times6 - 3\times5) - 8(1\times6 - 3\times4) + 9(1\times5 - 2\times4)$$
$$= 7(12-15) - 8(6-12) + 9(5-8)$$
$$= -21 + 48 - 27$$
$$= -48 + 48$$
$$= 0$$

Expanding $|A|$ using Sarrus Rule

$$\begin{array}{ccccc} 1 & 4 & 7 & 1 & 4 \\ 2 & 5 & 8 & 2 & 5 \\ 3 & 6 & 9 & 3 & 6 \end{array}$$

$$= 1\times5\times9 + 4\times8\times3 + 7\times2\times6 - 3\times5\times7 - 6\times8\times1 - 9\times2\times4$$
$$= 45 + 96 + 84 - 105 - 48 - 72$$
$$= 0$$
$$\therefore |A| = 0$$

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c) $\begin{vmatrix} 1 & 4 & 6 \\ 2 & 1 & 8 \\ 1 & 2 & 5 \end{vmatrix}$

Solution

$$\text{Let } |A| = \begin{vmatrix} 1 & 4 & 6 \\ 2 & 1 & 8 \\ 1 & 2 & 5 \end{vmatrix}$$

Expanding $|A|$ using R_1

$$= +1\begin{vmatrix} 1 & 8 \\ 2 & 5 \end{vmatrix} - 4\begin{vmatrix} 2 & 8 \\ 1 & 5 \end{vmatrix} + 6\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 1(1\times5 - 2\times8) - 4(2\times5 - 1\times8) + 6(2\times2 - 1\times1)$$

$$= 1(5-16) - 4(10-8) + 6(4-1)$$

$$= -11 - 8 + 18$$

$$= -19 + 18$$

$$= -1$$

$$\therefore |A| = -1$$

d) $\begin{vmatrix} 12 & 0 & 0 \\ 4 & 3 & 0 \\ 2 & 2 & -3 \end{vmatrix}$

Solution

$$\text{Let } |A| = \begin{vmatrix} 12 & 0 & 0 \\ 4 & 3 & 0 \\ 2 & 2 & -3 \end{vmatrix}$$

Expanding $|A|$ using R_1

$$= +12\begin{vmatrix} 3 & 0 \\ 2 & -3 \end{vmatrix} - 0\begin{vmatrix} 4 & 0 \\ 2 & -3 \end{vmatrix} + 0\begin{vmatrix} 4 & 3 \\ 2 & 2 \end{vmatrix}$$

$$= 12(3\times(-3) - 2\times 0) - 0(4\times(-3) - 2\times 0) + 0(4\times 2 - 2\times 3)$$
$$= 12(-9-0) - 0 - 0$$
$$= 12\times(-9)$$
$$= -108$$

$$\therefore |A| = -108$$

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e) $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

Solution

Let $|A| = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

Expanding $|A|$ using R_1

$$= +1\begin{vmatrix} b & c+a \\ c & a+b \end{vmatrix} - a\begin{vmatrix} 1 & c+a \\ 1 & a+b \end{vmatrix} + (b+c)\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$= 1[b(a+b) - c(c+a)] - a[(a+b) - (c+a)] + (b+c)[c-b]$$

$$= b(a+b) - c(c+a) - a(a+b) + a(c+a) + c(b+c) - b(b+c)$$

$$= \cancel{ab} + b^2 - \cancel{ac} - \cancel{bc} - \cancel{a^2} - \cancel{ab} + \cancel{ac} + \cancel{a^2} + \cancel{bc} + c^2 - b^2 - bc$$

$= 1\left[b(a+b) - c(c+a)\right] - a\left[(a+b) - (c+a)\right] + (b+c)\left[c-b\right]$

$\cancel{= ab + b^2 - c^2 + ac - a[a+b-c-a] + (b+c)(c-b)}$

$\cancel{= ab + b^2 - c^2 + ac - ab + ac + bc - b^2 + c^2 - bc}$

$= ab + b^2 - c^2 - ac - a[b-c] + (b+c)(c-b)$

$= ab + b^2 - c^2 - ac - ab + ac + bc - b^2 + c^2 - bc$

$= 0$

f) $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$

Solution

Let $|A| = \begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$

Expanding $|A|$ using R_1

$= -a^2 \begin{vmatrix} -b^2 & bc \\ bc & -c^2 \end{vmatrix} - ab \begin{vmatrix} ab & bc \\ ac & -c^2 \end{vmatrix} + ac \begin{vmatrix} ab & -b^2 \\ ac & bc \end{vmatrix}$

$= -a^2(b^2c^2 - bc \times bc) - ab(ab \times (-c^2) - ac \times bc)$
$+ ac(ab \times bc - ac \times (-b^2)))$

$= -a^2(b^2c^2 - b^2c^2) - ab(-abc^2 - abc^2) + ac(ab^2c + ab^2c)$

$= 0 - ab(-abc^2 - abc^2) + ac(2ab^2c)$

$= 0 - ab(-2abc^2) + ac(2ab^2c)$

$= +2a^2b^2c^2 + 2a^2b^2c^2$

$\cancel{= 2a^2b^2c^2}$ ~~4at~~

$= 4a^2b^2c^2$

$\therefore |A| = 4a^2b^2c^2$

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g) $\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix}$

Solution

Let $|A| = \begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix}$

Expanding $|A|$ using R_1

$= a\begin{vmatrix} 1 & c+a \\ 1 & a+b \end{vmatrix} - 1\begin{vmatrix} b & c+a \\ c & a+b \end{vmatrix} + (b+c)\begin{vmatrix} b & 1 \\ c & 1 \end{vmatrix}$

$= a[a+b-c-a] - 1[b(a+b) - c(c+a)] + (b+c)(b-c)$

$= a[ab-c] - 1[ab+b^2-c^2-ac] + b^2-c^2$

$= ab - ac - ab - b^2 + c^2 + ac + b^2 - c^2$

$= 0$

$\therefore |A| = 0$

h) $\begin{vmatrix} 77 & 78 & 79 \\ 75 & 74 & 73 \\ 76 & 75 & 74 \end{vmatrix}$

Solution

Let $|A| = \begin{vmatrix} 77 & 78 & 79 \\ 75 & 74 & 73 \\ 76 & 75 & 74 \end{vmatrix}$

Expanding $|A|$ using R_1

$$= 77\begin{vmatrix} 74 & 73 \\ 75 & 74 \end{vmatrix} - 78\begin{vmatrix} 75 & 73 \\ 76 & 74 \end{vmatrix} + 79\begin{vmatrix} 75 & 74 \\ 76 & 75 \end{vmatrix}$$

$= 77(74\times74 - 75\times73) - 78(75\times74 - 76\times73) + 79(75\times75 - 76\times74)$

$= 77(5476 - 5475) - 78(5550 - 5548) + 79(5625 - 5624)$

$= 77 - 156 + 79$

$= 156 - 156$

$= 0$

$\therefore |A| = 0$

2. Solve for 'x' when

a) $\begin{vmatrix} x & 2 \\ 3 & 5 \end{vmatrix} = 0$

Solution

Given,

$$\begin{vmatrix} x & 2 \\ 3 & 5 \end{vmatrix} = 0$$

Or, $x\times5 - 3\times2 = 0$

Or, $5x = 6$

$\therefore x = \frac{6}{5}$

b) $\begin{vmatrix} 3-x & 5 \\ 4 & 2 \end{vmatrix} = 0$

Solution
Given,

$$\begin{vmatrix} 3-x & 5 \\ 4 & 2 \end{vmatrix} = 0$$

or, $2(3-x) - 4\times5 = 0$
or, $6 - 2x = 20$
or, $2x = -14$
$\therefore x = -7$

c) $\begin{vmatrix} x+2 & 3 \\ 4 & x+1 \end{vmatrix} = 0$

Solution
Given,

$$\begin{vmatrix} x+2 & 3 \\ 4 & x+1 \end{vmatrix} = 0$$

or, $(x+2)(x+1) - 4\times3 = 0$
or, $x^2 + x + 2x + 2 = 12$
or, $x^2 + 3x + 2 - 12 = 0$
or, $x^2 + 3x - 10 = 0$
or, $x^2 + 5x - 2x - 10 = 0$
or, $x(x+5) - 2(x+5) = 0$
or, $(x+5)(x-2) = 0$
$\therefore x = 2, -5$

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d) $\begin{vmatrix} 2 & 3 & 4 \\ 0 & -1 & x \\ x & 4 & 0 \end{vmatrix} = 0$

Solution

Given,

$$\begin{vmatrix} 2 & 3 & 4 \\ 0 & -1 & x \\ x & 4 & 0 \end{vmatrix} = 0$$

or, $2\begin{vmatrix} -1 & x \\ 4 & 0 \end{vmatrix} - 3\begin{vmatrix} 0 & x \\ x & 0 \end{vmatrix} + 4\begin{vmatrix} 0 & -1 \\ x & 4 \end{vmatrix} = 0$

or, $2(-0-4x) - 3(0-x^2) + 4(0+x) = 0$

or, $-8x + 3x^2 + 4x = 0$

~~or,~~ $\cancel{3x^2 - 4x = 0}$

~~or,~~ $\cancel{x^2 - 4x = 0}$

~~or,~~ $\cancel{x(x-4) = 0}$

$\cancel{\therefore x = 0, 4}$

or, $3x^2 - 4x = 0$

or, $x(3x-4) = 0$

$\therefore x = 0, \frac{4}{3}$

e) $\begin{vmatrix} x+1 & 2 & 3 \\ 1 & x+2 & 3 \\ 1 & 2 & x+3 \end{vmatrix} = 0$

Solution

Given,

$$\begin{vmatrix} x+1 & 2 & 3 \\ 1 & x+2 & 3 \\ 1 & 2 & x+3 \end{vmatrix} = 0$$

or, $x+1 \begin{vmatrix} x+2 & 3 \\ 2 & x+3 \end{vmatrix} - 2 \begin{vmatrix} 1 & 3 \\ 1 & x+3 \end{vmatrix} + 3 \begin{vmatrix} 1 & x+2 \\ 1 & 2 \end{vmatrix} = 0$

or, $x+1 \left[(x+2)(x+3) - 3\times 2\right] - 2\left[(x+3) - 3\times 1\right] + 3\left[1\times 2 - (x+2)\right] = 0$

or, $x+1 \left[x^2+3x+2x+6 - 6\right] - 2\left[x+3-3\right] + 3\left[2 - x - 2\right] = 0$

or, $x+1 (x^2+5x) - 2(x) + 3(-x) = 0$

or, $x^3 + 5x^2 + x^2 + 5x - 2x - 3x = 0$

or, $x^3 + 5x^2 + x^2 + 5x - 5x = 0$

or, $x^3 + \cancel{5x^2}\ 6x^2 = 0$

$\therefore x = 0, -6$

f) $\begin{vmatrix} x & 2 & 2 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix} = 0$

Solution

Given,

$$\begin{vmatrix} x & 2 & 2 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix} = 0$$

or, $x \begin{vmatrix} x & 2 \\ 2 & x \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 \\ 2 & x \end{vmatrix} + 2 \begin{vmatrix} 2 & x \\ 2 & 2 \end{vmatrix} = 0$

or, $x(x \times x - 2\times2) - 2(2\times x - 2\times2) + 2(2\times2 - 2\times x) = 0$

or, $x(x^2 - 4) - 2(2x - 4) + 2(4 - 2x) = 0$

or, $x^3 - 4x - 4x + 8 + 8 - 4x = 0$

~~or, $x^3 - 4x^2 - 8x + 16 = 0$~~

or, $x^3 - 12x + 16 = 0$

$\therefore x = 2, -4$

34. Test whether the following matrices are singular or non-singular.

a) $\begin{bmatrix} 5 & 3 \\ 4 & 4 \end{bmatrix}$

Solution

Let $|A| = \begin{vmatrix} 5 & 3 \\ 4 & 4 \end{vmatrix}$

$= 5\times4 - 4\times3$

$= 20 - 12$

$\therefore |A| = 8$

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Since, $|A| \neq 0$, the matrix is non-singular

b) $\begin{bmatrix} 3 & 0 & 4 \\ 3 & 2 & 5 \\ 15 & 10 & 25 \end{bmatrix}$

Solution

Let $|A| = \begin{vmatrix} 3 & 0 & 4 \\ 3 & 2 & 5 \\ 15 & 10 & 25 \end{vmatrix}$

$= 3 \begin{vmatrix} 2 & 5 \\ 10 & 25 \end{vmatrix} - 0 \begin{vmatrix} 3 & 5 \\ 15 & 25 \end{vmatrix} + 4 \begin{vmatrix} 3 & 2 \\ 15 & 10 \end{vmatrix}$

$= 3(25 \times 2 - 10 \times 5) - 0 + 4(3 \times 10 - 15 \times 2)$

$= 3(50 - 50) - 0 + 4(30 - 30)$

$= 0 - 0 + 0$

$= 0$

Since, $|A| = 0$, the matrix is singular.

c) $\begin{bmatrix} 2 & 3 & 4 \\ 2 & -5 & 0 \\ 1 & 3 & 5 \end{bmatrix}$

Solution

Let $|A| = \begin{vmatrix} 2 & 3 & 4 \\ 2 & -5 & 0 \\ 1 & 3 & 5 \end{vmatrix}$

$= 2 \begin{vmatrix} -5 & 0 \\ 3 & 5 \end{vmatrix} - 3 \begin{vmatrix} 2 & 0 \\ 1 & 5 \end{vmatrix} + 4 \begin{vmatrix} 2 & -5 \\ 1 & 3 \end{vmatrix}$

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$= 2(-5\times5 - 3\times0) - 3(2\times5 - 1\times0) + 4(2\times3 - 1\times(-5))$

$= 2(-25 - 0) - 3(10 - 0) + 4(6 + 5)$

$= -50 - 30 + 44$

$= -36$

Since, $|A| \neq 0$, the matrix is non-singular.

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~~2. One unit of medicine A contains 10 units of chemical x and 6 units of chemical y unit of medicine B~~

Exercise 8(B)

5. Find the inverse of the following matrices.

a) $\begin{bmatrix} 5 & 1 \\ 2 & 4 \end{bmatrix}$

Solution

Let $A = \begin{bmatrix} 5 & 1 \\ 2 & 4 \end{bmatrix}$

$$|A| = \begin{vmatrix} 5 & 1 \\ 2 & 4 \end{vmatrix}$$
$$= 5 \times 4 - 2 \times 1$$
$$= 20 - 2$$
$= 18 \neq 0$. so, A^{-1} exists

Let the cofactor matrix be $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

$A_{11} = +4$

$A_{12} = -2$

$A_{21} = -1$

$A_{22} = +5$

Then, the cofactor matrix becomes,

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & 5 \end{bmatrix}$$

~~Adjor~~ Also,

Adjoint Matrix = (Cofactor matrix)^T

$$= \begin{bmatrix} 4 & -2 \\ -1 & 5 \end{bmatrix}^T$$

$$\therefore \text{Adj. } A = \begin{bmatrix} 4 & -1 \\ -2 & 5 \end{bmatrix}$$

Now,

Inverse Matrix $= \dfrac{\text{Adjoint matrix}}{|A|}$, $|A| \neq 0$

$$A^{-1} = \frac{\begin{bmatrix} 4 & -1 \\ -2 & 5 \end{bmatrix}}{18}$$

$$= \begin{bmatrix} 4/18 & -1/18 \\ -2/18 & 5/18 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 2/9 & -1/18 \\ -1/9 & 5/18 \end{bmatrix}$$

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b) $\begin{bmatrix} 2 & -1 \\ -3 & 3 \end{bmatrix}$

Solution

Let $A = \begin{bmatrix} 2 & -1 \\ -3 & 3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix}$$

$= 2 \times 3 - (-3) \times (-1)$

$= 6 - 3$

$= 3 \neq 0$. So, A^{-1} exists.

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Let the cofactor matrix be $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

$A_{11} = +3$

$A_{12} = -(-3) = 3$

~~$A_{13} = +$~~

$A_{21} = -(-1) = 1$

$A_{22} = +2$

Then, the cofactor matrix becomes,

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 1 & 2 \end{bmatrix}$$

Also,

Adjoint matrix = (cofactor matrix)^T

$$= \begin{bmatrix} 3 & 3 \\ 1 & 2 \end{bmatrix}^T$$

$$\therefore \text{Adj. } A = \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix}$$

Now,

Inverse matrix $= \frac{\text{Adj. } A}{|A|}$, $|A| \neq 0$

$$A^{-1} = \frac{\begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix}}{3}$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & 1/3 \\ 1 & 2/3 \end{bmatrix}$$

c) $\begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$

Solution

$$\text{Let } A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{vmatrix}$$

Expanding $|A|$ by using R_1

$$= 1 \begin{vmatrix} -1 & 4 \\ 2 & 1 \end{vmatrix} - (-2) \begin{vmatrix} 0 & 4 \\ -2 & 1 \end{vmatrix} + 3 \begin{vmatrix} 0 & -1 \\ -2 & 2 \end{vmatrix}$$

$$= 1(-1\times1 - 2\times4) + 2(0\times1 - (-2)\times4) + 3(0\times2 - (-2)\times(-1))$$

$$= 1(-1-8) + 2(0+8) + 3(0-2)$$

$$= -9 + 16 - 6$$

$= 1 \neq 0$. So, A^{-1} exists.

Let the cofactor matrix be $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

$$A_{11} = + \begin{vmatrix} -1 & 4 \\ 2 & 1 \end{vmatrix} = -1\times1 - 2\times4 = -9$$

$$A_{12} = - \begin{vmatrix} 0 & 4 \\ -2 & 1 \end{vmatrix} = -(0\times1 - (-2)\times4) = -8$$

$$A_{13} = + \begin{vmatrix} 0 & -1 \\ -2 & 2 \end{vmatrix} = 0\times2 - (-2)\times(-1) = -2$$

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$$A_{21} = -\begin{vmatrix} -2 & 3 \\ 2 & 1 \end{vmatrix} = -(-2\times1 - 2\times3) = 8$$

$$A_{22} = +\begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} = 1\times1 - (-2)\times3 = 7$$

$$A_{23} = -\begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix} = -(1\times2 - (-2)\times(-2)) = 2$$

$$A_{31} = +\begin{vmatrix} -2 & 3 \\ -1 & 4 \end{vmatrix} = -2\times4 - (-1)\times3 = -5$$

$$A_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -(1\times4 - 0\times3) = -4$$

$$A_{33} = +\begin{vmatrix} 1 & -2 \\ 0 & -1 \end{vmatrix} = 1\times(-1) - 0\times(-2) = -1$$

Then the cofactor matrix becomes,

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

Also,

Adjoint matrix = (cofactor matrix)^T

$$= \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}^T$$

$$\therefore \text{Adj. } A = \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}$$

Now,

$$\text{Inverse Matrix} = \frac{\text{Adj. }A}{|A|}, \quad |A| \neq 0$$

$$A^{-1} = \frac{\begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}}{1}$$

$$\therefore A^{-1} = \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}$$

d) $\begin{bmatrix} -1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$

Solution

$$\text{Let } A = \begin{bmatrix} -1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{vmatrix}$$

<u>Expanding $|A|$ by using R_1</u>

$$= +(-1)\begin{vmatrix} 0 & 2 \\ 1 & -3 \end{vmatrix} - 2\begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} + 1\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix}$$

$$= -1(0\times(-3) - 1\times 2) - 2((-1)\times(-3) - 2\times 2) + 1((-1)\times 1 - 2\times 0)$$

$$= 2 + 2 - 1$$

$= 3 \neq 0$. So A^{-1} exists

Let the cofactor matrix be $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

$$A_{11} = +\begin{vmatrix} 0 & 2 \\ 1 & -3 \end{vmatrix} = 0\times(-3) - 2\times1 = -2$$

$$A_{12} = -\begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = -((-1)\times(-3) - 2\times2) = 1$$

$$A_{13} = +\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} = (-1)\times1 - 2\times0 = -1$$

$$A_{21} = -\begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix} = -(2\times(-3) - 1\times1) = 7$$

$$A_{22} = +\begin{vmatrix} -1 & 1 \\ 2 & -3 \end{vmatrix} = (-1)\times(-3) - 2\times1 = 1$$

$$A_{23} = -\begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} = -(-1\times1 - 2\times2) = 5$$

$$A_{31} = +\begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} = 2\times2 - 0\times1 = 4$$

$$A_{32} = -\begin{vmatrix} -1 & 1 \\ -1 & 2 \end{vmatrix} = -(-1\times2 - (-1)\times1) = 1$$

$$A_{33} = +\begin{vmatrix} -1 & 2 \\ -1 & 0 \end{vmatrix} = -1\times0 - (-1)\times2 = 2$$

Then, the cofactor matrix becomes,

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 \\ 7 & 1 & 5 \\ 4 & 1 & 2 \end{bmatrix}$$

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Now,

Adjoint of A = (cofactor matrix)'

$$= \begin{bmatrix} -2 & 1 & -1 \\ 7 & 1 & 5 \\ 4 & 1 & 2 \end{bmatrix}^T$$

$$\therefore \text{Adj. } A = \begin{bmatrix} -2 & 7 & 4 \\ 1 & 1 & 1 \\ -1 & 5 & 2 \end{bmatrix}$$

Then,

$$\text{Inverse Matrix} = \frac{\text{Adj. } A}{|A|}, \quad |A| \neq 0$$

$$A^{-1} = \frac{\begin{bmatrix} -2 & 7 & 4 \\ 1 & 1 & 1 \\ -1 & 5 & 2 \end{bmatrix}}{3}$$

$$\therefore A^{-1} = \begin{bmatrix} -\frac{2}{3} & \frac{7}{3} & \frac{4}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{5}{3} & \frac{2}{3} \end{bmatrix}$$

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6. When will two matrices be inverse of each other? Show that the matrices $\begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix}$ and $\begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix}$ are inverse of each other.

⇒ Two square matrices A & B are inverses to each other if $AB = BA = I$, where I is the identity matrix of order A or B.

$$\text{Let } A = \begin{bmatrix} 3 & -1 \\ 5 & -2 \end{bmatrix} \text{ \& } B = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$$

Then,

$$AB = \begin{bmatrix} 3 & -1 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times2 + (-1)\times5 & 3\times(-1) + (-1)\times(-3) \\ 5\times2 + (-2)\times5 & 5\times(-1) + (-2)\times(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 6-5 & -3+3 \\ 10-10 & -5+6 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times3 + (-1)\times5 & 2\times(-1) + (-1)\times(-2) \\ 5\times3 + (-3)\times5 & 5\times(-1) + (-3)\times(-2) \end{bmatrix}$$

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$$= \begin{bmatrix} 6-5 & -2+2 \\ 15-15 & -5+6 \end{bmatrix}$$

$$\therefore BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Since $AB = BA = I$, the matrices are inverse of each other.

---

7. If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, verify that this matrix and its adjoint are inverse to each other.

Solution

Given,

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

Let the cofactor matrix be $\cancel{B} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

$$A_{11} = + \begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} = 3\times1 - (-2)\times0 = 3$$

$$A_{12} = - \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = -(-1\times1 - 0\times0) = 1$$

$$A_{13} = + \begin{vmatrix} -1 & 3 \\ 0 & -2 \end{vmatrix} = (-1)\times(-2) - 0\times3 = 2$$

$$A_{21} = - \begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} = -(2\times1 - (-2)\times(-2)) = 2$$

$$A_{22} = +\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1\times 1 - 0\times(-2) = 1$$

$$A_{23} = -\begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} = -(1\times(-2) - 0\times 2) = 2$$

$$A_{31} = +\begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} = 2\times 0 - 3\times(-2) = 6$$

$$A_{32} = -\begin{vmatrix} 1 & -2 \\ -1 & 0 \end{vmatrix} = -(1\times 0 - (-2)\times(-2)) = 2$$

$$A_{33} = +\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} = 1\times 3 - (-1)\times 2 = 5$$

~~Now~~, Then, the cofactor matrix becomes,

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}$$

Now,

Adjoint of A = (cofactor matrix)^T

$$= \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}^T$$

$$\therefore \text{Adj. } A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Now,

$$A \times Adj.\ A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times3+2\times1+(-2)\times2 & 1\times2+2\times1+(-2)\times2 & 1\times6+2\times2+(-2)\times5 \\ -1\times3+3\times1+0\times2 & -1\times2+3\times1+0\times2 & -1\times6+3\times2+0\times5 \\ 0\times3+(-2)\times1+1\times2 & 0\times2+(-2)\times1+1\times2 & 0\times6+(-2)\times2+1\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 3+2-4 & 2+2-4 & 6+4-10 \\ -3+3+0 & -2+3+0 & -6+6+0 \\ 0-2+2 & 0-2+2 & 0-4+5 \end{bmatrix}$$

$$\therefore \cancel{Adj}\ A \times Adj.\ A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Also,

$$Adj.\ A \times A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times1+2\times(-1)+6\times0 & 3\times2+2\times3+6\times(-2) & 3\times(-2)+2\times0+6\times1 \\ 1\times1+1\times(-1)+2\times0 & 1\times2+1\times3+2\times(-2) & 1\times(-2)+1\times0+2\times1 \\ 2\times1+2\times(-1)+5\times0 & 2\times2+2\times3+5\times(-2) & 2\times(-2)+2\times0+5\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2+0 & 6+6-12 & -6+0+6 \\ 1-1+0 & 2+3-4 & -2+0+2 \\ 2-2+0 & 4+6-10 & -4+0+5 \end{bmatrix}$$

$$\therefore Adj.\ A \times A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since $A \times Adj.\ A = Adj.\ A \times A = I$, the matrix and its adjoint are inverse to each other.

8. 8. If $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$, verify that $A(\text{Adj} A) = |A| I$

Solution

Given,

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{vmatrix}$$

Expanding |A| by using R_1

$$= 1 \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} + 5 \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix}$$

$$= 1(3\times1 - 1\times1) - 2(2\times1 - (-1)\times1) + 5(2\times1 - (-1)\times3)$$

$$= 1(3-1) - 2(2+1) + 5(2+3)$$

$$= 2 - 6 + 25$$

$$= 21$$

Let the cofactor matrix be $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

$$A_{11} = + \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 3\times1 - 1\times1 = 2$$

$$A_{12} = - \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = -(2\times1 - (-1)\times1) = -3$$

$$A_{13} = + \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} = +(2\times1 - 3\times(-1)) = 5$$

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$$A_{21} = -\begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = -(2\times1 - 5\times1) = 3$$

$$A_{22} = +\begin{vmatrix} 1 & 5 \\ -1 & 1 \end{vmatrix} = 1\times1 - (-1)\times5 = 6$$

$$A_{23} = -\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = -(1\times1 - (-1)\times2) = -3$$

$$A_{31} = +\begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix} = 2\times1 - 3\times5 = -13$$

$$A_{32} = -\begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix} = -(1\times1 - 2\times5) = 9$$

$$A_{33} = +\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1\times3 - 2\times2 = -1$$

Then, the cofactor matrix becomes,

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 6 & -3 \\ -13 & 9 & -1 \end{bmatrix}$$

Now,

Adjoint of A = (Cofactor matrix)^T

$$= \begin{bmatrix} 2 & -3 & 5 \\ 3 & 6 & -3 \\ -13 & 9 & -1 \end{bmatrix}^T$$

$$\therefore \text{Adj. } A = \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$$

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Now,

$$A \times Adj.\, A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times2+2\times(-3)+5\times5 & 1\times3+2\times6+5\times(-3) & 1\times(-13)+2\times9+5\times(-1) \\ 2\times2+3\times(-3)+1\times5 & 2\times3+3\times6+1\times(-3) & 2\times(-13)+3\times9+1\times(-1) \\ (-1)\times2+1\times(-3)+1\times5 & -1\times3+1\times6+1\times(-3) & -1\times(-13)+1\times9+1\times(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 2-6+25 & 3+12-15 & -13+18-5 \\ 4-9+5 & 6+18-3 & -26+27-1 \\ -2-3+5 & -3+6-3 & 13+9-1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

Also,

$$|A| \times I = 21 \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21\times1 & 21\times0 & 21\times0 \\ 21\times0 & 21\times1 & 21\times0 \\ 21\times0 & 21\times0 & 21\times0 \end{bmatrix}$$

$$\therefore |A| \times I = \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$\therefore A\,(Adj.\, A) = |A|\, I$ proved

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9. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 4 \\ 2 & 2 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$

Solution

Given,

$$A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} \ \& \ B = \begin{bmatrix} 5 & 4 \\ 2 & 2 \end{bmatrix}$$

Then,

$$AB = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 5 & 4 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times5+4\times2 & 3\times4+4\times2 \\ 5\times5+6\times2 & 5\times4+6\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 15+8 & 12+8 \\ 25+12 & 20+12 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 23 & 20 \\ 37 & 32 \end{bmatrix}$$

Let the cofactor matrix of AB be $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

$A_{11} = +32$

$A_{12} = -37$

$A_{21} = -20$

$A_{22} = +23$

Then, the cofactor matrix becomes

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 32 & -37 \\ -20 & 23 \end{bmatrix}$$

Adjoint of AB = (cofactor matrix)^T

$$= \begin{bmatrix} 32 & -37 \\ -20 & 23 \end{bmatrix}^T$$

$$\therefore \text{Adj. } AB = \begin{bmatrix} 32 & -20 \\ -37 & 23 \end{bmatrix}$$

Then,

$$\text{Inverse matrix} = \frac{\text{Adj. } AB}{|AB|}, \quad |AB| \neq 0$$

$$|AB| = \begin{vmatrix} 23 & 20 \\ 37 & 32 \end{vmatrix}$$

$$= 23 \times 32 - 37 \times 20$$

$$= 736 - 740$$

$$\therefore |AB| = -4$$

$$(AB)^{-1} = \frac{\begin{bmatrix} 32 & -20 \\ -37 & 23 \end{bmatrix}}{-4}$$

$$\therefore (AB)^{-1} = \begin{bmatrix} -8 & 5 \\ 37/4 & -23/4 \end{bmatrix}$$

Again,

$$|A| = \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix}$$

$$= 3 \times 6 - 5 \times 4$$

$$= 18 - 20$$

$= -2 \neq 0$. so, A^{-1} exists.

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Let the cofactor matrix be $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

$A_{11} = +6$

$A_{12} = -5$

$A_{21} = -4$

$A_{22} = +3$

The cofactor matrix becomes,

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 6 & -5 \\ -4 & 3 \end{bmatrix}$$

Adjoint of A = (cofactor matrix)^T

$$= \begin{bmatrix} 6 & -5 \\ -4 & 3 \end{bmatrix}^T$$

$$\therefore \text{Adj. } A = \begin{bmatrix} 6 & -4 \\ -5 & 3 \end{bmatrix}$$

Then,

Inverse matrix $= \dfrac{\text{Adj. } A}{|A|}$, $|A| \neq 0$

$$A^{-1} = \frac{\begin{bmatrix} 6 & -4 \\ -5 & 3 \end{bmatrix}}{\cancel{4}\ -2}$$

$$A^{-1} = \begin{bmatrix} 6/-2 & -4/-2 \\ -5/-2 & 3/-2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \cancel{3/2}\ -3 & 2 \\ 5/2 & \cancel{-4/3}\ -3/2 \end{bmatrix}$$

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$$|B| = \begin{vmatrix} 5 & 4 \\ 2 & 2 \end{vmatrix}$$
$= 5\times2 - 2\times4$
$= 10-8$
$= 2 \neq 0$ so, B^{-1} exists

let the cofactor matrix be $\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$

$B_{11} = +2$
$B_{12} = -2$
$B_{21} = -4$
$B_{22} = +5$

Then the cofactor matrix becomes,

$$\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -4 & 5 \end{bmatrix}$$

Adjoint of B = (cofactor matrix)^T

$$= \begin{bmatrix} 2 & -2 \\ -4 & 5 \end{bmatrix}^T$$

$$\therefore \text{Adj. } B = \begin{bmatrix} 2 & -4 \\ -2 & 5 \end{bmatrix}$$

Then,
Inverse matrix $= \dfrac{\text{Adj. } B}{|B|}$, $|B| \neq 0$

$$\therefore B^{-1} = \begin{bmatrix} 1 & -2 \\ -1 & 5/2 \end{bmatrix}$$

Finally,

$$B^{-1}A^{-1} = \begin{bmatrix} 1 & -2 \\ -1 & 5/2 \end{bmatrix} \begin{bmatrix} -3/2 & 1 \\ 5/4 & -4/3 \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} 1 & -2 \\ -1 & 5/2 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5/2 & -3/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times(-3) - 2\times(5/2) & 1\times 2 - 2\times(-3/2) \\ -1\times -3 + 5/2 \times 5/2 & -1\times 2 + 5/2 \times (-3/2) \end{bmatrix}$$

$$\therefore B^{-1}A^{-1} = \begin{bmatrix} -8 & 5 \\ 37/4 & -23/4 \end{bmatrix}$$

From the above result, we can see that $(AB)^{-1} = B^{-1}A^{-1}$ proved.

10. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$

Solution

To prove: $(AB)^{-1} = B^{-1}A^{-1}$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times3 + 2\times5 & 1\times4 + 2\times6 \\ 3\times3 + 4\times5 & 3\times4 + 4\times6 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 13 & 16 \\ 29 & 36 \end{bmatrix}$$

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$$|AB| = \begin{vmatrix} 13 & 16 \\ 29 & 36 \end{vmatrix} = 13 \times 36 - 29 \times 16 = 4$$

$$\text{Adj. of } AB = \begin{bmatrix} 36 & -16 \\ -29 & 13 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{4} \begin{bmatrix} 36 & -16 \\ -29 & 13 \end{bmatrix}$$

$$\therefore (AB)^{-1} = \begin{bmatrix} 9 & -4 \\ -29/4 & 13/4 \end{bmatrix}$$

Also,

$$|B| = \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix} = 3 \times 6 - 5 \times 4 = -2$$

$$\text{Adj. of } B = \begin{bmatrix} 6 & -4 \\ -5 & 3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{-2} \begin{bmatrix} 6 & -4 \\ -5 & 3 \end{bmatrix}$$

$$\therefore B^{-1} = \begin{bmatrix} -3 & 2 \\ 5/2 & -3/2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \times 4 - 2 \times 3 = -2$$

$$\text{Adj. of } A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$$

Then,

$$B^{-1} A^{-1} = \begin{bmatrix} -3 & 2 \\ 5/2 & -3/2 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$$

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$$= \begin{bmatrix} -3\times(-2) + 2\times 3/2 & -3\times 1 + 2\times(-1/2) \\ 5/2\times(-2) + (-3/2\times 3/2) & 5/2\times 1 + (-3/2 \times -1/2) \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -4 \\ -29/4 & 13/4 \end{bmatrix}$$

$\therefore (AB)^{-1} = B^{-1} A^{-1}$ proved

Exercise 8(B)

2. Prove the following:

a) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

Solution

$$LHS = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= \begin{vmatrix} a-b-c+2b+2c & 2a+b-c-a+2c & 2a+2b+c-a-b \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

[∵ $R_1 \rightarrow R_1+R_2+R_3$]

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

[∵ Taking $(a+b+c)$ common from R_1]

$$= (a+b+c)\begin{vmatrix} 1-1 & 1-1 & 1 \\ 2b-b+c+a & b-c-a-2b & 2b \\ 2c-2c & 2c-c+a+b & c-a-b \end{vmatrix}$$

[∵ $C_1 \rightarrow C_1-C_2$, $C_2 \rightarrow C_2-C_3$]

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ a+b+c & -(a+b+c) & 2b \\ 0 & a+b+c & c-a-b \end{vmatrix}$$

$$= (a+b+c)^2\begin{vmatrix} 0 & 0 & 1 \\ 1 & -(a+b+c) & 2b \\ 0 & a+b+c & c-a-b \end{vmatrix}$$

[$\because$ Taking $(a+b+c)$ common from C_1]

$$= (a+b+c)^3\begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 2b \\ 0 & 1 & c-a-b \end{vmatrix}$$

[$\because$ Taking $(a+b+c)$ common from C_2]

Expanding along R_1

$$= (a+b+c)^3\left[0\begin{vmatrix} -1 & 2b \\ 1 & c-a-b \end{vmatrix} - 0\begin{vmatrix} 1 & 2b \\ 0 & c-a-b \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix}\right]$$

$$= (a+b+c)^3\left[0 - 0 + 1(1\times 1 - 0\times(-1))\right]$$

$$= (a+b+c)^3 \times 1$$

$$= (a+b+c)^3$$

$\therefore$ LHS = RHS proved.

b) $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a)$

Solution

$$LHS = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

$$= \begin{vmatrix} 1-1 & a-b & bc-ca \\ 1-1 & b-c & ca-ab \\ 1 & c & ab \end{vmatrix}$$

$[\because R_1 \to R_1 - R_2 \; ; \; R_2 \to R_2 - R_3]$

$$= \begin{vmatrix} 0 & a-b & \cancel{c(b-a)}\ -c(a-b) \\ 0 & b-c & -a(b-c) \\ 1 & c & ab \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 1 & -c \\ 0 & 1 & -a \\ 1 & c & ab \end{vmatrix}$$

$[\because$ Taking $(a-b)$ common from R_1 & $(b-c)$ common from $R_2]$

Expanding by using R_1

$$= (a-b)(b-c)\left[0\begin{vmatrix} 1 & -a \\ c & ab \end{vmatrix} - 1\begin{vmatrix} 0 & -a \\ 1 & ab \end{vmatrix} + (c)\begin{vmatrix} 0 & 1 \\ 1 & c \end{vmatrix}\right]$$

$$= (a-b)(b-c)\left[0 - 1(0 \times ab - 1 \times (-a)) - c(0 \times c - 1 \times 1)\right]$$

$$= (a-b)(b-c)(-a+c)$$

$$= (a-b)(b-c)(c-a)$$

$\therefore$ LHS = RHS proved.

c) $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$

Solution

$$LHS = \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c-c & -c+a+b+c & -b-a \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

$[\because R_1 \to R_1 + R_2]$

$$= \begin{vmatrix} a+b & a+b & -(a+b) \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

$$= (a+b)\begin{vmatrix} 1 & 1 & -1 \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

[$\because$ Taking $(a+b)$ common from R_1]

$$= (a+b)\begin{vmatrix} 1 & 1-1 & -1 \\ -c & \overset{a+b+c-a}{\cancel{a+b-a}} & -a \\ -b & -a+a+b+c & a+b+c \end{vmatrix}$$

$[\because C_2 \to C_2 + C_3]$

$$= (a+b)\begin{vmatrix} 1 & 0 & -1 \\ -c & b+c & -a \\ -b & b+c & a+b+c \end{vmatrix}$$

$$= (a+b)(b+c)\begin{vmatrix} 1 & 0 & -1 \\ -c & 1 & -a \\ -b & 1 & a+b+c \end{vmatrix}$$

[$\because$ Taking $(b+c)$ common from C_2]

Expanding by using R_1

$$= (a+b)(b+c)\left[1\begin{vmatrix} 1 & -a \\ 1 & a+b+c \end{vmatrix} - 0\begin{vmatrix} -c & -a \\ -b & a+b+c \end{vmatrix} - 1\begin{vmatrix} -c & 1 \\ -b & 1 \end{vmatrix}\right]$$

$$= (a+b)(b+c)\left[(a+b+c+a) - 0 - 1(-c+b)\right]$$

$$= (a+b)(b+c)\left[2a+b+c+c-b\right]$$

$$= (a+b)(b+c)(2c+2a)$$

$$= 2(a+b)(b+c)(c+a)$$

$\therefore$ LHS = RHS proved.

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d) $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c)$

Solution

$$\text{LHS} = \begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}$$

$$= \begin{vmatrix} x+a+b+c & b & c \\ a+x+b+c & x+b & c \\ a+b+x+c & b & x+c \end{vmatrix}$$

[$\because C_1 \to C_1 + C_2 + C_3$]

$$= \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix}$$

$$= (x+a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix}$$

[$\because$ Taking $(x+a+b+c)$ common from C_1]

Expanding by using R_1.

$$= (x+a+b+c)\left[1 \begin{vmatrix} x+b & c \\ b & x+c \end{vmatrix} - b \begin{vmatrix} 1 & c \\ 1 & x+c \end{vmatrix} + c \begin{vmatrix} 1 & x+b \\ 1 & b \end{vmatrix}\right]$$

$$= (x+a+b+c)\left[(x+b)(x+c) - bc - b(x+c-c) + c(b-x-b)\right]$$

$$= (x+a+b+c)\left[x^2 + cx + bx + bc - bc - bx - cx\right]$$

$$= (x+a+b+c)(x^2)$$

$$= x^2(x+a+b+c)$$

$\therefore$ LHS = RHS proved.

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e) $\begin{vmatrix} a+x & b & c \\ a & b+y & c \\ a & b & c+z \end{vmatrix} = xyz\left(1 + \frac{a}{x} + \frac{b}{y} + \frac{c}{z}\right)$

Solution

$$\text{LHS} = \begin{vmatrix} a+x & b & c \\ a & b+y & c \\ a & b & c+z \end{vmatrix}$$

Expanding by using R_1

$$= (a+x)\begin{vmatrix} b+y & c \\ b & c+z \end{vmatrix} - b\begin{vmatrix} a & c \\ a & c+z \end{vmatrix} + c\begin{vmatrix} a & b+y \\ a & b \end{vmatrix}$$

$$= (a+x)\left[(b+y)(c+z) - bc\right] - b\left[a(c+z) - ac\right] + c\left[ab - a(b+y)\right]$$

$$= (a+x)\left[bc + bz + cy + yz - bc\right] - b\left[ac + az - ac\right] + c\left[ab - ab - ay\right]$$

$$= (a+x)(bz + cy + yz) - abz - acy$$

$$= abz + acy + ayz + xbz + xcy + xyz - abz - acy$$

$$= ayz + xbz + xcy + xyz$$

$$\cancel{= \frac{1}{xyz} \times xyz}$$

$$= xyz \times \frac{1}{xyz}\left[ayz + xbz + xcy + xyz\right]$$

$$= xyz\left(\frac{a}{x} + \frac{b}{y} + \frac{c}{z} + 1\right)$$

$$= xyz\left(1 + \frac{a}{x} + \frac{b}{y} + \frac{c}{z}\right)$$

$\therefore$ LHS = RHS proved.

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b) $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = (y-z)(z-x)(x-y)(x+y+z)$

Solution

$$\text{LHS} = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1-1 & 1-1 & 1 \\ x-y & y-z & z \\ x^3-y^3 & y^3-z^3 & z^3 \end{vmatrix}$$

$[\because C_1 \to C_1 - C_2,\ C_2 \to C_2 - C_3]$

$$= \begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ (x-y)(x^2+xy+y^2) & (y-z)(y^2+yz+z^2) & z^3 \end{vmatrix}$$

$$= (x-y)(y-z) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & z \\ x^2+xy+y^2 & y^2+yz+z^2 & z^3 \end{vmatrix}$$

[$\because$ Taking $(x-y)$ common from C_1 and $(y-z)$ common from C_2]

Now, Expanding by using R_1,

$$= (x-y)(y-z)\left[0\begin{vmatrix} 1 & z \\ y^2+yz+z^2 & z^3 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ x^2+xy+y^2 & z^3 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ x^2+xy+y^2 & y^2+yz+z^2 \end{vmatrix}\right]$$

$$= (x-y)(y-z)[0 - 0 + y^2+yz-z^2 - x^2 - xy - y^2]$$

$$= (x-y)(y-z)[yz - z^2 - x^2 - xy]$$

$$\cancel{= (x-y)(y-z)[z(y-z) - x(x+y)]}$$

Expanding by using R_1

$$= (x-y)(y-z)\left[0\begin{vmatrix} 1 & z^2 \\ y^2+yz+z^2 & z^3 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ x^2+xy+y^2 & z^3 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ x^2+xy+y^2 & y^2+yz+z^2 \end{vmatrix}\right]$$

$$= (x-y)(y-z)[0-0+y^2+yz+z^2-x^2-xy \pm y^2]$$

$$= (x-y)(y-z)(y^2+yz+z^2-x^2-xy \pm y^2)$$

$$= (x-y)(y-z)(yz+z^2-x^2-xy)$$

$$= (x-y)(y-z)[yz-xy+z^2-x^2]$$

$$= (x-y)(y-z)[y(z-x)+(z-x)(z+x)]$$

$$= (x-y)(y-z)(z-x)[y+z+x]$$

$$= (x-y)(y-z)(z-x)(x+y+z)$$

$$= (y-z)(z-x)(x-y)(x+y+z)$$

$\therefore$ LHS = RHS proved.

Signature of subject teacher:

Signature of Director: