## Exercise 7(B)

1. Find the scalar product (dot product) of each of the following vectors :

a) $(-1,3)$ & $(2,3)$

Solution

Let $\vec{a} = (-1,3)$

$\vec{b} = (2,3)$

Then,

$\vec{a}.\vec{b} = (-1,3).(2,3)$

$= -1\times2 + 3\times3$

$= -2+9$

$\therefore \vec{a}.\vec{b} = 7$

b) $3\vec{i} + 2\vec{j}$ and $-\vec{i} + 5\vec{k}$

Solution

Let $\vec{a} = 3\vec{i} + 2\vec{j} = (3,2)$

$\vec{b} = -\vec{i} + 5\vec{k} = (-1,5)$

Then,

$\vec{a}.\vec{b} = (3,2).(-1,5)$

$= 3\times(-1) + 2\times5$

$= -3+10$

$\therefore \vec{a}.\vec{b} = 7$

c) $(2,3,1)$ and $(1,-2,3)$

Solution

Let $\vec{a} = (2,3,1)$

$\vec{b} = (1,-2,3)$

Then,

$\vec{a}.\vec{b} = (2,3,1).(1,-2,3)$

$= 2\times1 + 3\times(-2) + 1\times3$

$= 2-6+3$

$\therefore \vec{a}.\vec{b} = -1$

d) $3\vec{i} + 2\vec{k}$ and $3\vec{i} - 2\vec{j} + 5\vec{k}$

Solution.

Let $\vec{a} = 3\vec{i} + 2\vec{k} = (3, 0, 2)$

$\vec{b} = 3\vec{i} - 2\vec{j} + 5\vec{k} = (3, -2, 5)$

Then,

$\vec{a} \cdot \vec{b} = (3, 0, 2) \cdot (3, -2, 5)$

$= 3 \times 3 + 0 \times (-2) + 2 \times 5$

$= 9 + 0 + 10$

$\therefore \vec{a} \cdot \vec{b} = 19$

2. Show that the following pair of vectors are orthogonal (perpendicular).

a) $(-3, 4)$ & $(8, 6)$

Solution

Let $\vec{a} = (-3, 4)$

$\vec{b} = (8, 6)$

Now,

$\vec{a} \cdot \vec{b} = (-3, 4) \cdot (8, 6)$

$= -3 \times 8 + 4 \times 6$

$= -24 + 24$

$\therefore \vec{a} \cdot \vec{b} = 0$

Since, $\vec{a} \cdot \vec{b} = 0$, so the vectors ~~ar~~ $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

proved

b) $-2\vec{i} + 3\vec{j} + \vec{k}$ and $6\vec{i} - 5\vec{j} + 27\vec{k}$

Solution

Let $\vec{a} = -2\vec{i} + 3\vec{j} + \vec{k} = (-2, 3, 1)$

$\vec{b} = 6\vec{i} - 5\vec{j} + 27\vec{k} = (6, -5, 27)$

Now,

$\vec{a} \cdot \vec{b} = (-2, 3, 1) \cdot (6, -5, 27)$

$= -2 \times 6 + 3 \times (-5) + 1 \times 27$

$= -12 - 15 + 27$

$= -27 + 27$

$\therefore \vec{a} \cdot \vec{b} = 0$

Since $\vec{a} \cdot \vec{b} = 0$. So, the vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

Proved

---

c) $(-1, 3, 4)$ and $(4, -4, 4)$

Solution

let $\vec{a} = (-1, 3, 4)$

$\vec{b} = (4, -4, 4)$

Now,

$\vec{a} \cdot \vec{b} = (-1, 3, 4) \cdot (4, -4, 4)$

$\cancel{= (-1 \times 4,}$

$= -1 \times 4 + 3 \times (-4) + 4 \times 4$

$= -4 - 12 + 16$

$= -16 + 16$

$\therefore \vec{a} \cdot \vec{b} = 0$

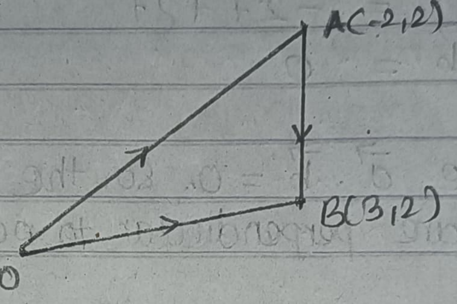
Since $\vec{a} \cdot \vec{b} = 0$. So, the vectors $\cancel{\vec{a} \cdot \vec{b}}$ $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

proved

3. a) Show that line joining points $(-2,2)$ & $(3,2)$ is perpendicular (orthogonal) to the line joining $(1,0)$ and $(1,4)$.

Solution

Let $A(-2,2)$ & $B(3,2)$ are the points. Let O be the origin. Join OA, OB and AB.



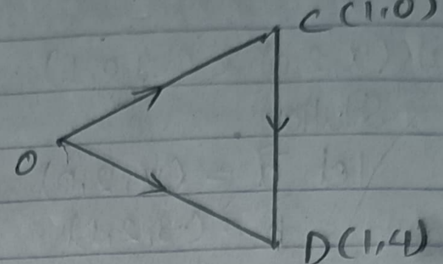

In $\Delta OAB$,

$\vec{OA} = (-2,2)$
$\vec{OB} = (3,2)$

$\vec{OA} + \vec{AB} = \vec{OB}$ [Using triangle law of vector addition]

or, $\vec{AB} = \vec{OB} - \vec{OA}$
or, $\vec{AB} = (3,2) - (-2,2)$
or, $\vec{AB} = (3+2, 2-2)$
$\therefore \vec{AB} = (5,0)$

Let $C(1,0)$ and $D(1,4)$ are the points. Let $O$ be the origin. Join $OC$, $OD$ and $CD$.




In $\Delta OCD$,

$\vec{OC} = (1,0)$
$\vec{OD} = (1,4)$

$\vec{OC} + \vec{CD} = \vec{OD}$ [using triangle law of vector addition]

or $\vec{CD} = \vec{OD} - \vec{OC}$
or, $\vec{CD} = (1,4) - (1,0)$
or $\vec{CD} = (1-1, 4-0)$
$\therefore \vec{CD} = (0,4)$

Now,

$\vec{AB} \cdot \vec{CD} = (5,0) \cdot (0,4)$
$= 5 \times 0 + 0 \times 4$
$= 0 + 0$
$\therefore \vec{AB} \cdot \vec{CD} = 0$

Since $\vec{AB} \cdot \vec{CD} = 0$. So, the vectors $\vec{AB}$ and $\vec{CD}$ are perpendicular to each other.

proved

b) Find the value of 'x' when following pair of vectors are perpendicular (orthogonal) to each other.

i) $(x, 2, 0)$ & $(-3, 2, 1)$

Solution

Let $\vec{a} = (x, 2, 0)$

$\vec{b} = (-3, 2, 1)$

Since $\vec{a}$ and $\vec{b}$ are perpendicular to each other,

$\vec{a} \cdot \vec{b} = 0$

or, $(x, 2, 0) \cdot (-3, 2, 1) = 0$

or, $x \times (-3) + 2 \times 2 + 0 \times 1 = 0$

or, $-3x + 4 + 0 = 0$

or, $-3x = -4$

$\therefore x = \frac{4}{3}$

---

ii) $3\vec{i} + x\vec{j}$, and $-7\vec{i} + \frac{21}{5}\vec{j}$

Solution

Let $\vec{a} = 3\vec{i} + x\vec{j} = (3, x, 0)$

$\vec{b} = -7\vec{i} + \frac{21}{5}\vec{j} = (-7, \frac{21}{5}, 0)$

Since $\vec{a}$ and $\vec{b}$ are perpendicular to each other,

$\vec{a} \cdot \vec{b} = 0$

or, $(3, x, 0) \cdot (-7, \frac{21}{5}, 0) = 0$

or, $3 \times (-7) + x \times \frac{21}{5} + 0 \times 0 = 0$

or, $x \times \frac{21}{5} = 21$

or, $x = \frac{21}{\frac{21}{5}}$

$\therefore x = 5$

iii) $(x, -2, 1)$ and $(x, x, 1)$

Solution

Let $\vec{a} = (x, -2, 1)$

$\vec{b} = (x, x, 1)$

Since $\vec{a}$ and $\vec{b}$ are perpendicular to each other,

$\vec{a} \cdot \vec{b} = 0$

or, $(x, -2, 1) \cdot (x, x, 1) = 0$

or, $x \times x + (-2) \times x + 1 \times 1 = 0$

or, $x^2 - 2x + 1 = 0$

or, $(x-1)^2 = 0$

or, $x - 1 = 0$

$\therefore x = 1$

4. a) Let $\vec{a} = (2\vec{i} + 3\vec{j} - \vec{k})$ and $\vec{b} = (1\vec{i} - 2\vec{j} + 4\vec{k})$ Find angle between $\vec{a}$ and $\vec{b}$.

Solution

Given,

$\vec{a} = (2\vec{i} + 3\vec{j} - \vec{k}) = (2, 3, -1)$

$\vec{b} = (1\vec{i} - 2\vec{j} + 4\vec{k}) = (1, -2, 4)$

Now,

$$\vec{a}\cdot\vec{b} = (2,3,-1)\cdot(1,-2,4)$$
$$= 2\times 1 + 3\times(-2) + (-1)\times 4$$
$$= 2-6-4$$
$$\therefore \vec{a}\cdot\vec{b} = -8$$

$$|\vec{a}| = \sqrt{(2)^2+(3)^2+(-1)^2} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{(1)^2+(-2)^2+(4)^2} = \sqrt{21}$$

We know,

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

Then,

$$\theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}\right)$$

$$= \cos^{-1}\left(\frac{\cancel{\sqrt{14}}\ -8}{\sqrt{14}\ \sqrt{21}}\right)$$

$$= \cos^{-1}\left(\frac{-8}{7\sqrt{6}}\right)$$

$$\therefore \theta = \cos^{-1}\left(\frac{-8}{\sqrt{294}}\right)$$

b) Find ~~#~~ cosine of angle between the vectors:

i) $\vec{a} = (2,2,-1)$ and $\vec{b} = (6,3,2)$

Solution

Given,

$\vec{a} = (2,2,-1)$

$\vec{b} = (6,3,2)$

Then,

$$\vec{a}\cdot\vec{b} = (2,2,-1)\cdot(6,3,2)$$

$$= \cancel{(2\times 6}$$

$$= 2\times 6 + 2\times 3 + (-1)\times 2$$

$$= 12 + 6 - 2$$

$$\therefore \vec{a}\cdot\vec{b} = 16$$

$$|\vec{a}| = \sqrt{(2)^2+(2)^2+(-1)^2} = 3$$

$$|\vec{b}| = \sqrt{(6)^2+(3)^2+(2)^2} = 7$$

We know,

$$\cos Q = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{16}{3\times 7}$$

$$\therefore \cos Q = \frac{16}{21}$$

ii) $\vec{a} = \vec{i} - 2\vec{j} - 2\vec{k}$, $\vec{b} = 2\vec{i} + \vec{j} - 2\vec{k}$

Solution

Given,

$\vec{a} = \vec{i} - 2\vec{j} - 2\vec{k} = (1, -2, -2)$

$\vec{b} = 2\vec{i} + \vec{j} - 2\vec{k} = (2, 1, -2)$

Then,

$$\vec{a} \cdot \vec{b} = (1, -2, -2) \cdot (2, 1, -2)$$
$$= 1 \times 2 + (-2) \times 1 + (-2) \times (-2)$$
$$= 2 - 2 + 4$$

$\therefore \vec{a} \cdot \vec{b} = 4$

$|\vec{a}| = \sqrt{(1)^2 + (-2)^2 + (-2)^2} = 3$

$|\vec{b}| = \sqrt{(2)^2 + (1)^2 + (-2)^2} = 3$

We know,

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$$= \frac{4}{3 \times 3}$$

$\therefore \cos\theta = \frac{4}{9}$

5. If $\vec{i}, \vec{j}, \vec{k}$ are three mutually perpendicular unit vectors and $\vec{a} = 3\vec{i} - 2\vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} - 3\vec{j} - 4\vec{k}$ are two vectors then find $|\vec{a}|$ $|\vec{b}|$, $\vec{a}\cdot\vec{b}$. Also, find the angle between two vectors.

Solution

Given,

$\vec{a} = 3\vec{i} - 2\vec{j} + \vec{k} = (3, -2, 1)$

$\vec{b} = 2\vec{i} - 3\vec{j} - 4\vec{k} = (2, -3, -4)$

Then,

$$|\vec{a}| = \sqrt{(3)^2 + (-2)^2 + (1)^2} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{(2)^2 + (-3)^2 + (-4)^2} = \sqrt{29}$$

$$\vec{a}\cdot\vec{b} = (3, -2, 1) \cdot (2, -3, -4)$$
$$= 3\times 2 + (-2)\times(-3) + 1\times(-4)$$
$$= 6 + 6 - 4$$
$$\therefore \vec{a}\cdot\vec{b} = 8$$

We know,

$$\cos Q = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

Then,

$$Q = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)$$

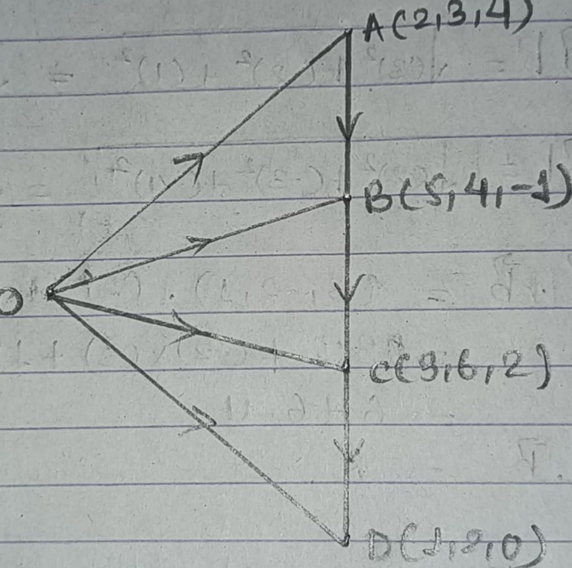
$$= \cos^{-1}\left(\frac{8}{\sqrt{14}\sqrt{29}}\right)$$

$$\therefore Q = \cos^{-1}\left(\frac{8}{\sqrt{406}}\right)$$

6. If A(2,3,4), B(5,4,-1), C(3,6,2) and D(1,2,0) be any points, express $\vec{AB}$ and $\vec{CD}$ in the form of $x\vec{i} + y\vec{j} + z\vec{k}$. Also show that $\vec{AB}$ is perpendicular to $\vec{CD}$.

Solution

Let A(2,3,4), B(5,4,-1), C(3,6,2) and D(1,2,0) are the points. Let O be the origin. Join OA, OB, OC, OD and AD.




In $\Delta OAB$,

$\vec{OA} + \vec{AB} = \vec{OB}$ [using triangle law of vector addition]

or, $\vec{AB} = \vec{OB} - \vec{OA}$

$= (5,4,-1) - (2,3,4)$

$= (5-2, 4-3, -1-4)$

$\therefore \vec{AB} = (3,1,-5)$

$\therefore \vec{AB} = 3\vec{i} + \vec{j} - 5\vec{k}$

In $\Delta OCD$

$\vec{OC} + \vec{CD} = \vec{OD}$ [Using triangle law of addition of vectors]

or $\vec{CD} = \vec{OD} - \vec{OC}$

$= (1,2,0) - (3,6,2)$

$= (1-3, 2-6, 0-2)$

$\therefore \vec{CD} = (-2, -4, -2)$

$\therefore \vec{CD} = -2\vec{i} - 4\vec{j} - 2\vec{k}$

Now,

To show the vectors $\vec{AB}$ and $\vec{CD}$ perpendicular to each other, we need to show $\vec{AB} \cdot \vec{CD} = 0$.

LHS $= \vec{AB} \cdot \vec{CD}$

~~Or~~ $= (3,1,-5) \cdot (-2,-4,-2)$ $\cancel{= 0}$

~~Or, (3~~ $3 \times (-2) + 1 \times (-4) + (-5) \times (-2)$ $\cancel{= 0}$

$= -6 - 4 + 10$

$= 0$ RHS.

Hence, $\vec{AB}$ and $\vec{CD}$ are perpendicular to each other.