Chapter-7

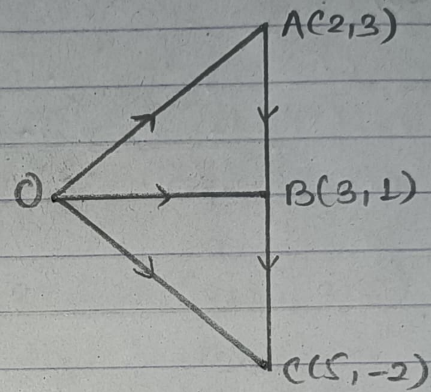
vectors

Exercise 7(A)

1. Find $\vec{AB}$, $\vec{AC}$, $\vec{BC}$, $\vec{CA}$ and $\vec{BA}$ when $A(2,3)$, $B(3,1)$ & $C(5,-2)$.

Solution

Here, $A(2,3)$, $B(3,1)$ & $C(5,-2)$ are the points.
Let O be the origin. Join OA, OB, OC & AC.



$\vec{OA} = (2,3)$
$\vec{OB} = (3,1)$
$\vec{OC} = (5,-2)$

In ΔOAB,

$\vec{OA} + \vec{AB} = \vec{OB}$
or, $\vec{AB} = \vec{OB} - \vec{OA}$
or, $\vec{AB} = (3,1) - (2,3)$
or, $\vec{AB} = (3-2, 1-3)$
$\therefore \vec{AB} = (1,-2)$

~~In ΔOAC~~, ~~In ΔOBC~~

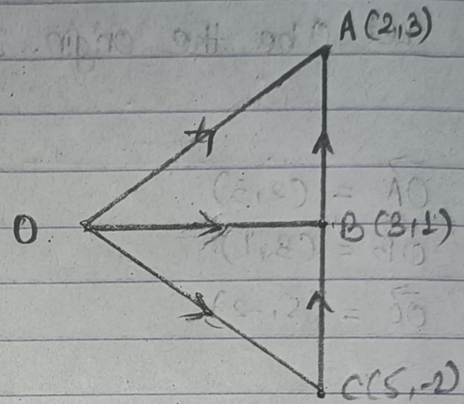
~~$\vec{OA}$ +~~ In ΔOBC,

$\vec{OB} + \vec{BC} = \vec{OC}$
or, $\vec{BC} = \vec{OC} - \vec{OB}$
or, $\vec{BC} = (5,-2) - (3,1)$
or, $\vec{BC} = (5-3, -2-1)$
$\therefore \vec{BC} = (2,-3)$

In ΔOAC,

$$\vec{OA} + \vec{AC} = \vec{OC}$$
or, $\vec{AC} = \vec{OC} - \vec{OA}$
or, $\vec{AC} = (5, -2) - (2, 3)$
or, $\vec{AC} = (5-2, -2-3)$
$\therefore \vec{AC} = (3, -5)$

Also,



In ΔOAB

$$\vec{OB} + \vec{BA} = \cancel{\vec{AO}}\ \vec{OA}$$
or, $\vec{BA} = \vec{OA} - \vec{OB}$
or, $\vec{BA} = (2, 3) - (3, 1)$
$\therefore \vec{BA} = (-1, 2)$

In ΔAOC

$$\vec{OC} + \vec{CA} = \vec{OA}$$
or, $\vec{CA} = \vec{OA} - \vec{OC}$
or, $\vec{CA} = (2, 3) - (5, -2)$
$\therefore \vec{CA} = (-3, 5)$

2. If $\vec{a} = (2,1)$, $\vec{b} = (3,5)$ and $\vec{c} = (-2,3)$, find $2\vec{a}$, $3\vec{a} + 2\vec{b}$, $3\vec{b} - 2\vec{c}$, $2\vec{a} + \vec{b} - \vec{c}$ and $8\vec{b} - 2\vec{c} + \vec{a}$.

Solution

Given,

$\vec{a} = (2,1)$

$\vec{b} = (3,5)$

$\vec{c} = (-2,3)$

Then,

~~$2\vec{a} = 2 \times (2,1) = ($~~

$\therefore 2\vec{a} = (2\times2, 2\times1) = (4,2)$

$3\vec{a} + 2\vec{b} = (3\times2, 3\times1) + (2\times3, 2\times5)$

$= (6,3) + (6,10)$

$= (6+6, 3+10)$

$\therefore 3\vec{a} + 2\vec{b} = (12, 13)$

$3\vec{b} - 2\vec{c} = (3\times3, 3\times5) - (2\times(-2), 2\times3)$

$= (9, 15) - (-4, 6)$

$= (9+4, 15-6)$

$\therefore 3\vec{b} - 2\vec{c} = (13, 9)$

$2\vec{a} + \vec{b} - \vec{c} = (2\times2, 2\times1) + (3,5) - (-2,3)$

$= (4,2) + (3,5) - (-2,3)$

$= (4+3, 2+5) - (-2,3)$

$= (7,7) - (-2,3)$

$= (7+2, 7-3)$

$2\vec{a} + \vec{b} - \vec{c} = (9,4)$

$3\vec{b} - 2\vec{c} + \vec{a} = (13, 9) + (2, 1)$

$\cancel{3\vec{b} - 2\vec{c} + \vec{a}} = (13 + 2, 9 + 1)$

$\therefore 3\vec{b} - 2\vec{c} + \vec{a} = (15, 10)$

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3. (i) If $\vec{a} = (3, -2)$ and $6\vec{a} + 5\vec{b} = (-2, 3)$, find $\vec{b}$.

Solution.

$\vec{a} = (3, -2)$

$6\vec{a} + 5\vec{b} = (-2, 3)$

or, $6(3, -2) + 5\vec{b} = (-2, 3)$

or, $(18, -12) + 5\vec{b} = (-2, 3)$

or, $5\vec{b} = (-2, 3) - (18, -12)$

or, $5\vec{b} = (-20, 15)$

or, $\vec{b} = \frac{(-20, 15)}{5}$

$\therefore \vec{b} = (-4, 3)$

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ii) If $\vec{a} = (2, -3)$ and $2\vec{a} + 3\vec{b} = (4, 3)$, find $\vec{b}$.

Solution

$\vec{a} = (2, -3)$

$2\vec{a} + 3\vec{b} = (4, 3)$

or, $2(2, -3) + 3\vec{b} = (4, 3)$

or, $(4, -6) + 3\vec{b} = (4, 3)$

or, $\vec{b} = \frac{(4, 3) - (4, -6)}{3}$

or, $\vec{b} = \frac{(0, 9)}{3}$

$\therefore \vec{b} = (0, 3)$

4. Let $\vec{a} = (2,3,5)$, $\vec{b} = (0,1,2)$ and $\vec{c} = (-1,2,-5)$ and $x\vec{a} + y\vec{b} + z\vec{c} = (3,5,8)$. Find value of x, y and z.

Solution

Given,

$\vec{a} = (2,3,5)$

$\vec{b} = (0,1,2)$

$\vec{c} = (-1,2,-5)$

$x\vec{a} + y\vec{b} + z\vec{c} = (3,5,8)$

or, $x(2,3,5) + y(0,1,2) + z(-1,2,-5) = (3,5,8)$

or, $(2x,3x,5x) + (0,y,2y) + (-z,2z,-5z) = (3,5,8)$

or, $(2x+0-z,\ 3x+y+2z,\ 5x+2y-5z) = (3,5,8)$

Equating the corresponding element, we get.

$2x - z = 3$

or, $2x = 3+z$

$\therefore x = \frac{3+z}{2}$ ———— (i)

$3x+y+2z = 5$

or, $3\left(\frac{3+z}{2}\right) + y + 2z = 5$

or, $9 + 3z + 2y + 4z = 10$

or, $9 + 2y + 7z = 10$

or, $2y = 10 - 9 - 7z$

$\therefore y = \frac{1-7z}{2}$ ———— (ii)

$5x + 2y - 5z = 8$

or, $5\left(\frac{3+z}{2}\right) + 2\left(\frac{1-7z}{2}\right) - 5z = 8$

or, $\frac{15+5z}{2} + 1 - 7z - 5z = 8$

or, $15 + 05z + 2 - 14z - 10z = 16$

or, $17 - 19z = 16$

or, $17 - 16 = 19z$

$\therefore z = \frac{1}{19}$

Substituting the value of z in equation (i) and (ii),

$$x = \frac{3+z}{2}$$

or, $x = \dfrac{3 + \frac{1}{19}}{2}$

or, $x = \dfrac{\frac{57+1}{19}}{2}$

or, $x = \frac{58}{2 \times 19}$

$\therefore x = \frac{29}{19}$

Equation (ii):

$$y = \frac{1 - 7 \times \frac{1}{19}}{2}$$

or, $y = \frac{19-7}{19 \times 2}$

or, $y = \frac{12}{19 \times 2}$

$\therefore y = \frac{6}{19}$

5. Check whether following vector are linearly independent or dependent.

i) $\vec{a} = (7, -4, 1)$, $\vec{b} = (-1, 1, 2)$ and $\vec{c} = (3, 2, -1)$

Solution

Given;

$\vec{a} = (7, -4, 1)$

$\vec{b} = (-1, 1, 2)$

$\vec{c} = (3, 2, -1)$

Now,

$$\begin{vmatrix} 7 & -4 & 1 \\ -1 & 1 & 2 \\ 3 & 2 & -1 \end{vmatrix}$$

Expanding along R_1

$$= 7\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} - (-4)\begin{vmatrix} -1 & 2 \\ 3 & -1 \end{vmatrix} + 1\begin{vmatrix} -1 & 1 \\ 3 & 2 \end{vmatrix}$$

$= 7(1\times(-1) - 2\times2) + 4(-1\times-1 - 3\times2) + 1(-1\times2 - 3\times1)$

$= 7(-1-4) + 4(1-6) + 1(-2-3)$

$= -35 - \cancel{28}\ 20 - 5$

$= -60$

$= -60 \neq 0$

So, the vectors are linearly independent.

ii) $4\vec{i} + 2\vec{j} + \vec{k}$, $\vec{i} + 2\vec{j} + 3\vec{k}$ and $3\vec{i} - 2\vec{j} - 3\vec{k}$

Solution

Let $\vec{a} = 4\vec{i} + 2\vec{j} + \vec{k} = (4, 2, 1)$
$\vec{b} = \vec{i} + 2\vec{j} + 3\vec{k} = (1, 2, 3)$
$\vec{c} = 3\vec{i} - 2\vec{j} - 3\vec{k} = (3, -2, -3)$

Now,

$$\begin{vmatrix} 4 & 2 & 1 \\ 1 & 2 & 3 \\ 3 & -2 & -3 \end{vmatrix}$$

Expanding along R_1

$$= 4\begin{vmatrix} 2 & 3 \\ -2 & -3 \end{vmatrix} - 2\begin{vmatrix} 1 & 3 \\ 3 & -3 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix}$$

$= 4(2\times -3 - (-2)\times 3) - 2(1\times(-3) - 3\times 3) + 1(1\times(-2) - 3\times 2)$
$= 4(-6 + 6) - 2(-3 - 9) + 1(-2 - 6)$
$= 0 + 24 - 8$
$= 16 \neq 0$

So, the vectors $\vec{a}$, $\vec{b}$ & $\vec{c}$ are linearly independent.

iii) $3\vec{i} + \frac{1}{2}\vec{j} - \vec{k}$, $\frac{3}{2}\vec{i} + \vec{j}$ and $\vec{k}$

Solution

Let $\vec{a} = 3\vec{i} + \frac{1}{2}\vec{j} - \vec{k} = (3, \frac{1}{2}, -1)$

$\vec{b} = \frac{3}{2}\vec{i} + \vec{j} + 0\vec{k} = (\frac{3}{2}, 1, 0)$

$\vec{c} = 0\vec{i} + 0\vec{j} + 1\vec{k} = (0, 0, 1)$

Now,

$$\begin{vmatrix} 3 & \frac{1}{2} & -1 \\ \frac{3}{2} & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along R_1,

$$= 3\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - \frac{1}{2}\begin{vmatrix} \frac{3}{2} & 0 \\ 0 & 1 \end{vmatrix} - 1\begin{vmatrix} \frac{3}{2} & 1 \\ 0 & 0 \end{vmatrix}$$

$$= 3(1\times1 - 0\times0) - \frac{1}{2}\times\left(\frac{3}{2}\times1 - 0\times0\right) - 1\times\left(\frac{3}{2}\times0 - 0\times1\right)$$

$$= 3 - \frac{3}{4} - 0$$

$$= \frac{9}{4} \neq 0.$$

So, the vectors $\vec{a}$, $\vec{b}$ & $\vec{c}$ are linearly independent.

6. Test whether the following vectors are co-planar or not.

i) $-\vec{a}+4\vec{b}+3\vec{c}$, $2\vec{a}-3\vec{b}-5\vec{c}$ and $2\vec{a}+7\vec{b}-3\vec{c}$

Solution

let $\vec{r_1} = -\vec{a}+4\vec{b}+3\vec{c}$

$\vec{r_2} = 2\vec{a}-3\vec{b}-5\vec{c}$

$\vec{r_3} = 2\vec{a}+7\vec{b} \cancel{-11\vec{c}} -3\vec{c}$

To show the vectors $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$ coplanar, we need to show $\vec{r_3}$ can be expressed as a linear combination of $\vec{r_1}$ and $\vec{r_2}$ i.e.

$\vec{r_3} = x\vec{r_1} + y\vec{r_2}$ where x & y are the scalars.

or, $2\vec{a}+7\vec{b}-3\vec{c} = x(-\vec{a}+4\vec{b}+3\vec{c}) + y(2\vec{a}-3\vec{b}-5\vec{c})$

or, $2\vec{a}+7\vec{b}-3\vec{c} = -x\vec{a}+4x\vec{b}+3x\vec{c} + 2y\vec{a}-3y\vec{b}-5y\vec{c}$

or, $2\vec{a}+7\vec{b}-3\vec{c} = -x\vec{a}+2y\vec{a}+4x\vec{b}-3y\vec{b} + 3x\vec{c}-5y\vec{c}$

or, $2\vec{a}+7\vec{b}-3\vec{c} = -\vec{a}(x-2y)+\vec{b}(4x-3y) + \vec{c}(3x-5y)$

By the definition of equality of ~~matr~~ vectors,

$2\vec{a} = -\vec{a}(x-2y)$ i.e. $x-2y=-2$ — (i)
$7\vec{b} = \vec{b}(4x-3y)$ i.e. $4x-3y=7$ — (ii) &
$-3\vec{c} = \vec{c}(3x-5y)$ i.e. $3x-5y=-3$ — (iii)

Solving equations (i) & (ii)

Multiplying equation (i) by 4, $4x-8y=-8$
Multiplying equation (ii) by 1, $4x-3y=7$

On substraction, $-5y=-15$
$\therefore y=3$

Substituting the value of y in equation (i)
$x-2\times3=-2$
or, $x-6=-2$
or, $x=-2+6$
$\therefore x=4$
$\therefore x=4, y=3$

Substituting the values of x & y in equation (iii), we get,

$3\times4-5\times3=-3$
or, $12-15=-3$
or, $-3=-3$ (true)

Therefore, the vectors $\vec{r_1}$, $\vec{r_2}$ & $\vec{r_3}$ are co-planar and hence, it can be expressed as a linear combination of other two vectors i.e.

$\vec{r_3} = x\vec{r_1} + y\vec{r_2}$
i.e. $\vec{r_3} = 4\vec{r_1} + 3\vec{r_2}$

~~Hence proved.~~

Hence $-\vec{a}+4\vec{b}+3\vec{c}$, $2\vec{a}-3\vec{b}-5\vec{c}$ and $2\vec{a}+7\vec{b}-3\vec{c}$ are co-planar.

ii) $2\vec{a} + 3\vec{b} - 4\vec{c}$, $-\vec{a} + 2\vec{b} + \vec{c}$ and $\vec{a} + 19\vec{b} - 7\vec{c}$

Solution

Let $\vec{r_1} = 2\vec{a} + 3\vec{b} - 4\vec{c}$
$\vec{r_2} = -\vec{a} + 2\vec{b} + \vec{c}$
$\vec{r_3} = \vec{a} + 19\vec{b} - 7\vec{c}$

To show the vectors $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$ co-planar, we need to show $\vec{r_3}$ can be expressed as a linear combination of $\vec{r_1}$ and $\vec{r_2}$ i.e.

$\vec{r_3} = x\vec{r_1} + y\vec{r_2}$, where x & y are scalars.

or, $\vec{a} + 19\vec{b} - 7\vec{c} = x(2\vec{a} + 3\vec{b} - 4\vec{c}) + y(-\vec{a} + 2\vec{b} + \vec{c})$

or, $\vec{a} + 19\vec{b} - 7\vec{c} = 2x\vec{a} + 3x\vec{b} - 4x\vec{c} - y\vec{a} + 2y\vec{b} + y\vec{c}$

or, $\vec{a} + 19\vec{b} - 7\vec{c} = 2x\vec{a} - y\vec{a} + 3x\vec{b} + 2y\vec{b} - 4x\vec{c} + y\vec{c}$

or, $\vec{a} + 19\vec{b} - 7\vec{c} = \vec{a}(2x-y) + \vec{b}(3x+2y) - \vec{c}(4x-y)$

By the definition of equality of vectors,

$\vec{a} = \vec{a}(2x-y)$ i.e. $2x - y = 1$ — (i)
$19\vec{b} = \vec{b}(3x+2y)$ i.e. $3x + 2y = 19$ — (ii)
$-7\vec{c} = -\vec{c}(4x-y)$ i.e. $4x - y = 7$ — (iii)

Solving equations (i) & (ii)

Multiplying equation (i) by 3, $6x-3y=3$
Multiplying equation (ii) by 2, $6x+4y=38$
On substraction, $-7y=-35$
$\therefore y=5$

Substituting the value of y in equation (i), we get,

$2x-5=1$
or, $2x=5+1$
$\therefore x=3$

$\therefore x=3, y=5$

Substituting the value of x & y in equation (iii) we get,

$4x-y=7$
or, $4\times3-5=7$
on $12-5=7$
~~or, 10 = 7 (False)~~
on $7=7$ (true)

Therefore the vectors $\vec{r_1}$, $\vec{r_2}$ & $\vec{r_3}$ are co-planar and hence it can be expressed as a linear combination of other two vectors i.e.

$\vec{r_3}=x\vec{r_1}+y\vec{r_2}$
i.e. $\vec{r_3}=3\vec{r_1}+5\vec{r_2}$

Hence, $2\vec{a}+3\vec{b}-4\vec{c}$, $-\vec{a}+2\vec{b}+\vec{c}$ and $\vec{a}+19\vec{b}-7\vec{c}$ are co-planar.

iii) $\vec{a} - 3\vec{b} + 5\vec{c}$, $\vec{a} - 2\vec{b} + 3\vec{c}$ and $-2\vec{a} + 3\vec{b} - 4\vec{c}$

Solution

Let $\vec{r_1} = \vec{a} - 3\vec{b} + 5\vec{c}$

$\vec{r_2} = \vec{a} - 2\vec{b} + 3\vec{c}$

$\vec{r_3} = -2\vec{a} + 3\vec{b} - 4\vec{c}$

To show the vectors $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$ are co-planar, we need to show that $\vec{r_3}$ can be expressed as a linear combination of $\vec{r_1}$ and $\vec{r_2}$ i.e.

$\vec{r_3} = x\vec{r_1} + y\vec{r_2}$, where x & y are the scalar.

or, $-2\vec{a} + 3\vec{b} - 4\vec{c} = x(\vec{a} - 3\vec{b} + 5\vec{c}) + y(\vec{a} - 2\vec{b} + 3\vec{c})$

or, $-2\vec{a} + 3\vec{b} - 4\vec{c} = x\vec{a} - 3x\vec{b} + 5x\vec{c} + y\vec{a} - 2y\vec{b} + 3y\vec{c}$

or, $-2\vec{a} + 3\vec{b} - 4\vec{c} = x\vec{a} + y\vec{a} - 3x\vec{b} - 2y\vec{b} + 5x\vec{c} + 3y\vec{c}$

or, $-2\vec{a} + 3\vec{b} - 4\vec{c} = \vec{a}(x+y) - \vec{b}(3x+2y) + \vec{c}(5x+3y)$

~~or,~~ By the definition of equality of vectors

$-2\vec{a} = \vec{a}(x+y)$ i.e. $x+y = -2$ — (i)

$3\vec{b} = -\vec{b}(3x+2y)$ i.e. $3x+2y = -3$ — (ii)

$-4\vec{c} = \vec{c}(5x+3y)$ i.e. $5x+3y = -4$ — (iii)

Solving ① & ⑪

Multiplying equation ① by 3, $3x + 3y = -6$
Multiplying equation ⑪ by 1, $3x + 2y = -3$
~~On substraction,~~
On subtraction, $y = -3$

Substituting the value of y in equation ①,

$$x - 3 = -2$$
$$\text{~~or,~~} \therefore x = 1$$
$$\therefore x = 1, \; y = -3$$

Substituting the value of x & y in equation ⑪⑪
we get,

$$x + y = -2$$
$$\text{or, } 1 - 3 = -2$$
$$\text{or } -2 = -2 \text{ (true)}$$

Therefore, the vector $\vec{r_1}$, $\vec{r_2}$ & $\vec{r_3}$ are co-planar and hence, it can be expressed as a linear combination of other two vectors i.e.

$$\vec{r_3} = x\vec{r_1} + y\vec{r_2}$$
i.e. $\vec{r_3} = 1\vec{r_1} - 3\vec{r_2}$

Hence $\vec{a} - 3\vec{b} + 5\vec{c}$, $\vec{a} - 2\vec{b} + 3\vec{c}$ and $-2\vec{a} + 3\vec{b} - 4\vec{c}$ are co-planar

iv) $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-\vec{b} + 2\vec{c}$

Solution.

Let $\vec{r_1} = \vec{a} - 2\vec{b} + 3\vec{c}$

$\vec{r_2} = -2\vec{a} + 3\vec{b} - 4\vec{c}$

$\vec{r_3} = -\vec{b} + 2\vec{c}$

To show vectors $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$ coplanar, we need to show $\vec{r_3}$ can be expressed as a linear combination of $\vec{r_2}$ and $\vec{r_1}$ i.e.

$\vec{r_3} = x\vec{r_1} + y\vec{r_2}$, where x & y are the scalars

or, $-\vec{b} + 2\vec{c} = x(\vec{a} - 2\vec{b} + 3\vec{c}) + y(-2\vec{a} + 3\vec{b} - 4\vec{c})$

or, $-\vec{b} + 2\vec{c} = x\vec{a} - 2x\vec{b} + 3x\vec{c} - 2y\vec{a} + 3y\vec{b} - 4y\vec{c}$

or, $-\vec{b} + 2\vec{c} = x\vec{a} - 2y\vec{a} - 2x\vec{b} + 3y\vec{b} + 3x\vec{c} - 4y\vec{c}$

or $-\vec{b} + 2\vec{c} = \vec{a}(x - 2y) - \vec{b}(2x - 3y) + \vec{c}(3x - 4y)$

By the definition of equality of vectors,

$0 \times \vec{a} = \vec{a}(x - 2y)$ i.e. $x = 2y$ — (i)

$-\vec{b} = -\vec{b}(2x - 3y)$ i.e. $2x - 3y = 1$ — (ii)

$2\vec{c} = \vec{c}(3x - 4y)$ i.e. $3x - 4y = 2$ — (iii)

Solving (I) & (II)

Multiplying equation (I) by 2, $2x - 4y = 0$
Multiplying equation (II) by 1, $2x - 3y = 1$
On subtraction, $-y = -1$
$\therefore y = 1$

Substituting the value of y in equation (I),

$x = 2y$
or, $x = 2 \times 1$
$\therefore x = 2$

$\therefore x = 2$ & $y = 1$

Substituting the value of x & y in equation (III), we get,

$3x - 4y = 2$
or, $3 \times 2 - 4 \times 1 = 2$
or, $6 - 4 = 2$
or, $2 = 2$ (true)

Therefore, the vectors $\vec{r_1}$, $\vec{r_2}$ & $\vec{r_3}$ are co-planar and hence, it can be expressed as a linear combination of other two vectors i.e.

$\vec{r_3} = x\vec{r_1} + y\vec{r_2}$

i.e. $\vec{r_3} = 2\vec{r_1} + 1\vec{r_2}$

Hence, $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-\vec{b} + 2\vec{c}$ are co-planar.

v) $4\vec{a} + 5\vec{b} + \vec{c}$, $-\vec{b} - \vec{c}$ and $\vec{a} + 9\vec{b} + 4\vec{c}$

Solution

Let $\vec{r_1} = 4\vec{a} + 5\vec{b} + \vec{c}$

$\vec{r_2} = -\vec{b} - \vec{c}$

~~$\vec{r_3} = 2\vec{a}$~~

$\vec{r_3} = \vec{a} + 9\vec{b} + 4\vec{c}$

To show the vectors $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$ coplanar, we need to show $\vec{r_3}$ can be expressed as linear combination of $\vec{r_1}$ and $\vec{r_3}$ i.e.

$\vec{r_3} = x\vec{r_1} + y\vec{r_2}$, where x & y are the scalars.

or, $\vec{a} + 9\vec{b} + 4\vec{c} = x(4\vec{a} + 5\vec{b} + \vec{c}) + y(-\vec{b} - \vec{c})$

or, $\vec{a} + 9\vec{b} + 4\vec{c} = 4x\vec{a} + 5x\vec{b} + x\vec{c} - y\vec{b} - y\vec{c}$

or, $\vec{a} + 9\vec{b} + 4\vec{c} = 4x\vec{a} + 5x\vec{b} - y\vec{b} + x\vec{c} - y\vec{c}$

or, $\vec{a} + 9\vec{b} + 4\vec{c} = 4x\vec{a} + \vec{b}(5x - y) + \vec{c}(x - y)$

~~or, $\vec{a}$~~

By the definition of equality of vectors,

$\vec{a} = 4x\vec{a}$ i.e. $4x = 1$ —— (I)

$9\vec{b} = \vec{b}(5x - y)$ i.e. $5x - y = 9$ —— (II)

$4\vec{c} = \vec{c}(x - y)$ i.e. $x - y = 4$ —— (III)

Solving (i) & (ii)

Multiplying equation (i) by 5, $20x - 0y = 5$
Multiplying equation (ii) by 4, $20x - 4y = 36$
on subtraction, $4y = -31$

$$\therefore y = -\frac{31}{4}$$

Substituting the value of y in equation (ii),

$$5x + \frac{31}{4} = 9$$

$$\text{or, } 5x = 9 - \frac{31}{4}$$

$$\text{or, } 5x = \frac{5}{4}$$

$$\therefore x = \frac{1}{4}$$

$$\therefore x = \frac{1}{4},\ y = -\frac{31}{4}$$

Substituting the value of x & y in equation (iii), we get,

$$x - y = 4$$

$$\text{or, } \frac{1}{4} + \frac{31}{4} = 4$$

$$\text{or, } \frac{32}{4} = 4$$

$$\text{or, } 8 = 4 \text{ (False)}$$

Hence, $4\vec{a} + 5\vec{b} + \vec{c}$, $-\vec{b} - \vec{c}$ and $\vec{a} + 9\vec{b} + 4\vec{c}$ are not co-planar.

or, $\vec{AC} = (1, 0)$

$\therefore \vec{AC} = 3\vec{AB}$

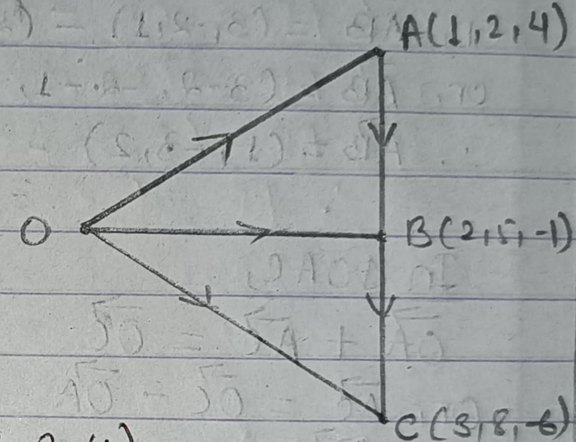
So, the vectors are collinear.

---

ii) $\vec{i} + 2\vec{j} + 4\vec{k}$, $2\vec{i} + 5\vec{j} - \vec{k}$ and $3\vec{i} + 8\vec{j} - 6\vec{k}$

Solution

Let A, B, C are the points. Let O be the origin. Join OA, OB, OC and AC.



<u>In ΔOAB,</u>

$\vec{OA} + \vec{AB} = \vec{OB}$

or, $\vec{AB} = \vec{OB} - \vec{OA}$

or, $\vec{AB} = (2, 5, -1) - (1, 2, 4)$

or, $\vec{AB} = (2-1, 5-2, -1-4)$

or, $\vec{AB} = (1, 3, -5)$

$\therefore \vec{AB} = \vec{i} + 3\vec{j} - 5\vec{k}$

<u>In ΔOAC,</u>

$\vec{OA} + \vec{AC} = \vec{OC}$

or, $\vec{AC} = \vec{OC} - \vec{OA}$

or, $\vec{AC} = (3, 8, -6) - (1, 2, 4)$

or, $\vec{AC} = (3-1, 8-2, -6-4)$

or, $\vec{AC} = (2, 6, -10)$

or, $\vec{AC} = 2\vec{i} + 6\vec{j} - 10\vec{k}$

or, $\vec{AC} = 2(\vec{i} + 3\vec{j} - 5\vec{k})$

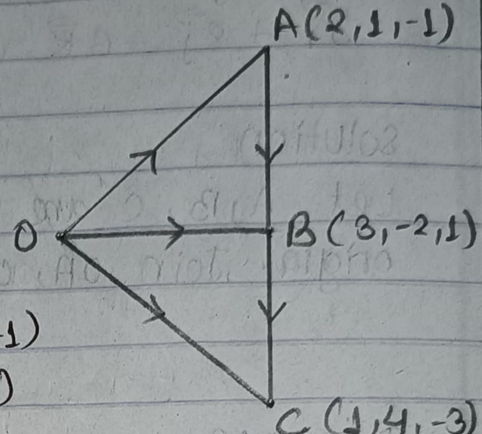
$\therefore \vec{AC} = 2\vec{AB}$ $[\because \vec{AB} = \vec{i} + 3\vec{j} - 5\vec{k}]$

So, the vectors are collinear.

iii) (2, 1, -1), (3, -2, 1) and (1, 4, -3)

Solution

Let A, B, C are the points. Let O be the origin. Join OA, OB, OC and AC.



In ΔOAB

$\vec{OA} + \vec{AB} = \vec{OB}$

or, $\vec{AB} = \vec{OB} - \vec{OA}$

or, $\vec{AB} = (3, -2, 1) - (2, 1, -1)$

or, $\vec{AB} = (3-2, -2-1, 1+1)$

$\therefore \vec{AB} = (1, -3, 2)$

In ΔOAC,

$\vec{OA} + \vec{AC} = \vec{OC}$

or, $\vec{AC} = \vec{OC} - \vec{OA}$

or, $\vec{AC} = (1, 4, -3) - (2, 1, -1)$

on $\vec{AC} = (1-2, 4-1, -3+1)$

on $\vec{AC} = (-1, 3, -2)$

or, $\vec{AC} = -1(1, -3, 2)$

$\therefore \vec{AC} = -1\vec{AB}$ [$\because \vec{AB} = (1, -3, 2)$]

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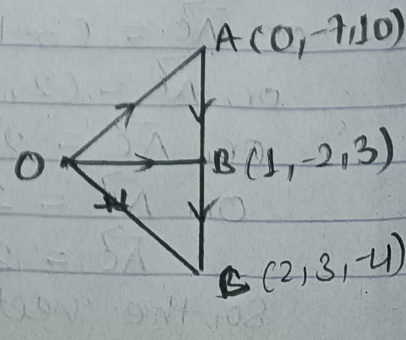
iv) (0, -7, 10), (1, -2, 3) and (2, 3, -4)

Solution

Let A, B, C are the points. Let O be the origin. Join OA, OB, OC and AC.



In $\triangle OAB$

$\vec{OA} + \vec{AB} = \vec{OB}$

or, $\vec{AB} = \vec{OB} - \vec{OA}$

or, $\vec{AB} = (1, -2, 3) - (0, -7, 10)$

or, $\vec{AB} = (1-0, -2+7, 3-10)$

$\therefore \vec{AB} = (1, 5, -7)$

In $\triangle OAC$,

$\vec{OA} + \vec{AC} = \vec{OC}$

or, $\vec{AC} = \vec{OC} - \vec{OA}$

~~or, $\vec{AC} = (1, 4, -3) - (2, 1, -1)$~~

~~or, $\vec{AC} = (1-2, 4-1, -3+1)$~~

~~or, $\vec{AC} = (-1, 3, -2)$~~

or, $\vec{AC} = (2, 3, -4) - (0, -7, 10)$

or, $\vec{AC} = (2-0, 3+7, -4-10)$

or, $\vec{AC} = (2, 10, -14)$

or, $\vec{AC} = 2(1, 5, -7)$

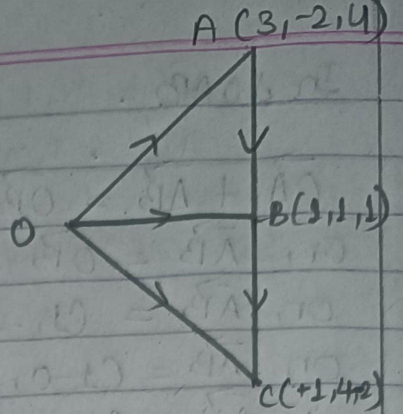
$\therefore \vec{AC} = 2\vec{AB}$ $[\because \vec{AB} = (1, 5, -7)]$

So, the vectors are collinear.

v) $3\vec{i} - 2\vec{j} + 4\vec{k}$, $\vec{i} + \vec{j} + \vec{k}$ and $-\vec{i} + 4\vec{j} - 2\vec{k}$

Solution

Let A, B and C are the points. Let O be the origin. Join OA, OB, OC and AC.



In ΔOAB

$\overrightarrow{OA} + \overrightarrow{AB} = \cancel{\overrightarrow{AB}}\ \overrightarrow{OB}$

or, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

or, $\overrightarrow{AB} = (1,1,1) - (3,-2,4)$

or, $\overrightarrow{AB} = (1-3,\ 1+2,\ 1-4)$

$\therefore\ \overrightarrow{AB} = (-2, 3, -3)$

In ΔOAC

$\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$

or, $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

or, $\overrightarrow{AC} = (-1,4,-2) - (3,-2,4)$

or, $\overrightarrow{AC} = (-1-3,\ 4+2,\ -2-4)$

or, $\overrightarrow{AC} = (-4, 6, -6)$

or, $\overrightarrow{AC} = 2(-2, 3, -3)$

$\therefore\ \overrightarrow{AC} = 2\overrightarrow{AB}$ $[\because \overrightarrow{AB} = (-2,3,-3)]$

Hence, ~~A~~ the vectors are collinear.

Exercise 7(A)

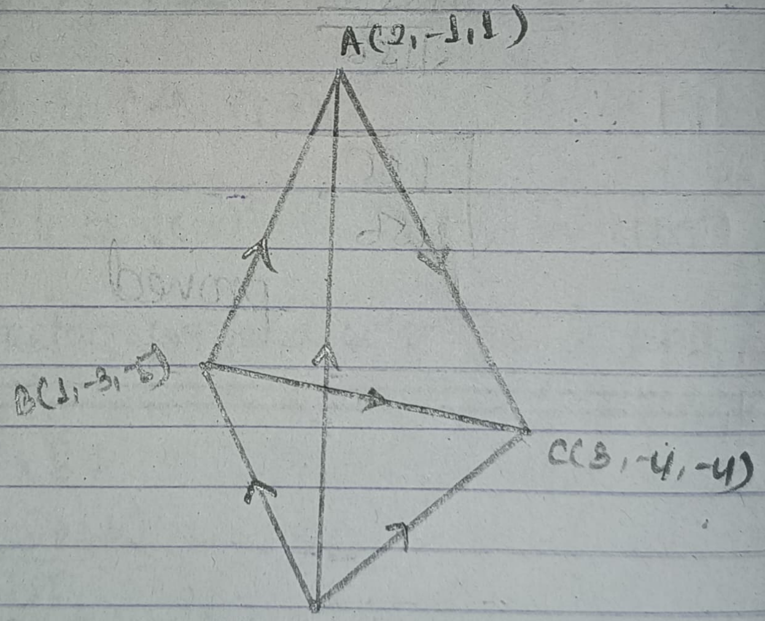
8. i) Show that three points $(2,-1,1)$, $(1,-3,-5)$ and $(3,-4,-4)$ are vertices of right-angled triangle.

Solution

Let ABC be a triangle whose vertices are $A(2,-1,1)$, $B(1,-3,-5)$ and $C(3,-4,-4)$ respectively.

To prove: ABC is a right angled triangle.

Proof:



Let O be the origin. Join OA, OB, and OC.

$\vec{OA} = (2,-1,1)$
$\vec{OB} = (1,-3,-5)$
$\vec{OC} = (3,-4,-4)$

In ΔOAB,

$\vec{OB} + \vec{BA} = \vec{OA}$ [using triangle law of addition of vector]

or, $\vec{BA} = \vec{OA} - \vec{OB}$

or, $\vec{BA} = (2, -1, 1) - (1, -3, -5)$

$\therefore \vec{BA} = (1, 2, 6)$

$|\vec{BA}| = \sqrt{(1)^2 + (2)^2 + (6)^2} = \sqrt{1+4+36}$

$\therefore |\vec{BA}| = \sqrt{41}$ units

In ΔOAC,

$\vec{OA} + \vec{AC} = \vec{OC}$

or, $\vec{AC} = \vec{OC} - \vec{OA}$

or, $\vec{AC} = (3, -4, -4) - (2, -1, 1)$

$\therefore \vec{AC} = (1, -3, -5)$

$|\vec{AC}| = \sqrt{(1)^2 + (-3)^2 + (-5)^2} = \sqrt{1+9+25}$

$\therefore |\vec{AC}| = \sqrt{35}$ units

In ΔOBC

$\vec{OB} + \vec{BC} = \vec{OC}$ [using triangle law of vector]

or, $\vec{BC} = \vec{OC} - \vec{OB}$

or, $\vec{BC} = (3, -4, -4) - (1, -3, -5)$

$= (3-1, -4+3, -4+5)$

$\therefore \vec{BC} = (2, -1, 1)$

$\therefore |\vec{BC}| = \sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{6}$ units

$\therefore |\vec{BC}| = BC = CB = \sqrt{6}$ units

To show a right anged triangle we need to show, $h^2 = p^2 + b^2$

or $(\sqrt{41})^2 = (\sqrt{35})^2 + (\sqrt{6})^2$

or $41 = 35 + 6$

i.e. $41 = 41$

Hence, proved.

---

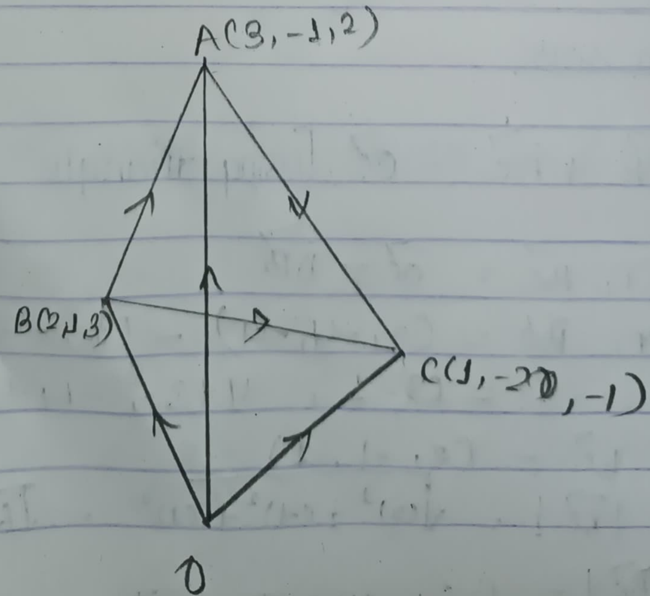
ii) Show that the vectors $3\vec{a} - \vec{b} + 2\vec{c}$, $2\vec{a} + \vec{b} + 3\vec{c}$ and $\vec{a} - 2\vec{b} - \vec{c}$ form the sides of right-angled triangle.

Solution

Let ABC be a triangle where vertices are A(3, -1, 2), B(2, 1, 3) & C(1, -2, -1) respectively.

To prove: ABC is a right angled triangle.

Proof:



~~$\vec{OA} = (2,-1,1)$~~
~~$\vec{OB} = (1,-3,-5)$~~

$\vec{OA} = (3,-1,2)$
$\vec{OB} = (2,1,3)$
$\vec{OC} = (1,-2,-1)$

<u>In $\triangle OAB$</u>

$\vec{OB} + \vec{BA} = \vec{OA}$ [Using triangle law of vector addition]

or $\vec{BA} = \vec{OA} - \vec{OB}$
$= (3,-1,2) - (2,1,3)$
$\therefore \vec{BA} = (1,-2,-1)$

$\therefore |\vec{BA}| = \sqrt{(1)^2+(-2)^2+(-1)^2} = \sqrt{6}$ units

<u>In $\triangle OAC$</u>

$\vec{OA} + \vec{AC} = \vec{OC}$ [Using triangle law of vector addition]

or, $\vec{AC} = \vec{OC} - \vec{OA}$
$= (1,-2,-1) - (3,-1,2)$
$\therefore \vec{AC} = (-2,-1,-3)$

$\therefore |\vec{AC}| = \sqrt{(-2)^2+(-1)^2+(-3)^2} = \sqrt{14}$ units

In ΔOBC,

$\vec{OB} + \vec{BC} = \vec{OC}$

or $\vec{BC} = \vec{OC} - \vec{OB}$

$= (1, -2, -1) - (2, 1, 3)$

$\therefore \vec{BC} = (-1, -3, -4)$

$\therefore |\vec{BC}| = \sqrt{(-1)^2 + (-3)^2 + (-4)^2} = \sqrt{26}$ units

To show a right angled triangle, we need to show $h^2 = p^2 + b^2$

or, $(\sqrt{26})^2 = (\sqrt{6})^2 + (\sqrt{14})^2$

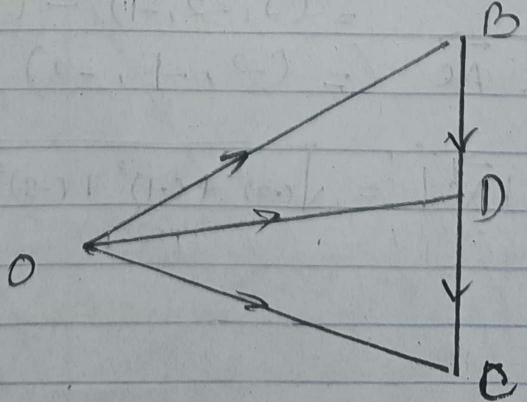
or, $26 = 6 + 14$

i.e. $26 = 20$

Hence, disproved.

9. OB and OC are two straight lines. D is a point on BC such that $\vec{BD} : \vec{DC} = m : n$, show that $\vec{OD} = \dfrac{n\vec{OB} + m\vec{OC}}{m+n}$

Solution



OB and OC are two straight lines. O is a point on BC such that;

$$\frac{\vec{BD}}{\vec{DC}} = \frac{\vec{m}}{n}$$

To prove:

$$\vec{OD} = \frac{n\vec{OB} + m\vec{OC}}{m+n}$$

<u>Proof</u>

<u>In ΔOBD</u>

$\vec{OB} + \vec{BD} = \vec{OD}$ [By using triangle law of vector addition]

$\therefore \vec{BD} = \vec{OD} - \vec{OB}$

<u>In ΔODC</u>

$\vec{OD} + \vec{DC} = \vec{OC}$ [By using triangle law of vector addition]

$\therefore \vec{DC} = \vec{OC} - \vec{OD}$

Now,

$$\frac{\vec{BD}}{\vec{DC}} = \frac{m}{n}$$

or, $$\frac{\vec{OD} - \vec{OB}}{\vec{OC} - \vec{OD}} = \frac{m}{n}$$

or, $n\vec{OD} - n\vec{OB} = m\vec{OC} - m\vec{OD}$

or, $n\vec{OD} + m\vec{OD} = m\vec{OC} + n\vec{OB}$

or, $\vec{OD}(n+m) = m\vec{OC} + n\vec{OB}$

$$\therefore \vec{OD} = \frac{m\vec{OC} + n\vec{OB}}{(m+n)}$$

proved

Signature of the subject teacher:

Signature of Director: