Exercise 4(A)

1. Find, from first principle, the derivatives with respect to x of the followings:

a) $2x+5$

Solution

Let $y = 2x+5$

Let Δy and Δx be small increment in y and x. such that,

$$y + \Delta y = 2(x+\Delta x) + 5$$

or, $y+\Delta y = 2x + 2\Delta x + 5$

or, $\Delta y = 2x + 2\Delta x + 5 - y$

or, $\Delta y = 2x + 2\Delta x + 5 - 2x - 5$ $[\because y = 2x+5]$

or, $\Delta y = 2\Delta x$

or, $\dfrac{\Delta y}{\Delta x} = 2$

Taking $\lim_{\Delta x \to 0}$ on both sides,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} (2)$$

$$\therefore \frac{d}{dx}(2x+5) = 2$$

b) $3x^2 - 5x$

Solution

Let $y = 3x^2 - 5x$

Let Δy and Δx be small increment in y and x such that,

$$y + \Delta y = 3(x+\Delta x)^2 - 5(x+\Delta x)$$

or, $y + \Delta y = 3\{x^2 + 2x\Delta x + (\Delta x)^2\} - 5x - 5\Delta x$

or, $y + \Delta y = 3x^2 + 6x\Delta x + 3(\Delta x)^2 - 5x - 5\Delta x$

or, $\Delta y = 3x^2 + 6x\Delta x + 3(\Delta x)^2 - 5x - 5\Delta x - y$

or, $\Delta y = 3x^2 + 6x\Delta x + 3(\Delta x)^2 - 5x - 5\Delta x - 3x^2 + 5x$

$[\because y = 3x^2 - 5x]$

or, $\Delta y = 6x\Delta x + 3(\Delta x)^2 - 5\Delta x$

or, $\Delta y = \Delta x\,(6x + 3\Delta x - 5)$

or, $\dfrac{\Delta y}{\Delta x} = (6x + 3\Delta x - 5)$

Taking $\lim\limits_{\Delta x \to 0}$ on both sides,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} (6x + 3\Delta x - 5)$$

or, $\dfrac{dy}{dx} = 6x + 3\times 0 - 5$

~~or~~ $\therefore \dfrac{d}{dx}(3x^2 - 5x) = 6x - 5$

() $\dfrac{1}{5x+3}$

Solution

Let $y = \dfrac{1}{5x+3}$

Let Δy and Δx be small increment in y and x. such that,

$$y + \Delta y = \frac{1}{5(x+\Delta x)+3}$$

$$\text{or, } y + \Delta y = \frac{1}{5x+5\Delta x+3}$$

$$\text{or, } \Delta y = \frac{1}{5x+5\Delta x+3} - \frac{1}{5x+3} \quad \left[\because y = \frac{1}{5x+3}\right]$$

$$\text{or, } \Delta y = \frac{5x+3-5x-5\Delta x-3}{(5x+5\Delta x+3)(5x+3)}$$

$$\text{or, } \Delta y = \frac{-5\Delta x}{(5x+5\Delta x+3)(5x+3)}$$

$$\text{or, } \frac{\Delta y}{\Delta x} = -\frac{5}{(5x+5\Delta x+3)(5x+3)}$$

Taking $\lim_{\Delta x \to 0}$ on both sides,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left[-\frac{5}{(5x+5\Delta x+3)(5x+3)}\right]$$

$$\text{or, } \frac{dy}{dx} = -\frac{5}{(5x+5\times 0+3)(5x+3)}$$

$$\therefore \frac{d}{dx}\left(\frac{1}{5x+3}\right) = -\frac{5}{(5x+3)^2}$$

d) $\frac{1}{x}$

Solution

Let $y = \frac{1}{x}$.

Let Δy and Δx be small increments in y and x, such that,

$$\cancel{y + \Delta y = \frac{1}{x}}$$

$$y + \Delta y = \frac{1}{x + \Delta x}$$

$$\text{or, } \Delta y = \frac{1}{x + \Delta x} - y$$

$$\text{or, } \Delta y = \frac{1}{x + \Delta x} - \frac{1}{x} \quad \left[\because y = \frac{1}{x}\right]$$

$$\text{or, } \Delta y = \frac{x - x - \Delta x}{x(x + \Delta x)}$$

$$\text{or, } \Delta y = \frac{-\Delta x}{x(x + \Delta x)}$$

$$\text{or, } \frac{\Delta y}{\Delta x} = \frac{-1}{x(x + \Delta x)}$$

Taking $\lim_{\Delta x \to 0}$ on both sides,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left[\frac{-1}{x(x + \Delta x)}\right]$$

or, $\frac{dy}{dx} = -\frac{1}{x(x+0)}$

$\therefore$ ~~$\frac{dy}{}$~~ $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

e) $x + \sqrt{x}$

Solution

Let $y = x + \sqrt{x}$

Let Δy and Δx be small increments in y and x, such that,

$$y + \Delta y = x + \Delta x + \sqrt{x + \Delta x}$$

or, $\Delta y = x + \Delta x + \sqrt{x + \Delta x} - x - \sqrt{x}$

$[\because y = x + \sqrt{x}]$

or, $\Delta y = \Delta x + \left[\sqrt{x + \Delta x} - \sqrt{x}\right]$

or, $\Delta y = \Delta x + \left[\sqrt{x+\Delta x} - \sqrt{x} \times \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}\right]$

or, $\Delta y = \Delta x + \left[\frac{x + \Delta x - x}{\sqrt{x+\Delta x} + \sqrt{x}}\right]$

or, $\Delta y = \Delta x + \frac{\Delta x}{\sqrt{x+\Delta x} + \sqrt{x}}$

or, $\Delta y = \Delta x\left[1 + \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}\right]$

or $\frac{\Delta y}{\Delta x} = 1 + \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}$

Taking $\lim_{\Delta x \to 0}$ on both sides,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left[1 + \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} \right]$$

or, $\frac{dy}{dx} = 1 + \frac{1}{\sqrt{x+0} + \sqrt{x}}$

$\therefore \frac{d}{dx}(x+\sqrt{x}) = 1 + \frac{1}{2\sqrt{x}}$

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f) $\frac{1}{\sqrt{x+2}}$

Solution

Let $y = \frac{1}{\sqrt{x+2}}$

Let Δy and Δx be small increment in y and x, such that,

$$y + \Delta y = \frac{1}{\sqrt{x+\Delta x+2}}$$

or, $\Delta y = \frac{1}{\sqrt{x+\Delta x+2}} - \frac{1}{\sqrt{x+2}}$

or, $\Delta y = \frac{\sqrt{x+2} - \sqrt{x+\Delta x+2}}{(\sqrt{x+\Delta x+2} \cdot \sqrt{x+2})}$

$$\text{or, } \Delta y = \frac{\sqrt{x+2} - \sqrt{x+\Delta x+2}}{(\sqrt{x+\Delta x+2} \cdot \sqrt{x+2})} \times \frac{\sqrt{x+2} + \sqrt{x+\Delta x+2}}{\sqrt{x+2} + \sqrt{x+\Delta x+2}}$$

$$\text{or, } \Delta y = \frac{x+2-x-\Delta x-2}{(\sqrt{x+\Delta x+2} \cdot \sqrt{x+2})(\sqrt{x+2} + \sqrt{x+\Delta x+2})}$$

$$\text{or, } \Delta y = \frac{-\Delta x}{(\sqrt{x+\Delta x+2} \cdot \sqrt{x+2})(\sqrt{x+2} + \sqrt{x+\Delta x+2})}$$

$$\text{or, } \frac{\Delta y}{\Delta x} = \frac{-1}{(\sqrt{x+\Delta x+2} \cdot \sqrt{x+2})(\sqrt{x+2} + \sqrt{x+\Delta x+2})}$$

Taking $\lim_{\Delta x \to 0}$ on both sides,

$$\text{or, } \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left[\frac{-1}{(\sqrt{x+\Delta x+2} \cdot \sqrt{x+2})(\sqrt{x+2} + \sqrt{x+\Delta x+2})} \right]$$

$$\text{or, } \frac{dy}{dx} = \frac{-1}{(\sqrt{x+0+2} \cdot \sqrt{x+2})(\sqrt{x+2} + \sqrt{x+0+2})}$$

$$\therefore \frac{d}{dx}\left(\frac{1}{\sqrt{x+2}}\right) = -\frac{1}{(x+2)(2\sqrt{x+2})}$$

g) $\dfrac{ax+b}{x}$

Solution

Let $y = \dfrac{ax+b}{x}$

Let Δy and Δx be the small increment in y and x, such that.

$$y + \Delta y = \frac{a(x+\Delta x)+b}{x+\Delta x}$$

$$\text{or, } \Delta y = \cancel{a(x}\ \frac{ax + a\Delta x + b}{x+\Delta x} - \frac{ax+b}{x}$$

$$\text{or, } \Delta y = \frac{ax^2 + ax\Delta x + bx - ax^2 - ax\Delta x - bx - b\Delta x}{(x+\Delta x)x}$$

$$\text{or, } \Delta y = \frac{-b\Delta x}{(x+\Delta x)x}$$

$$\text{or, } \frac{\Delta y}{\Delta x} = \frac{-b}{(x+\Delta x)x}$$

Taking $\lim_{\Delta x \to 0}$ on both sides,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left[\frac{-b}{(x+\Delta x)x} \right]$$

or, $\frac{dy}{dx} = \frac{-b}{(x+0)x}$

$\therefore \frac{d}{dx}\left(\frac{ax+b}{x}\right) = -\frac{b}{x^2}.$

i) e^{ax}

Solution

Let $y = e^{ax}$

Let Δy and Δx be small incroment in y and x such that,

$y + \Delta y = e^{a(x+\Delta x)}$

or, $y + \Delta y = e^{ax + a\Delta x}$

or, $y + \Delta y = e^{ax} \cdot e^{a\Delta x}$

or, $\Delta y = e^{ax} \cdot e^{a\Delta x} - e^{ax}$ $[\because y = e^{ax}]$

or, $\Delta y = e^{ax}(e^{a\Delta x} - 1)$

Dividing both sides by Δx,

or, $\frac{\Delta y}{\Delta x} = \frac{a e^{ax}(e^{a\Delta x} - 1)}{a\Delta x}$

Taking $\lim\limits_{\Delta x \to 0}$ on both sides,

$\lim\limits_{\Delta x \to 0}\left[\frac{\Delta y}{\Delta x}\right] = a e^{ax} \cdot \lim\limits_{\Delta x \to 0}\left[\frac{e^{a\Delta x} - 1}{a\Delta x}\right]$

or $\frac{dy}{dx} = ae^{ax} \cdot 1 \left[\because \lim_{x\to 0} \left(\frac{e^x - 1}{x}\right) = 1 \right]$

$\therefore \frac{d}{dx}(e^{ax}) = ae^{ax}$

K) e^{2x+1}

Solution: let $y = e^{2x+1}$

Let Δy and Δx be small increment in y and x, such that:

$y + \Delta y = e^{2(x+\Delta x)+1}$

or $y + \Delta y = e^{2x+2\Delta x+1}$

or $\Delta y = e^{2x+2\Delta x+1} - e^{2x+1}$ $\left[\because y = e^{2x+1}\right]$

or, $\Delta y = e^{2x+1} \cdot e^{2\Delta x} - e^{2x+1}$

or, $\Delta y = e^{2x+1}(e^{2\Delta x} - 1)$

Dividing both sides by Δx & taking $\lim_{\Delta x\to 0}$ on both sides,

$\lim_{\Delta x\to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0} \left[\frac{e^{2x+1} \cdot (e^{2\Delta x} - 1)}{\Delta x}\right]$

or $\frac{dy}{dx} = e^{2x+1} \cdot \lim_{\Delta x\to 0} \left[\frac{e^{2\Delta x} - 1}{\Delta x}\right]$

or, $\frac{dy}{dx} = e^{2x+1} \cdot e \cdot \lim_{\Delta x \to 0} \left[\frac{e^{2\Delta x} - 1}{2\Delta x} \right]$

or, $\frac{dy}{dx} = 2e^{2x+1} \cdot 1 \quad \left[\because \lim_{x \to 0} \left(\frac{e^x - 1}{x} \right) = 1 \right]$

$\therefore \frac{d}{dx}(e^{2x+1}) = 2e^{2x+1}$

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h) $\ln 3x$

Solution
Let $y = \cancel{e^{2x+1}}\ \ln 3x$
Let Δy and Δx be small increment in y and x, such that:

$y + \Delta y = \ln 3(x + \Delta x)$
or, $y + \Delta y = \ln(3x + 3\Delta x)$
or, $\Delta y = \ln(3x + 3\Delta x) - y$
or, $\Delta y = \ln(3x + 3\Delta x) - \ln 3x \quad [\because y = \ln 3x]$

or, $\Delta y = \ln\left(\frac{3x + 3\Delta x}{3x}\right) \quad \left[\because \ln m - \ln n = \ln\left(\frac{m}{n}\right)\right]$

or, $\Delta y = \ln\left(\frac{3x}{3x} + \frac{3\Delta x}{3x}\right)$

or, $\Delta y = \ln\left(1 + \frac{\Delta x}{x}\right)$

Dividing both sides by Δx and taking $\lim_{\Delta x \to 0}$ on both sides,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left[\frac{\ln\left(1 + \frac{\Delta x}{x}\right)}{\Delta x} \right]$$

$$or,\ \frac{dy}{dx} = \lim_{\frac{\Delta x}{x} \to 0} \left[\frac{\ln\left(1 + \frac{\Delta x}{x}\right)}{\frac{\Delta x}{x} \times x} \right]$$

$$or,\ \frac{dy}{dx} = \frac{1}{x} \left[\lim_{\frac{\Delta x}{x} \to 0} \left\{ \frac{\ln\left(1 + \frac{\Delta x}{x}\right)}{\frac{\Delta x}{x}} \right\} \right]$$

$$or,\ \frac{dy}{dx} = \frac{1}{x} \times 1 \left[\because \lim_{x \to 0} \left\{ \frac{\ln(1+x)}{x} \right\} = 1 \right]$$

$$\therefore \frac{d}{dx}(\ln x) = \frac{1}{x}$$

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j) $\ln x$

Solution

Let $y = \ln x$

Let Δy and Δx be small increments in y and x, such that:

$$\cancel{\Delta y}\ y + \Delta y = \ln(x + \Delta x)$$

$$or,\ \Delta y = \ln(x + \Delta x) - y$$

$$or,\ \Delta y = \ln(x + \Delta x) - \ln x \quad [\because y = \ln x]$$

$$or,\ \Delta y = \ln\left(\frac{x + \Delta x}{x}\right) \left[\because \ln m - \ln n = \ln\left(\frac{m}{n}\right)\right]$$

$$or,\ \Delta y = \ln\left(\frac{x}{x} + \frac{\Delta x}{x}\right)$$

$$or,\ \Delta y = \ln\left(1 + \frac{\Delta x}{x}\right)$$

Taking $\lim\limits_{\Delta x \to 0}$ on both sides and dividing both sides by Δx,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left[\ln \left(1 + \frac{\Delta x}{x} \right) \right]$$

$$\text{or, } \frac{dy}{dx} = \lim_{\Delta x \to 0} \left[\ln \left(\frac{1 + \frac{\Delta x}{x}}{\Delta x} \right) \right]$$

$$\text{or, } \frac{dy}{dx} = \lim_{\Delta x \to 0} \left[\frac{\ln \left(1 + \frac{\Delta x}{x} \right)}{\frac{\Delta x}{x} \times x} \right]$$

$$\text{or, } \frac{dy}{dx} = \frac{1}{x} \cdot \lim_{\frac{\Delta x}{x} \to 0} \left[\frac{\ln \left(1 + \frac{\Delta x}{x} \right)}{\frac{\Delta x}{x}} \right]$$

$$\text{or, } \frac{dy}{dx} = \frac{1}{x} \times 1 \quad \left[\because \lim_{x \to 0} \left\{ \frac{\ln (1+x)}{x} \right\} = 1 \right]$$

$$\therefore \frac{d}{dx} (\ln x) = \frac{1}{x}$$

l) $\ln(5x+2)$

Solution

Let $y = \ln(5x+2)$

let Δy and Δx be small increments in x such that:

$$y + \Delta y = \ln(5x + 5\Delta x + 2)$$

or, $\Delta y = \ln(5x + 5\Delta x + 2) - y$

or, $\Delta y = \ln(5x + 5\Delta x + 2) - \ln(5x+2)$

$[\because y = \ln(5x+2)]$

or, $\Delta y = \ln\left(\frac{5x + 5\Delta x + 2}{5x+2}\right)$

$[\because \ln m - \ln n = \ln\left(\frac{m}{n}\right)]$

or, $\Delta y = \ln\left(\frac{5x+2}{5x+2} + \frac{5\Delta x}{5x+2}\right)$

or, $\Delta y = \ln\left(1 + \frac{5\Delta x}{5x+2}\right)$

Dividing both sides by Δx, & Taking $\lim\limits_{\Delta x \to 0}$ on both sides,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left[\frac{\ln\left(1 + \frac{5\Delta x}{5x+2}\right)}{\Delta x}\right]$$

$$\text{or, } \frac{dy}{dx} = \lim_{\Delta x \to 0} \left[\frac{5}{5x+2} \cdot \frac{\ln\left(1 + \frac{5\Delta x}{5x+2}\right)}{\frac{5\Delta x}{5x+2}}\right]$$

or, $\frac{dy}{dx} = \frac{5}{5x+2} \cdot \lim_{\frac{5\Delta x}{5x+2} \to 0} \left[\frac{\ln\left(1 + \frac{5\Delta x}{5x+2}\right)}{\frac{5\Delta x}{5x+2}} \right]$

or, $\frac{dy}{dx} = \frac{5}{5x+2} \times 1$

$\left[\because \lim_{x \to 0} \left\{ \frac{\ln(1+x)}{x} \right\} = 1 \right]$

$\therefore \frac{d}{dx} [\ln(5x+2)] = \frac{5}{5x+2}$

Signature of the subject Teacher :

Signature of Director :

Chapter 4 : Differentiation and It's Application

Exercise 4(A)

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1. Find the derivatives with respect to 'x'.

a) x^2+x

$= \frac{d}{dx}(x^2+x)$

$= \frac{d}{dx}(x^2) + \frac{d}{dx}(x)$

$= 2x^{2-1} + 1 \quad \left[\because \frac{d}{dx}(x^n) = nx^{n-1},\ \frac{d}{dx}(x) = 1\right]$

$= 2x+1$

b) $4x^3 - 3x + 2$

$= \frac{d}{dx}(4x^3 - 3x + 2)$

$= \frac{d}{dx}(4x^3) - \frac{d}{dx}(3x) + \frac{d}{dx}(2)$

$= 4.\frac{d}{dx}(x^3) - 3.\frac{d}{dx}(x) + 0 \quad \left[\because \frac{d}{dx}(2) = 0\right]$

$= 4.3x^{3-1} - 3.1 \quad \left[\because \frac{d}{dx}(x) = 1\right]$

$= 12x^2 - 3$

c) $\frac{ax^2+bx+c}{x}$

$= \frac{d}{dx}\left(\frac{ax^2+bx+c}{x}\right)$

$= \frac{d}{dx}\left(\frac{ax^2}{x}\right) + \frac{d}{dx}\left(\frac{bx}{x}\right) + \frac{d}{dx}\left(\frac{c}{x}\right)$

$= \frac{d}{dx}(ax) + \frac{d}{dx}(b) + \frac{d}{dx}(cx^{-1})$

$= a.\frac{d}{dx}(x) + 0 + c\frac{d}{dx}(x^{-1}) \quad \left[\because \frac{d}{dx}(b) = 0\right]$

$= a.1 + c.-1x^{-1-1}$ $\left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$

$= a - cx^{-2}$

$= a - \frac{c}{x^2}$

d) $\frac{2}{x} - \frac{3}{x^2}$

$= \frac{d}{dx}\left(\frac{2}{x} - \frac{3}{x^2}\right)$

$= \frac{d}{dx}\left(2x^{-1} - 3x^{-2}\right)$

$= \frac{d}{dx}(2x^{-1}) - \frac{d}{dx}(3x^{-2})$

$= 2.\frac{d}{dx}(x^{-1}) - 3.\frac{d}{dx}(x^{-2})$

$= 2.(-1)x^{-1-1} - 3.(-2)x^{-2-1}$ $\left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$

$= -2x^{-2} + 6x^{-3}$

$= -\frac{2}{x^2} + \frac{6}{x^3}$

e) $\frac{1+x+x^2}{x}$

$= \frac{d}{dx}\left(\frac{1+x+x^2}{x}\right)$

$= \frac{d}{dx}\left(\frac{1}{x}\right) + \frac{d}{dx}\left(\frac{x}{x}\right) + \frac{d}{dx}\left(\frac{x^2}{x}\right)$

$= \frac{d}{dx}(x^{-1}) + \frac{d}{dx}(1) + \frac{d}{dx}(x)$

$= -1x^{-1-1} + 0 + 1$ $\left[\because \frac{d}{dx}(x^n) = nx^{n-1}, \frac{d}{dx}(x) = 1\right]$

$= -1x^{-2} + 1$

$= -\frac{1}{x^2} + 1$

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f) $e^x + \ln x$

$= \frac{d}{dx}(e^x + \ln x)$

$= \frac{d}{dx}(e^x) + \frac{d}{dx}(\ln x)$

$= e^x + \frac{1}{x}$ $\left[\because \frac{d}{dx}(e^x) = e^x, \frac{d}{dx}(\ln x) = \frac{1}{x}\right]$

Q2. Find $\frac{dy}{dx}$ using product rule when,

a) $y = (x+1)(x+3)$

Here,

$y = (x+1)(x+3)$

Differentiating both sides with respect to x, we get

$\frac{dy}{dx} = \frac{d}{dx}[(x+1)(x+3)]$

$= (x+1)\left[\frac{d}{dx}(x+3)\right] + (x+3)\left[\frac{d}{dx}(x+1)\right]$

$= (x+1)\left[\frac{d}{dx}(x) + \frac{d}{dx}(3)\right] + (x+3)\left[\frac{d}{dx}(x) + \frac{d(1)}{d(x)}\right]$

$= (x+1).(1+0) + (x+3)(1+0)$ $\left[\because \frac{d(3)}{dx} = 0, \frac{d(1)}{dx} = 0, \frac{d(x)}{dx} = 1\right]$

$= x+1 + x+3$

$= 2x+4$

$$\boxed{\therefore \frac{dy}{dx} = 2x+4}$$

(b) $y = x^2(x^3+1)$

Here,

$y = x^2(x^3+1)$

Differentiating both sides with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^2(x^3+1)\right]$$

$$= x^2\frac{d}{dx}(x^3+1) + (x^3+1)\frac{d}{dx}(x^2)$$

$$= x^2\left[\frac{d(x^3)}{dx} + \frac{d(1)}{dx}\right] + (x^3+1)\left[\frac{d(x^2)}{dx}\right]$$

$$= x^2(3x^{3-1}+0) + (x^3+1)(2x^{2-1}) \quad \left[\because \frac{d(1)}{dx} = 0, \ \frac{d(x^n)}{dx} = nx^{n-1}\right]$$

$$= x^2(3x^2) + (x^3+1)(2x)$$

$$= 3x^4 + 2x(x^3+1)$$

$$= 3x^4 + 2x^4 + 2x$$

$$= 5x^4 + 2x$$

$$\boxed{\therefore \frac{dy}{dx} = 5x^4 + 2x}$$

(c) $y = (2x+3)(3x+2)$

Here,

$y = (2x+3)(3x+2)$

Differentiating both sides with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{dx}\left[(2x+3)(3x+2)\right]$$

$$= (2x+3)\frac{d}{dx}(3x+2) + (3x+2)\frac{d}{dx}(2x+3)$$

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$$= (2x+3)\left[\frac{d(3x)}{dx} + \frac{d(2)}{dx}\right] + (3x+2)\left[\frac{d(2x)}{dx} + \frac{d(3)}{dx}\right]$$

$$= (2x+3)\left[3.\frac{d(x)}{dx} + 0\right] + (3x+2)\left[2.\frac{d(x)}{dx} + 0\right] \quad \left[\because \frac{d(2)}{dx} = 0,\ \because \frac{d(3)}{dx} = 0\right]$$

$$= (2x+3)(3\times1) + (3x+2)(2\times1) \quad \left[\because \frac{d(x)}{dx} = 1\right]$$

$$= 6x+9+6x+4$$

$$= 12x+13$$

$$\boxed{\therefore \frac{dy}{dx} = 12x+13}$$

(d) $y = x.e^x$

Here,

$y = x.e^x$

Differentiating both sides with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{dx}[x.e^x]$$

$$= x.\frac{d}{dx}(e^x) + e^x.\frac{d}{dx}(x)$$

$$= x.e^x + e^x.1 \quad \left[\because \frac{d}{dx}(e^x) = e^x,\ \frac{d}{dx}(x) = 1\right]$$

$$= x.e^x + e^x$$

$$= e^x(x+1)$$

$$\boxed{\therefore \frac{dy}{dx} = e^x(x+1)}$$

(e) $y = e^x.\ln x$

Given,

$y = e^x.\ln x$

Differentiating both sides with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{dx}[e^x.\ln x]$$

$$= e^x.\frac{d}{dx}(\ln x) + \ln x.\frac{d}{dx}(e^x)$$

$= e^x \cdot \frac{1}{x} + \ln x \cdot e^x$ $\left[\because \frac{d}{dx}(\ln x) = \frac{1}{x}, \frac{d}{dx}(e^x) = e^x\right]$

$= e^x\left(\frac{1}{x} + \ln x\right)$

$\boxed{\therefore \frac{dy}{dx} = e^x\left(\frac{1}{x} + \ln x\right)}$

(f) $y = x^3(1+\ln x)$

Given,

$y = x^3(1+\ln x)$

Differentiating both sides with respect to x, we get,

$\frac{dy}{dx} = \frac{d}{dx}\left[x^3(1+\ln x)\right]$

$= x^3 \frac{d}{dx}(1+\ln x) + (1+\ln x)\frac{d}{dx}(x^3)$

$= x^3\left[\frac{d(1)}{dx} + \frac{d(\ln x)}{dx}\right] + (1+\ln x) \cdot 3x^{3-1}$ $\left[\because \frac{d(x^n)}{dx} = nx^{n-1}\right]$

$= x^3\left[0 + \frac{1}{x}\right] + (1+\ln x) \cdot 3x^2$ $\left[\begin{array}{l}\because \frac{d}{dx}(1) = 0 \\ \because \frac{d}{dx}(\ln x) = \frac{1}{x}\end{array}\right]$

$= \frac{x^3}{x} + (1+\ln x) \cdot 3x^2$

$= x^2 + (1+\ln x) \cdot 3x^2$

$= x^2 + 3x^2 + 3x^2 \ln x$

$= 4x^2 + 3x^2 \ln x$

$\boxed{\therefore \frac{dy}{dx} = 4x^2 + 3x^2 \ln x}$

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3. Find $\frac{dy}{dx}$ using quotient rule when,

a) $y = \frac{x}{x-3}$

Given,

$$y = \frac{x}{x-3}$$

Differentiating both sides with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{x}{x-3}\right]$$

$$= \frac{(x-3)\frac{d}{dx}(x) - x.\frac{d}{dx}(x-3)}{(x-3)^2} \quad \left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{v^2}\right]$$

$$= \frac{(x-3)\times 1 - x.\left[\frac{d}{dx}(x) - \frac{d}{dx}(3)\right]}{(x-3)^2} \quad \left[\because \frac{d}{dx}(x) = 1\right]$$

$$= \frac{(x-3) - x.\left[1-0\right]}{(x-3)^2} \quad \left[\because \frac{d}{dx}(x) = 1, \frac{d}{dx}(3) = 0\right]$$

$$= \frac{(x-3) - x}{(x-3)^2}$$

$$= \frac{x-3-x}{(x-3)^2}$$

$$= \frac{-3}{(x-3)^2}$$

$$\boxed{\therefore \frac{dy}{dx} = -\frac{3}{(x-3)^2}}$$

b) $y = \frac{x^2}{x-1}$

Given,

$$y = \frac{x^2}{x-1}$$

Differentiating both sides with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{x^2}{x-1}\right]$$

$$= \frac{(x-1)\frac{d}{dx}(x^2) - x^2 \cdot \frac{d}{dx}(x-1)}{(x-1)^2} \quad \left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{d(u)}{dx} - u\frac{d(v)}{dx}}{v^2}\right]$$

$$= \frac{(x-1)\, 2x^{2-1} - x^2\left[\frac{d}{dx}(x) - \frac{d}{dx}(1)\right]}{(x-1)^2} \quad \left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$$

$$= \frac{(x-1)\, 2x - x^2(1-0)}{(x-1)^2} \quad \left[\because \frac{d}{dx}(x) = 1,\ \frac{d}{dx}(1) = 0\right]$$

$$= \frac{(x-1)\, 2x - x^2}{(x-1)^2}$$

$$= \frac{2x^2 - 2x - x^2}{(x-1)^2}$$

$$= \frac{x^2 - 2x}{(x-1)^2}$$

$$= \frac{x(x-2)}{(x-1)^2}$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{x(x-2)}{(x-1)^2}}$$

i) $y = \frac{x^2+x-1}{x^2-x+1}$

Given,

$$y = \frac{x^2+x-1}{x^2-x+1}$$

Differentiating both sides with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{x^2+x-1}{x^2-x+1}\right]$$

$$= \frac{(x^2-x+1).\frac{d}{dx}(x^2+x-1) - (x^2+x-1).\frac{d}{dx}(x^2-x+1)}{(x^2-x+1)^2}$$

$$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{v^2}\right]$$

$$= \frac{(x^2-x+1).\left[\frac{d}{dx}(x^2) + \frac{d}{dx}(x) - \frac{d}{dx}(1)\right] - (x^2+x-1)\frac{d}{dx}(x^2-x+1)}{(x^2-x+1)^2}$$

$$= \frac{(x^2-x+1)\left[2x^{2-1}+1-0\right] - (x^2+x-1)\left[\frac{d}{dx}(x^2) - \frac{d}{dx}(x) + \frac{d}{dx}(1)\right]}{(x^2-x+1)^2}$$

$$= \frac{(x^2-x+1)(2x+1) - (x^2+x-1)(2x-1)}{(x^2-x+1)^2}$$

$$= \frac{2x^3+x^2-2x^2-x+2x+1-(2x^3-x^2+2x^2-x-2x+1)}{(x^2-x+1)^2}$$

$$= \frac{2x^3+x^2-2x^2-x+2x+1-2x^3+x^2-2x^2+x+2x-1}{(x^2-x+1)^2}$$

$$= \frac{4x-2x^2}{(x^2-x+1)^2}$$

$$= \frac{-2x^2+4x}{(x^2-x+1)^2}$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{-2x^2+4x}{(x^2-x+1)^2}}$$

d) $y = \frac{x^3+3x+1}{x^2-1}$

Here,

$$y = \frac{x^3+3x+1}{x^2-1}$$

Differentiating both sides with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{x^3+3x+1}{x^2-1}\right]$$

$$= \frac{(x^2-1)\frac{d}{dx}(x^3+3x+1) - (x^3+3x+1)\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{v^2}\right]$$

$$= \frac{(x^2-1)\left[\frac{d}{dx}(x^3) + \frac{d}{dx}(3x) + \frac{d}{dx}(1)\right] - (x^3+3x+1)\left[\frac{d}{dx}(x^2) - \frac{d}{dx}(1)\right]}{(x^2-1)^2}$$

$$= \frac{(x^2-1)\left[3x^{3-1} + 3.\frac{d}{dx}(x) + 0\right] - (x^3+3x+1)\left[2x^{2-1} - 0\right]}{(x^2-1)^2}$$

$$\left[\because \frac{d}{dx}(x^n) = nx^{n-1}, \ \frac{d}{dx}(1) = 0,\right]$$

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$$= \frac{(x^2-1)(3x^2+3) - (x^3+3x+1)(2x)}{(x^2-1)^2} \quad \left[\because \frac{d}{dx}(x) = 1\right]$$

$$= \frac{3x^4 + 3x^2 - 3x^2 - 3 - (2x^4 + 6x^2 + 2x)}{(x^2-1)^2}$$

$$= \frac{3x^4 + 3x^2 - 3x^2 - 3 - 2x^4 - 6x^2 - 2x}{(x^2-1)^2}$$

$$= \frac{x^4 - 6x^2 - 2x - 3}{(x^2-1)^2}$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{x^4 - 6x^2 - 2x - 3}{(x^2-1)^2}}$$

e) $y = \frac{e^x}{x+1}$

Here,

$$y = \frac{e^x}{x+1}$$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{e^x}{x+1}\right]$$

$$= \frac{(x+1)\frac{d}{dx}(e^x) - e^x \frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{(x+1)e^x - e^x\left[\frac{d}{dx}(x) + \frac{d}{dx}(1)\right]}{(x+1)^2} \quad \left[\because \frac{d}{dx}(e^x) = e^x\right]$$

$$= \frac{(x+1)e^x - e^x[1+0]}{(x+1)^2} \quad \left[\because \frac{d}{dx}(x) = 1, \frac{d}{dx}(1) = 0\right]$$

$$= \frac{(x+1)e^x - e^x}{(x+1)^2}$$

$$= \frac{e^x[x+1-1]}{(x+1)^2}$$

$$= \frac{e^x . x}{(x+1)^2}$$

$$= \frac{x . e^x}{(x+1)^2}$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{xe^x}{(x+1)^2}}$$

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f) $y = \frac{e^x}{\ln x}$

Given,

$$y = \frac{e^x}{\ln x}$$

Differentiating both sides with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{e^x}{\ln x}\right]$$

$$= \frac{\ln x . \frac{d}{dx}(e^x) - e^x . \frac{d}{dx}(\ln x)}{(\ln x)^2}$$

$$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v . \frac{d}{dx}(u) - u \frac{d}{dx}(v)}{v^2}\right]$$

$$= \frac{\ln x . e^x - e^x . \frac{1}{x}}{(\ln x)^2} \quad \left[\because \frac{d}{dx}(e^x) = e^x, \frac{d}{dx}(\ln x) = \frac{1}{x}\right]$$

$$= \frac{e^x\left(\ln x - \frac{1}{x}\right)}{(\ln x)^2}$$

$$= \frac{e^x\left(\frac{x\ln x - 1}{x}\right)}{(\ln x)^2}$$

$$= \frac{e^x(x\ln x - 1)}{x(\ln x)^2}$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{e^x(x\ln x - 1)}{x(\ln x)^2}}$$

4. Find the slope of a tangent to the following curves at the point mentioned.

a) $y = 3x^2 - 2x + 1$ at $x = 0$

Given,

$y = 3x^2 - 2x + 1$

Differentiating both sides with respect to x, we get,

$$\frac{d}{dx}(y) = \frac{d}{dx}(3x^2 - 2x + 1)$$

Or, $$\frac{dy}{dx} = \frac{d}{dx}(3x^2) - \frac{d}{dx}(2x) + \frac{d}{dx}(1)$$

$$= 3.\frac{d}{dx}(x^2) - 2.\frac{d}{dx}(x) + 0 \quad \left[\because \frac{d}{dx}(1) = 0\right]$$

$$= 3 \times 2x^{2-1} - 2 \times 1 \quad \left[\because \frac{d}{dx}(x^n) = nx^{n-1}, \frac{d}{dx}(x) = 1\right]$$

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$\therefore \frac{dy}{dx} = 6x - 2$ $\quad \therefore f'(x) = 6x - 2$

When $x = 0$,

$f'(0) = 6 \times 0 - 2 = -2$

$\therefore$ Slope of a tangent of the curve is -2.

b) $y = \frac{1}{8}x^3 - 2x$ at $x = 1$

Given,

$y = \frac{1}{8}x^3 - 2x$

Differentiating both sides with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{1}{8}x^3 - 2x\right]$$

Or, $f'(x) = \frac{1}{8} \cdot \frac{d}{dx}(x^3) - 2 \cdot \frac{d}{dx}(x)$

$= \frac{1}{8} \times 3x^2 - 2 \times 1$ $\left[\because \frac{d(x^n)}{dx} = nx^{n-1}, \frac{d(x)}{dx} = 1\right]$

$\therefore f'(x) = \frac{3x^2}{8} - 2$

When $x = 1$,

$f'(1) = \frac{3 \times 1^2}{8} - 2$

$= \frac{3}{8} - 2$

$= \frac{3 - 16}{8}$

$= -\frac{13}{8}$

$\therefore$ Slope of tangent of the curve is $-\frac{13}{8}$.

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c) $y = \frac{3}{4}x^4 - 7x$ at $x = 2$

Given,

$y = \frac{3}{4}x^4 - 7x$

Differentiating both sides with respect to x, we get,

$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{3}{4}x^4 - 7x\right]$

or, $f'(x) = \frac{3}{4} \cdot \frac{d}{dx}(x^4) - 7 \cdot \frac{d}{dx}(x)$

$= \frac{3}{4} \times 4x^3 - 7 \times 1 \quad \left[\because \frac{d(x^n)}{dx} = nx^{n-1}, \frac{d(x)}{dx} = 1\right]$

$\therefore f'(x) = 3x^3 - 7$

When $x = 2$,

$f'(2) = 3 \times (2)^3 - 7$

$= 3 \times 8 - 7$

$= 24 - 7$

$= 17$

$\therefore$ Slope of tangent of the curve is 17.

5. Show that the tangent of the following curves at the point mentioned are parallel to x-axis.

a) $y = 5x^2 - 4x + 1$ at $x = 2/5$

Given,

$y = 5x^2 - 4x + 1$

Differentiating both sides with respect to x, we get,

$\frac{dy}{dx} = \frac{d}{dx}\left[5x^2 - 4x + 1\right]$

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Or, $f'(x) = \frac{d}{dx}(5x^2) - \frac{d}{dx}(4x) + \frac{d}{dx}(1)$

$= 5.\frac{d}{dx}(x^2) - 4.\frac{d}{dx}(x) + 0 \left[\because \frac{d}{dx}(1) = 0\right]$

$= 5 \times 2x^{2-1} - 4 \times 1 \left[\because \frac{d}{dx}(x^n) = nx^{n-1}, \frac{d}{dx}(x) = 1\right]$

$\therefore f'(x) = 10x - 4$

When $x = 2/5$,

$f'(2/5) = 10 \times 2/5 - 4$

$= 4 - 4$

$= 0$

Since, the slope of the curve is 0. The tangent of the curve is parallel to x-axis.

b) $y = x^3 - 3x^2 + 7$ at $x = 0$ and $x = 2$

Given,

$y = x^3 - 3x^2 + 7$

Differentiating both sides with respect to x,

$\frac{dy}{dx} = \frac{d}{dx}(x^3 - 3x^2 + 7)$

or, $f'(x) = \frac{d}{dx}(x^3) - \frac{d}{dx}(3x^2) + \frac{d}{dx}(7)$

$= 3x^{3-1} - 3.\frac{d}{dx}(x^2) + 0 \left[\because \frac{d}{dx}(x^n) = nx^{n-1}, \frac{d}{dx}(7) = 0\right]$

$= 3x^2 - 3 \times 2x$

$\therefore f'(x) = 3x^2 - 6x$

When $x = 0$

$f'(0) = 3 \times (0)^2 - 6 \times 0$

$= 0$

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When $x=2$

$$f'(2) = 3\times(2)^2 - 6\times 2$$
$$= 12-12$$
$$=0$$

Since, $f'(0)$ & $f'(2)=0$. The tangent of the curve is parallel to x-axis.

6. a) Differentiate $2x+3$ with respect to $x+4$.

$$= \frac{d(2x+3)}{d(x+4)}$$

$$= \frac{\frac{d}{dx}(2x+3)}{\frac{d}{dx}(x+4)}$$

$$= \frac{\frac{d}{dx}(2x) + \frac{d}{dx}(3)}{\frac{d}{dx}(x) + \frac{d}{dx}(4)}$$

$$= \frac{2.\frac{d}{dx}(x) + 0}{\frac{d}{dx}(x) + 0} \quad \left[\because \frac{d}{dx}(3)=0,\ \frac{d}{dx}(4)=0\right]$$

$$= \frac{2\times 1+0}{1+0} \quad \left[\because \frac{d}{dx}(x)=1\right]$$

$$= 2$$

b) Differentiate x^2+5x+6 with respect to x^2-3x+2 at $x=1$.

$$= \left.\frac{d(x^2+5x+6)}{d(x^2-3x+2)}\right|_{x=1}$$

$$= \left.\frac{\frac{d}{dx}(x^2+5x+6)}{\frac{d}{dx}(x^2-3x+2)}\right|_{x=1}$$

$$= \left.\left[\frac{\frac{d}{dx}(x^2)+\frac{d}{dx}(5x)+\frac{d}{dx}(6)}{\frac{d}{dx}(x^2)-\frac{d}{dx}(3x)+\frac{d}{dx}(2)}\right]\right|_{x=1}$$

$$= \left.\left[\frac{2x+5+0}{2x-3+0}\right]\right|_{x=1}$$

$$= \left.\left[\frac{2x+5}{2x-3}\right]\right|_{x=1}$$

$$= \frac{2\times1+5}{2\times1-3}$$

$$= \frac{2+5}{2-3}$$

$$= -\frac{7}{1}$$

$$= -7$$

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7. Find $f'(x)$, $f''(x)$ and $f'''(x)$ when

a) $y = 3x^2 + 5x + 1$

Given,

$y = 3x^2 + 5x + 1$

Differentiating both sides with respect to x,

$\frac{dy}{dx} = \frac{d}{dx}(3x^2 + 5x + 1)$

i.e. $f'(x) = \frac{d}{dx}(3x^2) + \frac{d}{dx}(5x) + \frac{d}{dx}(1)$

$= 3.\frac{d}{dx}(x^2) + 5.\frac{d}{dx}(x) + 0 \quad [\because \frac{d}{dx}(1) = 0]$

$= 3 \times 2x^{2-1} + 5 \times 1 \quad [\because \frac{d}{dx}(x^n) = nx^{n-1}, \frac{d}{dx}(x) = 1]$

$\therefore f'(x) = 6x + 5$

Again, Differentiating both sides with respect to x,

$f''(x) = \frac{d}{dx}[f'(x)]$

$= \frac{d}{dx}(6x + 5)$

$= \frac{d}{dx}(6x) + \frac{d}{dx}(5)$

$= 6.\frac{d}{dx}(x) + 0 \quad [\because \frac{d}{dx}(5) = 0]$

$= 6 \times 1 \quad [\because \frac{d}{dx}(x) = 1]$

$\therefore f''(x) = 6$

Also, Differentiating both sides with respect to x,

$f'''(x) = \frac{d}{dx}[f''(x)]$

$= \frac{d}{dx}(6)$

$\therefore f'''(x) = 0 \quad [\because \frac{d}{dx}(6) = 0]$

b) $y = \frac{-3}{x} + \frac{1}{x^2} + \frac{2}{x^3}$

Given,

$$y = -\frac{3}{x} + \frac{1}{x^2} + \frac{2}{x^3}$$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{-3}{x} + \frac{1}{x^2} + \frac{2}{x^3}\right)$$

i.e. $f'(x) = \frac{d}{dx}(-3x^{-1} + x^{-2} + 2x^{-3})$

$$= \frac{d}{dx}(-3x^{-1}) + \frac{d}{dx}(x^{-2}) + \frac{d}{dx}(2x^{-3})$$

$$= -3\frac{d}{dx}(x^{-1}) + \frac{d}{dx}(x^{-2}) + 2\frac{d}{dx}(x^{-3})$$

$$= -3\times(-1)\,x^{-1-1} + (-2)\,x^{-2-1} + 2\times(-3)\,x^{-3-1}$$

$$\left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$$

$\therefore f'(x) = 3x^{-2} - 2x^{-3} - 6x^{-4}$

Again, Differentiating both sides with respect to x,

$$f''(x) = \frac{d}{dx}\left[f'(x)\right]$$

$$= \frac{d}{dx}(3x^{-2} - 2x^{-3} - 6x^{-4})$$

$$= \frac{d}{dx}(3x^{-2}) - \frac{d}{dx}(2x^{-3}) - \frac{d}{dx}(6x^{-4})$$

$$= 3.\frac{d}{dx}(x^{-2}) - 2\frac{d}{dx}(x^{-3}) - 6\frac{d}{dx}(x^{-4})$$

$$= 3\times(-2)\,x^{-2-1} - 2\times(-3)\,x^{-3-1} - 6\times(-4)\,x^{-4-1}$$

$\therefore f''(x) = -6x^{-3} + 6x^{-4} + 24x^{-5}$

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Also, Differentiating both sides with respect to x,

$$f'''(x) = \frac{d}{dx}[f''(x)]$$

$$= \frac{d}{dx}(-6x^{-3} + 6x^{-4} + 24x^{-5})$$

$$= \frac{d}{dx}(-6x^{-3}) + \frac{d}{dx}(6x^{-4}) + \frac{d}{dx}(24x^{-5})$$

$$= -6\frac{d}{dx}(x^{-3}) + 6\frac{d}{dx}(x^{-4}) + 24\frac{d}{dx}(x^{-5})$$

$$= -6\times(-3)x^{-3-1} + 6\times(-4)x^{-4-1} + 24\times(-5)x^{-5-1}$$

$$\therefore f'''(x) = 18x^{-4} - 24x^{-5} - 120x^{-6}$$

c) $y = \frac{1}{x+1}$

Given,

$$y = \frac{1}{x+1}$$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{x+1}\right)$$

i.e. $f'(x) = \frac{d}{dx}\left(\frac{1}{x+1}\right)$

$$= \frac{(x+1)\frac{d}{dx}(1) - 1.\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{v^2}\right]$$

$$= \frac{(x+1).0 - 1\left[\frac{d(x)}{dx} + \frac{d(1)}{dx}\right]}{(x+1)^2} \quad \left[\because \frac{d}{dx}(1) = 0\right]$$

$$= \frac{0 - 1(1+0)}{(x+1)^2} \quad \left[\because \frac{d}{dx}(x) = 1,\ \frac{d}{dx}(1) = 0\right]$$

$$\therefore f'(x) = -\frac{1}{(x+1)^2}$$

Again, Differentiating both sides with respect to x,

$$f''(x) = \frac{d}{dx}[f'(x)]$$

$$= \frac{d}{dx}\left[-1(x+1)^{-2}\right]$$

$$= \frac{d}{dx}\left[\frac{-1}{(x+1)^2}\right]$$

$$= \frac{(x+1)^2 \cdot \frac{d}{dx}(-1) - (-1) \cdot \frac{d}{dx}(x+1)^2}{[(x+1)^2]^2}$$

$$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{d}{dx}(u) - u \frac{d}{dx}(v)}{v^2}\right]$$

$$\cancel{(x+1)^2 \cdot 0 + 1 \cdot \left[\frac{d(x)}{dx} + \frac{d}{d}\right]}$$

$$= \frac{(x+1)^2 \cdot 0 + 1 \cdot 2(x+1)^{2-1}}{(x+1)^4} \quad \left[\because \frac{d}{dx}(-1) = 0,\ \frac{d}{dx}(x^n) = nx^{n-1}\right]$$

$$= \frac{2(x+1)}{(x+1)^4}$$

$$\therefore f''(x) = \frac{2}{(x+1)^3}$$

Also, Differentiating both sides with

$$f'''(x) = \frac{d}{dx}[f''(x)]$$

$$= \frac{d}{dx}\left[\frac{2}{(x+1)^3}\right]$$

$$= \frac{(x+1)^3 \cdot \frac{d}{dx}(2) - 2 \cdot \frac{d}{dx}(x+1)^3}{[(x+1)^3]^2}$$

$$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}\right]$$

$$= \frac{(x+1)^3 \cdot 0 - 2 \cdot 3(x+1)^{3-1}}{(x+1)^6} \quad \left[\because \frac{d}{dx}(2) = 0, \frac{d}{dx}(x^n) = nx^{n-1}\right]$$

$$= \frac{0 - 6(x+1)^2}{(x+1)^6}$$

$$= \frac{-6(x+1)^2}{(x+1)^6}$$

$$\therefore f'''(x) = -\frac{6}{(x+1)^4}$$

d) $y = \log 2x$

Here,

$y = \log 2x$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}[\log 2x]$$

$$f'(x) = \frac{d(\log 2x)}{d(2x)} \times \frac{d(2x)}{dx}$$

$$= \frac{1}{2x} \times 2 . \frac{d(x)}{dx} \quad \left[\because \frac{d(\log x)}{dx} = \frac{1}{x}\right]$$

$$= \frac{1}{2x} \times 2 \quad \left[\because \frac{d(x)}{dx} = 1\right]$$

$$\therefore f'(x) = \frac{1}{x}$$

Again, Differentiating both sides with respect to x,

$$f''(x) = \frac{d}{dx}[f'(x)]$$

$$= \frac{d}{dx}\left[\frac{1}{x}\right]$$

$$= \frac{d}{dx}(x^{-1})$$

$$= -1\, x^{-1-1} \quad \left[\because \frac{d}{dx}(x^n) = n x^{n-1}\right]$$

$$\therefore f''(x) = -x^{-2}$$

Also, Differentiating both sides with respect to x,

$$f'''(x) = \frac{d}{dx}[f''(x)]$$

$$= \frac{d}{dx}[-1 . x^{-2}]$$

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$= -1 \cdot \frac{d}{dx}(x^{-2})$

$= -1 \times (-2)\, x^{-2-1} \quad \left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$

$= 2x^{-3}$

$\therefore f'''(x) = 2x^{-3}$

8. Find $\frac{dy}{dx}$ when,

a) $y = (2x-1)^3$

Given,

$y = (2x-1)^3$

Differentiating both sides with respect to x,

$\frac{dy}{dx} = \cancel{df}\, \frac{d(2x-1)^3}{dx}$

$= \frac{d(2x-1)^3}{d(2x-1)} \times \frac{d(2x-1)}{dx}$

$= \cancel{(2x}\ 3 \times (2x-1)^{3-1} \times \frac{d(2x-1)}{dx} \quad \left[\because \frac{d(x^n)}{dx} = nx^{n-1}\right]$

$= 3(2x-1)^2 \times \left[\frac{d(2x)}{dx} - \frac{d(1)}{dx}\right]$

$= 3(2x-1)^2 \times \left[2 \cdot \frac{d(x)}{dx} - 0\right] \quad \left[\because \frac{d}{dx}(1) = 0\right]$

$= 3(2x-1)^2 \times 2 \quad \left[\because \frac{d}{dx}(x) = 1\right]$

$= 6(2x-1)^2$

$$\boxed{\therefore \frac{d}{dx} = 6(2x-1)^2}$$

b) $y = (x^2 - 4x + 1)^5$

Given,

$y = (x^2 - 4x + 1)^5$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 - 4x + 1)^5$$

$$= \frac{d(x^2-4x+1)^5}{d(x^2-4x+1)} \times \frac{d(x^2-4x+1)}{dx}$$

$$= 5(x^2-4x+1)^{5-1} \times \frac{d(x^2-4x+1)}{dx} \left[\because \frac{d(x^n)}{dx} = nx^{n-1}\right]$$

$$= 5(x^2-4x+1)^4 \times \left[\frac{d(x^2)}{dx} - \frac{d(4x)}{dx} + \frac{d(1)}{dx}\right]$$

$$= 5(x^2-4x+1)^4 \times (2x - 4 + 0) \left[\because \frac{d(x^n)}{dx} = nx^{n-1}, \frac{d(4x)}{dx} = 4, \because \frac{d}{dx}(1) = 0\right]$$

$$= 5(x^2-4x+1)^4 \times (2x-4)$$

$$= 5(x^2-4x+1)^4 (2x-4)$$

$$\boxed{\therefore \frac{d}{dx} = 5(x^2-4x+1)^4 (2x-4)}$$

---

c) $y = \sqrt{2x^2 + 3x + 1}$

Given,

$y = \sqrt{2x^2 + 3x + 1}$

or, $y = (2x^2 + 3x + 1)^{1/2}$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}(2x^2 + 3x + 1)^{1/2}$$

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$$= \frac{d(2x^2+3x+1)^{1/2}}{d(2x^2+3x+1)} \times \frac{d(2x^2+3x+1)}{dx}$$

$$= \frac{1}{2}(2x^2+3x+1)^{\frac{1}{2}-1} \times \left[\frac{d}{dx}(2x^2) + \frac{d}{dx}(3x) + \frac{d}{dx}(1)\right] \left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$$

$$= \frac{1}{2}(2x^2+3x+1)^{-\frac{1}{2}} \times \left[2\frac{d}{dx}(x^2) + 3 \cdot \frac{d}{dx}(x) + 0\right] \left[\because \frac{d}{dx}(1) = 0\right]$$

$$= \frac{1}{2}(2x^2+3x+1)^{-\frac{1}{2}} \times \left[2 \times 2x^{2-1} + 3 \times 1\right] \left[\because \frac{d}{dx}(x^n) = nx^{n-1}, \frac{d}{dx}(x) = 1\right]$$

$$= \frac{1}{2}(2x^2+3x+1)^{-1/2}(4x+3)$$

$$= \frac{(4x+3)}{2(2x^2+3x+1)^{1/2}}$$

$$= \frac{(4x+3)}{2\sqrt{2x^2+3x+1}}$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{4x+3}{2\sqrt{2x^2+3x+1}}}$$

d) $y = \frac{1}{\sqrt[3]{(x^2-2x+1)}}$

Given,

$$y = \frac{1}{\sqrt[3]{(x^2-2x+1)}}$$

or, $y = \frac{1}{(x^2-2x+1)^{1/3}}$

or, $y = (x^2-2x+1)^{-1/3}$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}(x^2-2x+1)^{-1/3}$$

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$$= \frac{d(x^2-2x+1)^{-\frac{1}{3}}}{d(x^2-2x+1)} \times \frac{d(x^2-2x+1)}{dx}$$

$$= \cancel{-\frac{1}{3}}$$

$$= -\frac{1}{3}(x^2-2x+1)^{-\frac{1}{3}-1} \times \left[\frac{d(x^2)}{dx} - \frac{d(2x)}{dx} + \frac{d(1)}{dx}\right]$$

$$\left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$$

$$= -\frac{1}{3}(x^2-2x+1)^{\cancel{\frac{-1-3}{3}}\,-\frac{4}{3}} \times \left[2x^{2-1} - 2 + 0\right] \quad \left[\because \frac{d(x^n)}{dx} = nx^{n-1}, \frac{d}{dx}(1) = 0\right]$$

$$= -\frac{1}{3}(x^2-2x+1)^{-\frac{4}{3}}(2x-2)$$

$$= -\frac{1}{3}(x^2-2x+1)^{-\frac{4}{3}} \cdot 2(x-1)$$

$$= -\frac{2}{3}(x^2-2x+1)^{-\frac{4}{3}}(x-1)$$

$$\boxed{\therefore \frac{dy}{dx} = -\frac{2}{3}(x^2-2x+1)^{-\frac{4}{3}}(x-1)}$$

(e) $y = \dfrac{1}{\sqrt{x+a} - \sqrt{x-b}}$

Given,

$$y = \frac{1}{\sqrt{x+a} - \sqrt{x-b}}$$

$$\text{or, } y = \frac{1}{\sqrt{x+a} - \sqrt{x-b}} \times \frac{\sqrt{x+a} + \sqrt{x-b}}{\sqrt{x+a} + \sqrt{x-b}}$$

$$\text{or, } y = \frac{\sqrt{x+a} + \sqrt{x-b}}{(\sqrt{x-a})^2 - (\sqrt{x-b})^2}$$

$$\text{or, } y = \frac{\sqrt{x+a} + \sqrt{x-b}}{x+a-x+b}$$

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or, $y = \dfrac{\sqrt{x+a} + \sqrt{x-b}}{a+b}$

Differentiating both sides with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{(x+a)^{1/2} + (x-b)^{1/2}}{a+b}\right]$$

$$= \frac{(a+b)\frac{d}{dx}\left[(x+a)^{1/2} + (x-b)^{1/2}\right] - \left[(x+a)^{1/2} + (x-b)^{1/2}\right]\frac{d}{dx}(a+b)}{(a+b)^2}$$

$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}\right]$

$$= \frac{(a+b)\left[\frac{d(x+a)^{1/2}}{dx} + \frac{d(x-b)^{1/2}}{dx}\right] - \left[(x+a)^{1/2} + (x-b)^{1/2}\right]\left[\frac{d(a)}{dx} + \frac{d(b)}{dx}\right]}{(a+b)^2}$$

$$= \frac{(a+b)\left[\frac{d(x+a)^{1/2}}{d(x+a)} \times \frac{d(x+a)}{dx} + \frac{d(x-b)^{1/2}}{d(x-b)} \times \frac{d(x-b)}{dx}\right] - 0}{(a+b)^2}$$

$\left[\because \frac{d}{dx}(a) = 0, \frac{d}{dx}(b) = 0\right]$

$$= \frac{(a+b)\left[\frac{1}{2}(x+a)^{1/2-1} \times \left\{\frac{d(x)}{dx} + \frac{d(a)}{dx}\right\} + \frac{1}{2}(x-b)^{1/2-1} \times \left\{\frac{d(x)}{dx} - \frac{d(b)}{dx}\right\}\right]}{(a+b)^2}$$

$$= \frac{\left[\frac{1}{2}(x+a)^{-1/2} \times \{1+0\}\right] + \left[\frac{1}{2}(x-b)^{-1/2} \times \{1-0\}\right]}{(a+b)}$$

$\left[\because \frac{d}{dx}(x) = 1, \frac{d}{dx}(a) = 0, \frac{d}{dx}(b) = 0\right]$

$$= \frac{\frac{1}{2}(x+a)^{-1/2} + \frac{1}{2}(x-b)^{-1/2}}{a+b}$$

$$= \frac{\frac{1}{2}\left[(x+a)^{-1/2} + (x-b)^{-1/2}\right]}{a+b}$$

$$= \frac{\left[(x+a)^{-1/2} + (x-b)^{-1/2}\right]}{2(a+b)}$$

$$= \frac{1}{2(a+b)}\left[\frac{1}{(x+a)^{1/2}} + \frac{1}{(x-b)^{1/2}}\right]$$

$$= \frac{1}{2(a+b)}\left[\frac{1}{\sqrt{x+a}} + \frac{1}{\sqrt{x-b}}\right]$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{1}{2(a+b)}\left[\frac{1}{\sqrt{x+a}} + \frac{1}{\sqrt{x-b}}\right]}$$

---

f) $y = \dfrac{1}{\sqrt{2x-3} - \sqrt{2x-5}}$

Given,

$$y = \frac{1}{\sqrt{2x-3} - \sqrt{2x-5}}$$

or, $y = \dfrac{1}{\sqrt{2x-3} - \sqrt{2x-5}} \times \dfrac{\sqrt{2x-3} + \sqrt{2x-5}}{\sqrt{2x-3} + \sqrt{2x-5}}$

or, $y = \dfrac{\sqrt{2x-3} + \sqrt{2x-5}}{(\sqrt{2x-3})^2 - (\sqrt{2x-5})^2}$

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or $y = \dfrac{\sqrt{2x-3} + \sqrt{2x-5}}{2x-3-2x+5}$

or, $y = \dfrac{\sqrt{2x-3} + \sqrt{2x-5}}{2}$

Differentiating both sides with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\sqrt{2x-3} + \sqrt{2x-5}}{2}\right]$$

$$= \frac{2 \times \frac{d}{dx}\left[(2x-3)^{1/2} + (2x-5)^{1/2}\right] - \left[(2x-3)^{1/2} + (2x-5)^{1/2}\right] \times \frac{d}{dx}(2)}{(2)^2}$$

$$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}\right]$$

$$= \frac{2 \times \left[\frac{d}{dx}(2x-3)^{1/2} + \frac{d}{dx}(2x-5)^{1/2}\right] - \left[(2x-3)^{1/2} + (2x-5)^{1/2}\right] \times 0}{4}$$

$$\left[\because \frac{d}{dx}(2) = 0\right]$$

$$= \frac{\frac{d}{dx}(2x-3)^{1/2} + \frac{d}{dx}(2x-5)^{1/2}}{2}$$

$$= \frac{\left[\frac{d(2x-3)^{1/2}}{d(2x-3)} \times \frac{d(2x-3)}{dx}\right] + \left[\frac{d(2x-5)^{1/2}}{d(2x-5)} \times \frac{d(2x-5)}{dx}\right]}{2}$$

$$= \frac{\left[\frac{1}{2}(2x-3)^{1/2-1} \times \left\{\frac{d(2x)}{dx} - \frac{d(3)}{dx}\right\}\right] + \left[\frac{1}{2}(2x-5)^{1/2-1} \times \left\{\frac{d(2x)}{dx} - \frac{d(5)}{dx}\right\}\right]}{2}$$

$$= \frac{\left[\frac{1}{2}(2x-3)^{-1/2} \times \{2-0\}\right] + \left[\frac{1}{2}(2x-5)^{-1/2} \times \{2-0\}\right]}{2}$$

$$\left[\because \frac{d(x)}{dx} = 1,\ \frac{d(3)}{dx} = 0,\ \frac{d(5)}{dx} = 0\right]$$

$$= \frac{(2x-3)^{-1/2} + (2x-5)^{-1/2}}{2}$$

$$= \frac{1}{2}\left[\frac{1}{(2x-3)^{1/2}} + \frac{1}{(2x-5)^{1/2}}\right]$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2x-3}} + \frac{1}{\sqrt{2x-5}}\right]$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{\sqrt{2x-3}} + \frac{1}{\sqrt{2x-5}}\right]}$$

---

g) $y = \frac{1}{x + \sqrt{a^2+x^2}}$

Given,

$$y = \frac{1}{x+\sqrt{a^2+x^2}}$$

$$\text{or, } y = \frac{1}{x+\sqrt{a^2+x^2}} \times \frac{x-\sqrt{a^2+x^2}}{x-\sqrt{a^2+x^2}}$$

$$\text{or, } y = \frac{x-\sqrt{a^2+x^2}}{(x)^2 - (\sqrt{a^2+x^2})^2}$$

or, $y = \dfrac{x - \sqrt{a^2 + x^2}}{x^2 - a^2 - x^2}$

$\therefore y = \dfrac{x - \sqrt{a^2 + x^2}}{-a^2}$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{x - \sqrt{a^2 + x^2}}{-a^2}\right]$$

$$= -\frac{1}{a^2}\frac{d}{dx}\left(x - \sqrt{a^2 + x^2}\right)$$

$$= -\frac{1}{a^2}\frac{d}{dx}\left[x - (a^2 + x^2)^{1/2}\right]$$

$$= -\frac{1}{a^2}\left[\frac{d}{dx}(x) - \frac{d(a^2 + x^2)^{1/2}}{dx}\right]$$

$$= -\frac{1}{a^2}\left[1 - \frac{d(a^2 + x^2)^{1/2}}{d(a^2 + x^2)} \times \frac{d(a^2 + x^2)}{dx}\right]$$

$$= -\frac{1}{a^2}\left[1 - \frac{1}{2}(a^2 + x^2)^{-1/2} \times \left\{\frac{d(a^2)}{dx} + \frac{d(x^2)}{dx}\right\}\right]$$

$$= -\frac{1}{a^2}\left[1 - \frac{1}{2}(a^2 + x^2)^{-1/2} \times \{0 + 2x\}\right]$$

$$= -\frac{1}{a^2}\left[1 - \frac{1}{\cancel{2}}(a^2 + x^2)^{-1/2} \times \cancel{2}x\right]$$

$$= -\frac{1}{a^2}\left[1 - x(a^2 + x^2)^{-1/2}\right]$$

$$= -\frac{1}{a^2}\left[1 - \frac{x}{\sqrt{a^2+x^2}}\right]$$

$$= -\frac{1}{a^2}\left[\frac{\sqrt{a^2+x^2} - x}{\sqrt{a^2+x^2}}\right]$$

$$\boxed{\therefore \frac{dy}{dx} = -\frac{1}{a^2}\left[\frac{\sqrt{a^2+x^2} - x}{\sqrt{a^2+x^2}}\right]}$$

---

h) $y = \frac{1}{\sqrt{3x^2-4x-1}}$

Given,

$$y = \frac{1}{\sqrt{3x^2-4x-1}}$$

or $y = \frac{1}{(3x^2-4x-1)^{1/2}}$

or $y = (3x^2-4x-1)^{-1/2}$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}(3x^2-4x-1)^{-1/2}$$

$$= \frac{d(3x^2-4x-1)^{-1/2}}{d(3x^2-4x-1)} \times \frac{d(3x^2-4x-1)}{dx}$$

$$= -\frac{1}{2}(3x^2-4x-1)^{-\frac{1}{2}-1} \times \left[\frac{d}{d(x)}\frac{d(3x^2)}{dx} - \frac{d(4x)}{dx} - \frac{d(1)}{dx}\right]$$

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$$= -\frac{1}{2}(3x^2-4x-1)^{-\frac{3}{2}} \times \left[3.\frac{d}{dx}(x^2) - 4.\frac{d}{dx}(x) - 0\right]$$

$$= -\frac{1}{2}(3x^2-4x-1)^{-\frac{3}{2}} (6x-4)$$

$$= -\frac{1}{2}(3x^2-4x-1)^{-\frac{3}{2}} 2(3x-2)$$

$$= -1 (3x^2-4x-1)^{-\frac{3}{2}} (3x-2)$$

$$= -\frac{(3x-2)}{(3x^2-4x-1)^{3/2}}$$

$$\boxed{\therefore \frac{dy}{dx} = -\frac{(3x-2)}{(3x^2-4x-1)^{3/2}}}$$

i) $y = \log(x^2+2x+5)$

Given,

$y = \log(x^2+2x+5)$

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Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}[\log(x^2+2x+5)]$$

$$= \frac{d[\log(x^2+2x+5)]}{\cancel{dx}\, d(x^2+2x+5)} \times \frac{d(x^2+2x+5)}{dx}$$

$$= \frac{1}{x^2+2x+5} \times \left[\frac{d}{dx}(x^2) + \frac{d}{dx}(2x) + \frac{d}{dx}(5)\right]$$

$$= \frac{1}{x^2+2x+5} \times [2x+2]$$

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$$= \frac{2x+2}{x^2+2x+5}$$

$$= \frac{2(x+1)}{x^2+2x+5}$$

$$\boxed{\therefore \frac{d}{dx} = \frac{2(x+1)}{x^2+2x+5}}$$

j) $y = \log[\sqrt{x-a} + \sqrt{x-b}]$

Given,

$$y = \log[\sqrt{x-a} + \sqrt{x-b}]$$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}[\log(\sqrt{x-a} + \sqrt{x-b})]$$

$$= \frac{d[\log(\sqrt{x-a} + \sqrt{x-b})]}{d(\sqrt{x-a} + \sqrt{x-b})} \times \frac{d(\sqrt{x-a} + \sqrt{x-b})}{dx}$$

$$= \frac{1}{(\sqrt{x-a} + \sqrt{x-b})} \times \left[\frac{d(\sqrt{x-a})}{dx} + \frac{d\sqrt{x-b}}{dx}\right]$$

$$= \frac{1}{(\sqrt{x-a} + \sqrt{x-b})} \times \left[\frac{d(x-a)^{1/2}}{d(x-a)} \times \frac{d(x-a)}{dx} + \frac{d(x-b)^{1/2}}{d(x-b)} \times \frac{d(x-b)}{dx}\right]$$

$$= \frac{1}{(\sqrt{x-a} + \sqrt{x-b})} \times \left[\frac{1}{2}(x-a)^{-1/2} \times \{1-0\} + \frac{1}{2}(x-b)^{-1/2} \times \{1-0\}\right]$$

$$= \frac{1}{(\sqrt{x-a} + \sqrt{x-b})}\left[\frac{1}{2\sqrt{x-a}} + \frac{1}{2\sqrt{x-b}}\right]$$

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$$= \frac{1}{(\sqrt{x-a} + \sqrt{x-b})} \left[\frac{1}{2\sqrt{x-a}} + \frac{1}{2\sqrt{x-b}}\right]$$

$$= \frac{1}{(\sqrt{x-a} + \sqrt{x-b})} \left[\frac{\sqrt{x-b} + \sqrt{x-a}}{(2\sqrt{x-a})(2\sqrt{x-b})}\right]$$

$$= \frac{1}{2(\sqrt{x-a}\sqrt{x-b})}$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x-a}\sqrt{x-b}}}$$

---

k) $y = \log(x + \sqrt{x^2+1})$

Given,

$$y = \log(x + \sqrt{x^2+1})$$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log(x + \sqrt{x^2+1})\right]$$

$$= \frac{d[\log(x+\sqrt{x^2+1})]}{d(x+\sqrt{x^2+1})} \times \frac{d(x+\sqrt{x^2+1})}{dx}$$

$$= \frac{1}{x+\sqrt{x^2+1}} \times \left[\frac{d(x)}{dx} + \left\{\frac{d}{dx}(x^2+1)^{1/2}\right\}\right]$$

$$= \frac{1}{x+\sqrt{x^2+1}} \times \left[1 + \left\{\frac{d(x^2+1)^{1/2}}{d(x^2+1)} \times \frac{d(x^2+1)}{dx}\right\}\right]$$

$$= \frac{1}{x+\sqrt{x^2+1}} \times \left[1 + \left\{\frac{1}{2}(x^2+1)^{-\frac{1}{2}} \times \left(\frac{d(x)^2}{dx} + \frac{d(1)}{dx}\right)\right\}\right]$$

$$= \frac{1}{x+\sqrt{x^2+1}}\left[1+\left\{\frac{1}{2}(x^2+1)^{-1/2}\times 2x\right\}\right]$$

$$= \frac{1}{x+\sqrt{x^2+1}}\left[1+\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}\right]$$

$$= \frac{1}{x+\sqrt{x^2+1}}\left[\frac{\cancel{2}\sqrt{x^2+1}+\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}\right]$$

$$= \frac{1}{\cancel{2}\sqrt{x^2+1}}$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}}}$$

6. Find $\frac{dy}{dx}$ when,

a) $x = 3at^2, y = at^3$

Given,

$$x = 3at^2$$

Differentiating both sides with respect to "t".

$$\frac{dx}{dt} = \frac{d}{dt}(3at^2)$$

$$= 3a.\frac{d}{dt}(t^2)$$

$$= 3a.2t^{2-1}$$

$$\therefore \frac{dx}{dt} = 6at$$

$$\therefore \frac{dt}{dx} = \frac{1}{6at}$$

Again,

$y = at^3$

Differentiating both sides with respect to "t";

$\frac{dy}{dt} = \frac{d}{dt}(at^3)$

$= a.\frac{d}{dt}(t^3)$

$= a.3t^2$

$\therefore \frac{dy}{dt} = 3at^2$

We know,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= 3at^2 \times \frac{1}{6at}$

$= \frac{t}{2}$

$\boxed{\therefore \frac{dy}{dx} = \frac{t}{2}}$

---

b) $ax = u-b$, $y = \frac{3}{4}u^2$

Given,

$ax = u-b$

or, $x = \frac{u-b}{a}$

Differentiating both sides with respect to "u".

$\frac{dx}{du} = \frac{d}{du}\left[\frac{u-b}{a}\right]$

$= \frac{1}{a}\left[\frac{d}{du}(u-b)\right]$

$$= \frac{1}{a}\left[\frac{d(u)}{du} - \frac{d}{du}(b)\right]$$

$$= \frac{1}{a}(1-0)$$

$$= \frac{1}{a}$$

$$\therefore \frac{dx}{du} = \frac{1}{a}$$

$$\therefore \frac{du}{dx} = a$$

Again,

$$y = \frac{3}{4}u^2$$

Differentiating both sides with respect to "u"

$$\frac{dy}{du} = \frac{d}{du}\left[\frac{3}{4}u^2\right]$$

$$= \frac{3}{4}\frac{d}{du}(u^2)$$

$$= \frac{3}{4} \times 2u$$

$$= \frac{6u}{4}$$

$$\therefore \frac{dy}{du} = \frac{3u}{2}$$

we know,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \frac{3u}{2} \times a$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{3au}{2}}$$

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c) $x = t^2 - 1, y = t^4 - 1$

Given,

$$x = t^2 - 1$$

Differentiating both sides with respect to "t".

$$\frac{dx}{dt} = \frac{d}{dt}[t^2 - 1]$$

$$= \frac{d}{dt}(t^2) - \frac{d}{dt}(1)$$

$$= 2t - 0$$

$$\therefore \frac{dx}{dt} = 2t$$

$$\therefore \frac{dt}{dx} = \frac{1}{2t}$$

Again,

$$y = t^4 - 1$$

Differentiating both sides with respect to "t".

$$\frac{dy}{dt} = \frac{d}{dt}[t^4 - 1]$$

$$= \frac{d}{dt}(t^4) - \frac{d}{dt}(1)$$

$$= 4t^3 - 0$$

$$\therefore \frac{dy}{dt} = 4t^3$$

we know,

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= 4t^3 \times \frac{1}{2t}$$

$$\boxed{\therefore \frac{dy}{dx} = 2t^2}$$

d) $x = z^2 + 2$, $y = z^3 + 2z + 1$

Given,

$x = z^2 + 2$

Differentiating both sides with respect to "z".

$$\frac{dx}{dz} = \frac{d}{dz}(z^2 + 2)$$

$$= \frac{d}{dz}(z^2) + \frac{d}{dz}(2)$$

$$= 2z + 0$$

$$\therefore \frac{dx}{dz} = 2z$$

$$\therefore \frac{dz}{dx} = \frac{1}{2z}$$

Again,

$y = z^3 + 2z + 1$

Differentiating both sides with respect to "z".

$$\frac{dy}{dz} = \frac{d}{dz}(z^3 + 2z + 1)$$

$$= \frac{d}{dz}(z^3) + 2 \cdot \frac{d}{dz}(z) + \frac{d}{dz}(1)$$

$$= 3z^2 + 2 + 0$$

$$\therefore \frac{dy}{dz} = 3z^2 + 2$$

We know,

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= (3z^2 + 2) \times \frac{1}{2z}$$

$$= \frac{3z^2 + 2}{2z}$$

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$$= \frac{3z^2}{2z} + \frac{2}{2z}$$

$$= \frac{3z^2}{2z} + \frac{1}{z}$$

$$\therefore \frac{dy}{dx} = \frac{3z^{\cancel{2}}}{2\cancel{z}} + \frac{1}{z}$$

$$= \frac{3z}{2} + \frac{1}{z}$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{3z}{2} + \frac{1}{z}}$$

e) $x = \frac{2a}{t}$, $y = \frac{a}{t^2}$

Given,

$$x = \frac{2a}{t}$$

or, $x = 2at^{-1}$

Differentiating both sides with respect to "t".

$$\frac{dx}{dt} = \frac{d}{dt}(2at^{-1})$$

$$= 2a.\frac{d}{dt}(t^{-1})$$

$$= 2a \times (-1)\, t^{-1-1}$$

$$\therefore \frac{dx}{dt} = -2at^{-2}$$

$$\therefore \frac{dt}{dx} = -\frac{1}{2at^{-2}}$$

Again,

$$y = \frac{a}{t^2}$$

Or, $y = at^{-2}$

Differentiating both sides with respect to "t".

$$\frac{dy}{dt} = \frac{d}{dt}(at^{-2})$$

$$= a.\frac{d}{dt}(t^{-2})$$

$$= a.(-2)t^{-2-1}$$

$$\therefore \frac{dy}{dt} = -2at^{-3}$$

we know;

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= -2at^{-3} \times -\frac{1}{2at^{-2}}$$

$$= t^{-1}$$

$$= \frac{1}{t}$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{1}{t}}$$

f) $x = \frac{3at}{1+t^2}$, $y = \frac{3at^2}{1+t^2}$

Given,

$$x = \frac{3at}{1+t^2}$$

Differentiating both sides with respect to "t",

$$\frac{dx}{dt} = \frac{d}{dt}\left[\frac{3at}{1+t^2}\right]$$

$$= 3a \cdot \frac{d}{dt}\left[\frac{t}{1+t^2}\right]$$

~~= 3a.~~

$$= 3a \times \frac{(1+t^2)\frac{d}{dt}(t) - t \cdot \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \quad \left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{d}{dx}(u) - u \cdot \frac{d}{dx}(v)}{v^2}\right]$$

$$= 3a \times \frac{(1+t^2) \times 1 - t\,[0+2t]}{(1+t^2)^2}$$

$$= 3a\left[\frac{1+t^2-2t^2}{(1+t^2)^2}\right]$$

$$\therefore \frac{dx}{dt} = \frac{3a(1-t^2)}{(1+t^2)^2}, \qquad \therefore \frac{dt}{dx} = \frac{(1+t^2)^2}{3a(1-t^2)}$$

Again,

$$y = \frac{3at^2}{1+t^2}$$

Differentiating both sides with respect to "t",

$$\frac{dy}{dt} = \frac{d}{dt}\left[\frac{3at^2}{1+t^2}\right]$$

$$= 3a \cdot \frac{d}{dt}\left[\frac{t^2}{1+t^2}\right]$$

$$= 3a \times \frac{(1+t^2)\frac{d}{dt}(t^2) - t^2 \cdot \frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$= 3a \times \frac{(1+t^2)\times 2t - t^2(0+2t)}{(1+t^2)^2}$$

$$= 3a \times \left[\frac{2t + 2t^3 - 2t^3}{(1+t^2)^2}\right]$$

$$\therefore \frac{dy}{dt} = \frac{6at}{(1+t^2)^2}$$

we know,

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{6at}{(1+t^2)^2} \times \frac{(1+t^2)^2}{3a(1-t^2)}$$

$$= \frac{2t}{1-t^2}$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{2t}{1-t^2}}$$

10. Find $\frac{dy}{dx}$ when,

a) $x^2+y^2=a^2$

Given,

$$x^2+y^2=a^2$$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(x^2+y^2)=\frac{d}{dx}(a^2)$$

or, $\frac{d(x^2)}{dx}+\frac{d(y^2)}{dx}=0$ $\left[\because \frac{d}{dx}(a^2)=0\right]$

or, $2x^{2-1}+\frac{d(y^2)}{dy}\times\frac{dy}{dx}=0$ $\left[\because \frac{d}{dx}(x^n)=nx^{n-1}\right]$

or, $2x+(2y^{2-1})\times\frac{dy}{dx}=0$

or, $2x+2y\times\frac{dy}{dx}=0$

or, $2y\times\frac{dy}{dx}=-2x$

or, $\frac{dy}{dx}=\frac{-2x}{2y}$

$$\boxed{\therefore \frac{dy}{dx}=-\frac{x}{y}}$$

b) $x+y = xy$

Given,

$x+y = xy$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(x+y) = \frac{d}{dx}(xy)$$

$$\text{or, } \cancel{\frac{d(x)}{dx} + \frac{d(y)}{dx} = x.\frac{d(y)}{dx} +}$$

$$\text{or, } \frac{d(x)}{dx} + \frac{d(y)}{dx} = x.\frac{d(y)}{dx} + y.\frac{d}{dx}(x)$$

$$\text{or, } 1 + \frac{d(y)}{dy} \times \frac{dy}{dx} = x.\frac{dy}{dx} + y \times 1$$

$$\text{or, } 1 + 1 \times \frac{dy}{dx} = x.\frac{dy}{dx} + y$$

$$\text{or, } \frac{dy}{dx} - x.\frac{dy}{dx} = y - 1$$

$$\text{or, } \frac{dy}{dx}(1-x) = y-1$$

$$\text{or, } \frac{dy}{dx} = \frac{y-1}{1-x}$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{y-1}{1-x}}$$

c) $x^2+y^2=xy$

Given,

$x^2+y^2=xy$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(x^2+y^2)=\frac{d}{dx}(xy)$$

or, $$\frac{d(x^2)}{dx}+\frac{d(y^2)}{dx}=x.\frac{d(y)}{dx}+y.\frac{d(x)}{dx}$$

or, $$2x+\left[\frac{d(y^2)}{dy}\times\frac{dy}{dx}\right]=\left[x\times\frac{dy}{dx}\right]+y\times 1$$

or, $$2x+\left(2y\times\frac{dy}{dx}\right)=\left(x\times\frac{dy}{dx}\right)+y$$

or, $$\left(2y\times\frac{dy}{dx}\right)-\left(x\times\frac{dy}{dx}\right)=y-2x$$

or, $$\frac{dy}{dx}(2y-x)=y-2x$$

or, $$\frac{dy}{dx}=\frac{y-2x}{2y-x}$$

$$\boxed{\therefore\ \frac{dy}{dx}=\frac{y-2x}{2y-x}}$$

d) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Given,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Differentiating both sides with respect to x,

$$\frac{d}{dx}\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = \frac{d}{dx}(1)$$

or, $\frac{d}{dx}\left(\frac{x^2}{a^2}\right) + \frac{d}{dx}\left(\frac{y^2}{b^2}\right) = 0 \quad \left[\because \frac{d}{dx}(1) = 0\right]$

or, $\frac{1}{a^2} \cdot \frac{d}{dx}(x^2) + \frac{1}{b^2} \frac{d}{dx}(y^2) = 0$

or, $\frac{1}{a^2} \cdot 2x + \frac{1}{b^2} \cdot 2y = 0$

or, $\frac{2x}{a^2} + \frac{2y}{b^2} = 0$

or, $\frac{b^2 2x + a^2 2y}{a^2 + b^2} = 0$

or, $\frac{1}{a^2} \cdot 2x + \frac{1}{b^2}\left[\frac{d(y^2)}{dy} \times \frac{dy}{dx}\right] = 0$

or, $\frac{2x}{a^2} + \frac{1}{b^2}\left[2y \times \frac{dy}{dx}\right] = 0$

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or $\frac{2y}{b^2} \times \frac{dy}{dx} = -\frac{2x}{a^2}$

or $\frac{dy}{dx} = -\frac{2x}{a^2} \times \frac{b^2}{2y}$

or $\frac{dy}{dx} = -\frac{b^2x}{a^2y}$

$$\boxed{\therefore \frac{dy}{dx} = -\frac{b^2x}{a^2y}}$$

e) $x^3 + y^3 = 3axy$

Given,

$x^3 + y^3 = 3axy$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(3axy)$$

or, $\frac{d(x^3)}{dx} + \frac{d(y^3)}{dx} = 3a \cdot \frac{d}{dx}(xy)$

or, $3x^2 + \frac{d(y^3)}{dy} \times \frac{dy}{dx} = 3a\left[x \cdot \frac{d(y)}{dx} + y \cdot \frac{d(x)}{dx}\right]$

or, $3x^2 + 3y^2 \times \frac{dy}{dx} = 3a\left[x \cdot \frac{d(y)}{dx} + y \times 1\right]$

or, $3x^2 + 3y^2 \times \frac{dy}{dx} = 3a\left[x \cdot \frac{dy}{dx} + y\right]$

or, $3x^2 + 3y^2 \times \frac{dy}{dx} = 3ax \cdot \frac{dy}{dx} + 3ay$

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or, $\left[3y^2 \times \frac{dy}{dx}\right] - \left[3ax \times \frac{dy}{dx}\right] = 3ay - 3x^2$

or, $\frac{dy}{dx}(3y^2 - 3ax) = 3ay - 3x^2$

or, $\frac{dy}{dx} = \frac{3ay - 3x^2}{3y^2 - 3ax}$

or, $\frac{dy}{dx} = \frac{3(ay - x^2)}{3(y^2 - ax)}$

or, $\frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$

$$\boxed{\therefore \frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}}$$

f) $x^2 + y^2 = 2xy$

Given,

$x^2 + y^2 = 2xy$

Differentiating both sides with respect to x,

$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(2xy)$

or, $\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 2 \cdot \frac{d}{dx}(xy)$

or, $2x + \left[\frac{d(y^2)}{dy} \times \frac{dy}{dx}\right] = 2\left[x \cdot \frac{dy}{dx} + y \cdot \frac{d(x)}{dx}\right]$

or, $2x + \left[2y \times \frac{dy}{dx}\right] = 2\left[x \cdot \frac{dy}{dx} + y \times 1\right]$

or, $2x + \left(2y \times \frac{dy}{dx}\right) = \left(2x \times \frac{dy}{dx}\right) + 2y$

or, $\left(2y \times \frac{dy}{dx}\right) - \left(2x \times \frac{dy}{dx}\right) = 2y - 2x$

or, $\frac{dy}{dx}(2y - 2x) = 2y - 2x$

or, $\frac{dy}{dx} = \frac{2y - 2x}{2y - 2x}$

$$\boxed{\therefore \frac{dy}{dx} = 1}$$

g) $x^2y^2 = x^2 + y^2$

Given,

$x^2y^2 = x^2 + y^2$

Differentiating both sides with respect to x,

$\frac{d}{dx}(x^2y^2) = \frac{d}{dx}(x^2 + y^2)$

or, $x^2 \cdot \frac{d(y^2)}{dx} + y^2 \cdot \frac{d(x^2)}{dx} = \frac{d(x^2)}{dx} + \frac{d(y^2)}{dx}$

or, $\left(x^2 \cdot \frac{d(y^2)}{dy} \times \frac{dy}{dx}\right) + y^2 \times 2x = 2x + \left[\frac{d(y^2)}{dy} \times \frac{dy}{dx}\right]$

or, $\left(x^2 \cdot 2y \times \frac{dy}{dx}\right) + 2xy^2 = 2x + \left[2y \times \frac{dy}{dx}\right]$

or, $\left(x^2 \cdot 2y \times \frac{dy}{dx}\right) - \left(2y \times \frac{dy}{dx}\right) = 2x - 2xy^2$

Or, $\frac{dy}{dx}(2x^2y - 2y) = 2x - 2xy^2$

Or, $\frac{dy}{dx} = \frac{2x - 2xy^2}{2x^2y - 2y}$

or, $\frac{dy}{dx} = \frac{2x(1 - y^2)}{2y(x^2 - 1)}$

$$\boxed{\therefore \frac{dy}{dx} = \frac{x(1-y^2)}{y(x^2-1)}}$$