Exercise 9(B)

1. The following are the annual sales of certain goods in thousand units.

Years	2010	2011	2012	2013	2014	2015	2016
No of goods in '000'	35	47	54	~~63~~ 62	71	88	96

a) Determine the equation of trend line by the method of least squares.

b) Estimate the number of sales of goods in 2017.

c) Draw a scatter diagram.

Solution

Since, the number of years n is 7. So, it is odd case.

Let the trend line equation be $y = a + bx$ --- (I)

where b and a are constants and can be calculated by the following formulae:

$$b = \frac{\sum x \sum y - n\sum xy}{(\sum x)^2 - n\sum x^2} \quad \text{—— (II)}$$

$$a = \frac{\sum y - b\sum x}{n} \quad \text{—— (III)}$$

CALCULATION TABLE

Year (X)	Sal. No of paddy (y)	$x = X -$ Midyear $x = X - 2013$	x^2	xy
2010	35	$2010 - 2013 = -3$	9	-105
2011	47	$2011 - 2013 = -2$	4	-94
2012	54	$2012 - 2013 = -1$	1	-54
2013	62	$2013 - 2013 = 0$	0	0
2014	71	$2014 - 2013 = 1$	1	71
2015	88	$2015 - 2013 = 2$	4	$\cancel{352}$ 176
2016	96	$2016 - 2013 = 3$	9	$\cancel{864}$ 288
$n = 7$	$\Sigma y = 453$	$\Sigma x = 0$	$\Sigma x^2 = 28$	$\Sigma xy = \cancel{1142}$ 282

Substituting the value of n, Σy, Σx, Σx^2, Σxy in equation (ii) & (iii) we get,

$$b = \frac{\Sigma x \Sigma y - n\Sigma xy}{(\Sigma x)^2 - n\Sigma x^2}$$

$$= \frac{0 \times 453 - 7 \times \cancel{1142}\ 282}{(0)^2 - 7 \times 28}$$

$$= \cancel{\frac{-7994}{-196}} \quad \frac{-1974}{-196}$$

$$= \cancel{40.78}\ 10.07$$

Also,

$$a = \frac{\Sigma y - b\Sigma x}{n}$$

$$= \frac{453 - \cancel{40.78 \times 0}\ 10.07 \times 0}{7}$$

$$= 64.71$$

Now, the trend line equation ① becomes,

$$y = a + bx$$

$$y = 64.71 + \cancel{40.78x}\ 10.07x$$

When $X = 2017$, then $x = X - 2013 = 2017 - 2013 = 4$

When $x = 4$, then,

$$\cancel{y = 64.71 + 40.78 \times 4}$$

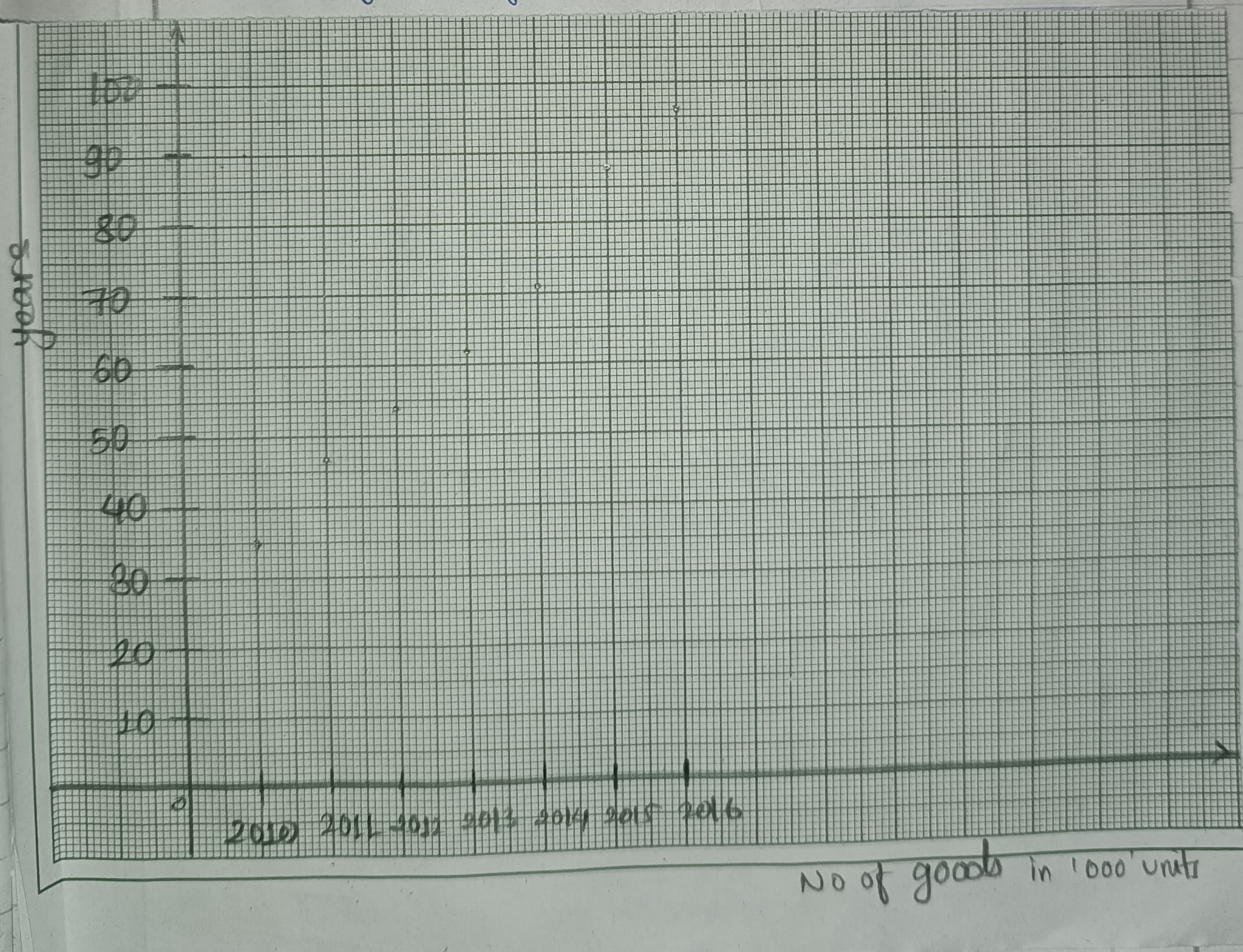
$$\cancel{\therefore y = 227.83}$$

$$y = 64.71 + 10.07 \times 4$$

$$\therefore y = 104.99$$

Therefore, the number of sales of goods in 2017 is 104.99 ('000 units)

Showing the information in a scatter diagram as:



2. The following are the annual sales of laptops in thousands unit:

Year	2009	2010	2011	2012	2013	2014	2015
No. of Laptops in '000'	90	95	88	100	105	102	110

a) Determine the equation of straight line by the method of least square.

b) Estimate the number of sales of laptops in 2017.

c) Draw a scatter diagram.

Solution

Since, the number of years n is 7. So, it is odd case.

Let the trend line equation be $y = a + bx$ --- (I)
where b and a are constants and can be calculated by the following formulae:

$$b = \frac{\Sigma x \Sigma y - n\Sigma xy}{(\Sigma x)^2 - n\Sigma x^2} \quad \text{(II)}$$

$$a = \frac{\Sigma y - b\Sigma x}{n} \quad \text{(III)}$$

CALCULATION TABLE

Year (X)	No of laptop (y)	$x = X -$ Mid year, $x = X - 2012$	x^2	xy
2009	90	2009−2012 = −3	9	−270
2010	95	2010 −2012 = −2	4	−190
2011	88	2011 − 2012 = −1	1	−88
2012	100	2012 −2012 = 0	0	0
2013	105	2013−2012 = 1	1	105
2014	102	2014−2012 = 2	4	204
2015	110	2015−2012 = 3	9	330
~~N = 7~~ n = 7	$\Sigma y = 690$	$\Sigma x = 0$	$\Sigma x^2 = 28$	$\Sigma xy = 91$

Substituting the values of n, Σy, Σx, Σx^2, Σxy in equations (II) & (III) we get,

$$b = \frac{\Sigma x \Sigma y - n \Sigma xy}{(\Sigma x)^2 - n \Sigma x^2}$$

$$= \frac{0 \times 690 - 7 \times 91}{(0)^2 - 7 \times 28}$$

$$= \frac{-637}{-196}$$

$$= 3.25$$

Also,

$$a = \frac{\Sigma y - b \Sigma x}{n}$$

$$= \frac{690 - 3.25 \times 0}{7}$$

$$= 98.57$$

Now, the trend line equation (I) becomes,

$$y = a + bx$$

$$\therefore y = 3.25x + 98.57$$

When $X = 2017$, $x = X - 2012 = 2017 - 2012 = 5$

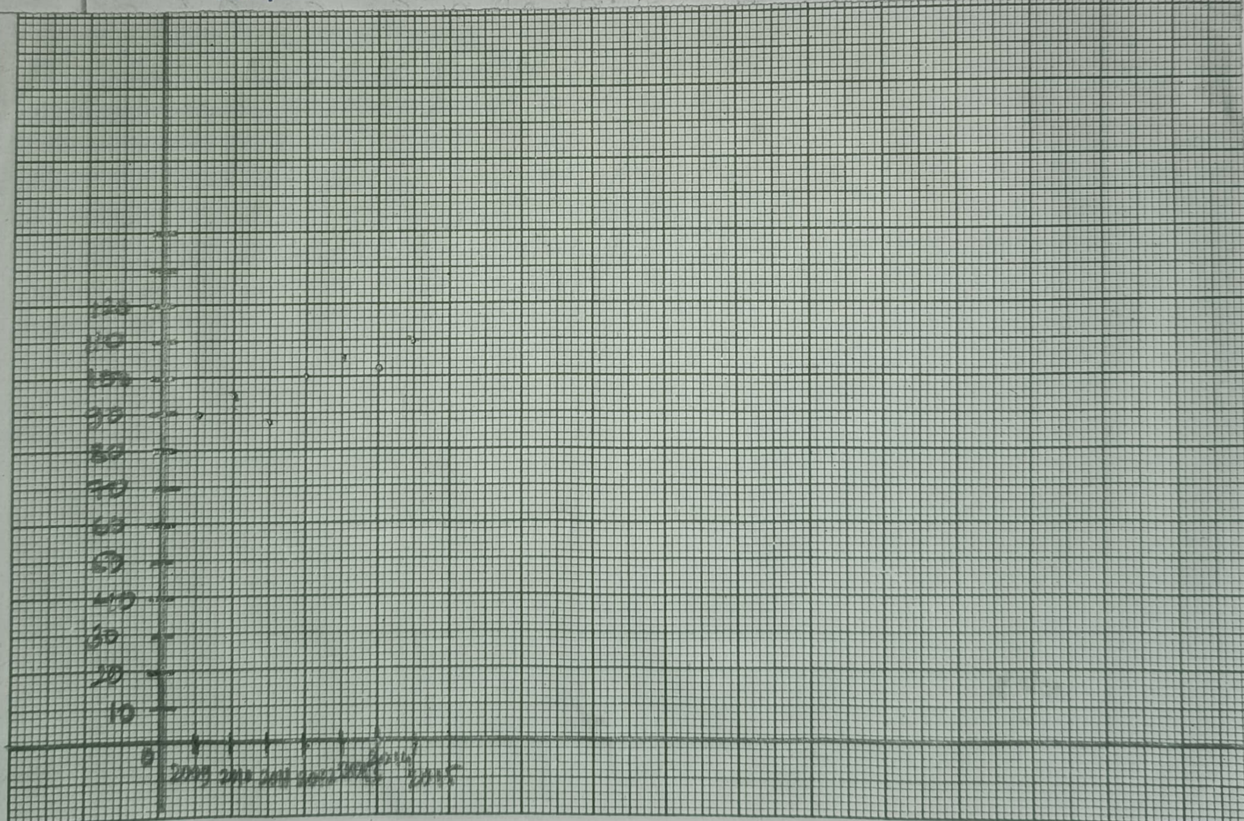
When $x = 5$, then

$y = 3.25 \times 5 + 98.57$

$\therefore y = 114.82$

Therefore the number of sales of laptops in 2017 is 114.82 (000's units)

Showing the information in a scatter diagram as:



3. Actual development expenditure on agriculture sector in Nepalese 7th development plan are as follows:

Year	Expenditure in agriculture sector (in million)
1985-1986	2100
1986-1987	2000
1987-1988	2300
1988-1989	3300
1989-1990	3000

Find the equation of line of best fit and estimate the expected expenditure in agriculture sector for the year 1990-91. Also, draw a scatter diagram

Solution

Since, the numbers of years n is 5. So, it is ~~called~~ odd case.

Let the trend line equation be $y = a + bx$ --(i)
where b and a are constants and can be calculated by the following formulae;

$$b = \frac{\Sigma x \Sigma y - n\Sigma xy}{(\Sigma x)^2 - n\Sigma x^2} \quad \text{------} \quad (ii)$$

$$a = \frac{\Sigma y - b\Sigma x}{n} \quad \text{------} \quad (iii)$$

CALCULATION TABLE

Year (X)	Expenditure (y)	$x = X - \text{Mid year}$ $x = X - 1987.5$	x^2	xy
1985.5	2100	~~1985.5~~ -2	4	-4200
1986.5	2000	-1	1	-2000
1987.5	2300	0	0	0
1988.5	3300	1	1	3300
1989.5	3000	2	4	6000
$n = 5$	$\Sigma y = 12700$	$\Sigma x = 0$	$\Sigma x^2 = 10$	$\Sigma xy = 3100$

Substituting the values of $n, \Sigma y, \Sigma x, \Sigma x^2$ & Σxy in equations (ii) & (iii), we get

$$b = \frac{\Sigma x \Sigma y - n \Sigma xy}{(\Sigma x)^2 - n \Sigma x^2}$$

$$= \frac{0 \times 12700 - 5 \times 3100}{(0)^2 - 5 \times 10}$$

$$= \frac{-15500}{-50}$$

$$= \cancel{-310}\ 310$$

Also,

$$a = \frac{\Sigma y - b \Sigma x}{n}$$

$$= \frac{12700 - 310 \times 0}{5}$$

$$= 2540$$

Now the trend line equation (i) becomes,

$$y = a + bx$$

$$\therefore y = 310x + 2540$$

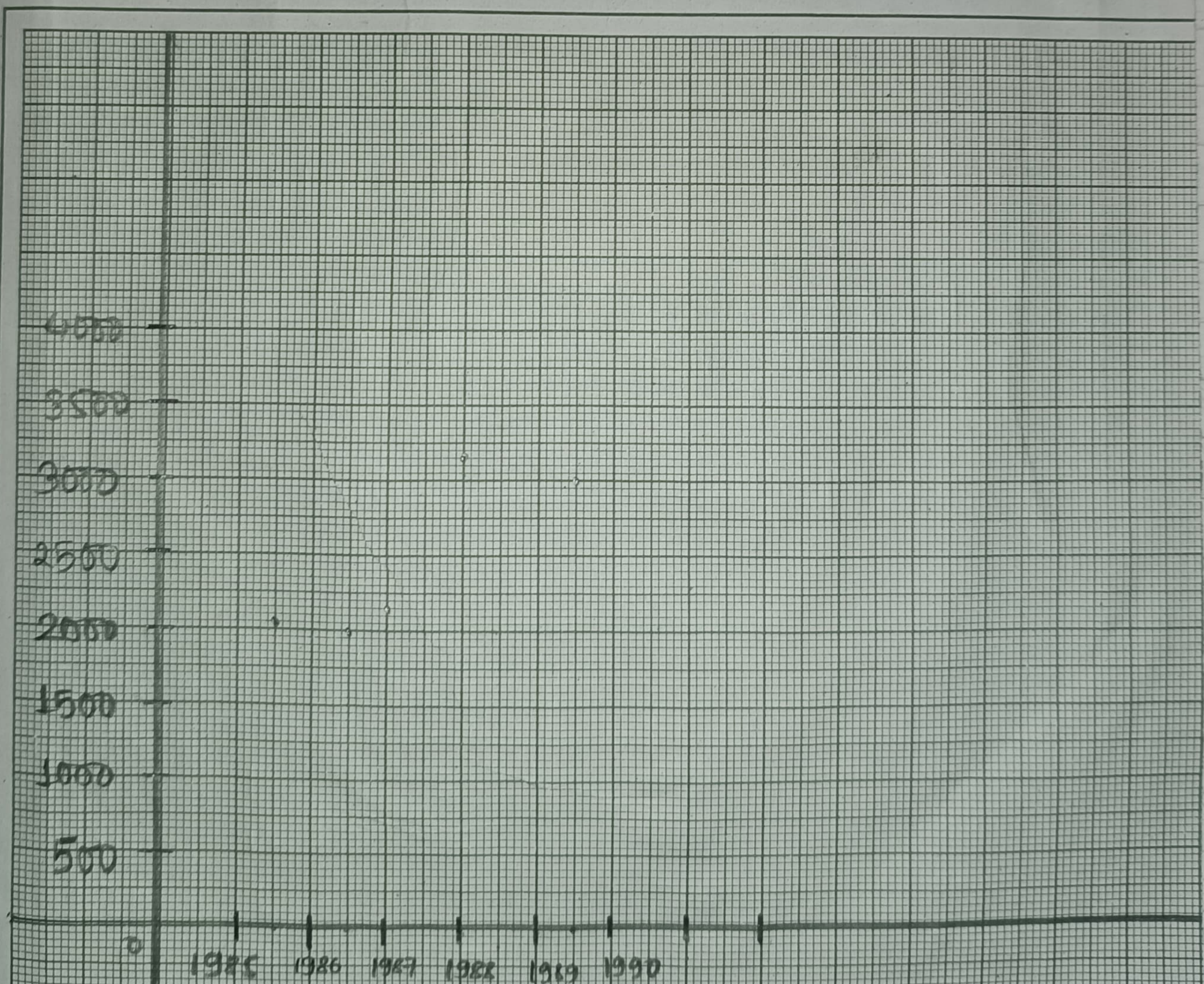
When $X = 1990.5$, $x = 1990.5 - 1987.5 = 3$

When $x = 3$, then,

$$y = 310 \times 3 + 2540$$
$$\therefore y = 3470$$

Therefore, the expected expenditure in agriculture sector for the year 1990-91 is Rs 3470 millions.

Showing the information in a scatter diagram as:



4. The following are the annual sales of computer in thousand units.

Year	2006	2007	2008	2009	2010	2011	2012
No. of computers	85	92	95	88	97	100	98

a) Determine the trend line by method of least squares.
b) Estimate the number of computers in 2014.
c) Draw a scatter diagram.

Solution
Since, the numbers of years n is 7. So, it is odd case.

Let the trend line equation be $y = a + bx$ --- (i)
where b and a are constants and can be calculated by ~~use~~ the following formulae:

$$b = \frac{\Sigma x \Sigma y - n \Sigma xy}{(\Sigma x)^2 - n\Sigma x^2} \text{ ——— (ii)}$$

$$a = \frac{\Sigma y - b\Sigma x}{n} \text{ ——— (iii)}$$

CALCULATION TABLE

Year (X)	No. of Computers (y)	$x = X -$ Mid Year $x = X - 2009$	x^2	xy
2006	85	-3	9	-255
2007	92	-2	4	-184
2008	95	-1	1	-95
2009	88	0	0	0
2010	97	1	1	97
2011	100	2	4	200
2012	98	3	9	294
$n = 7$	$\Sigma y = 655$	$\Sigma x = 0$	$\Sigma x^2 = 28$	$\Sigma xy = 57$

Substituting the values of $n, \Sigma y, \Sigma x, \Sigma x^2, \Sigma xy$ in equations (ii) & (iii), we get.

$$b = \frac{\Sigma x \Sigma y - n \Sigma xy}{(\Sigma x)^2 - n \Sigma x^2}$$

$$= \frac{0 \times 655 - 7 \times 57}{(0)^2 - 7 \times 28}$$

$$= \frac{-399}{-196}$$

$$= 2.03$$

Also,

$$a = \frac{\Sigma y - b \Sigma x}{n}$$

$$= \frac{655 - 2.03 \times 0}{7}$$

$$= 93.57$$

Now, the trend line equation (i) becomes,

$$y = a + bx$$

$$\therefore y = 93.57 + 2.03x$$

When $X = 2014$, $x = 2014 - 2009 = 5$

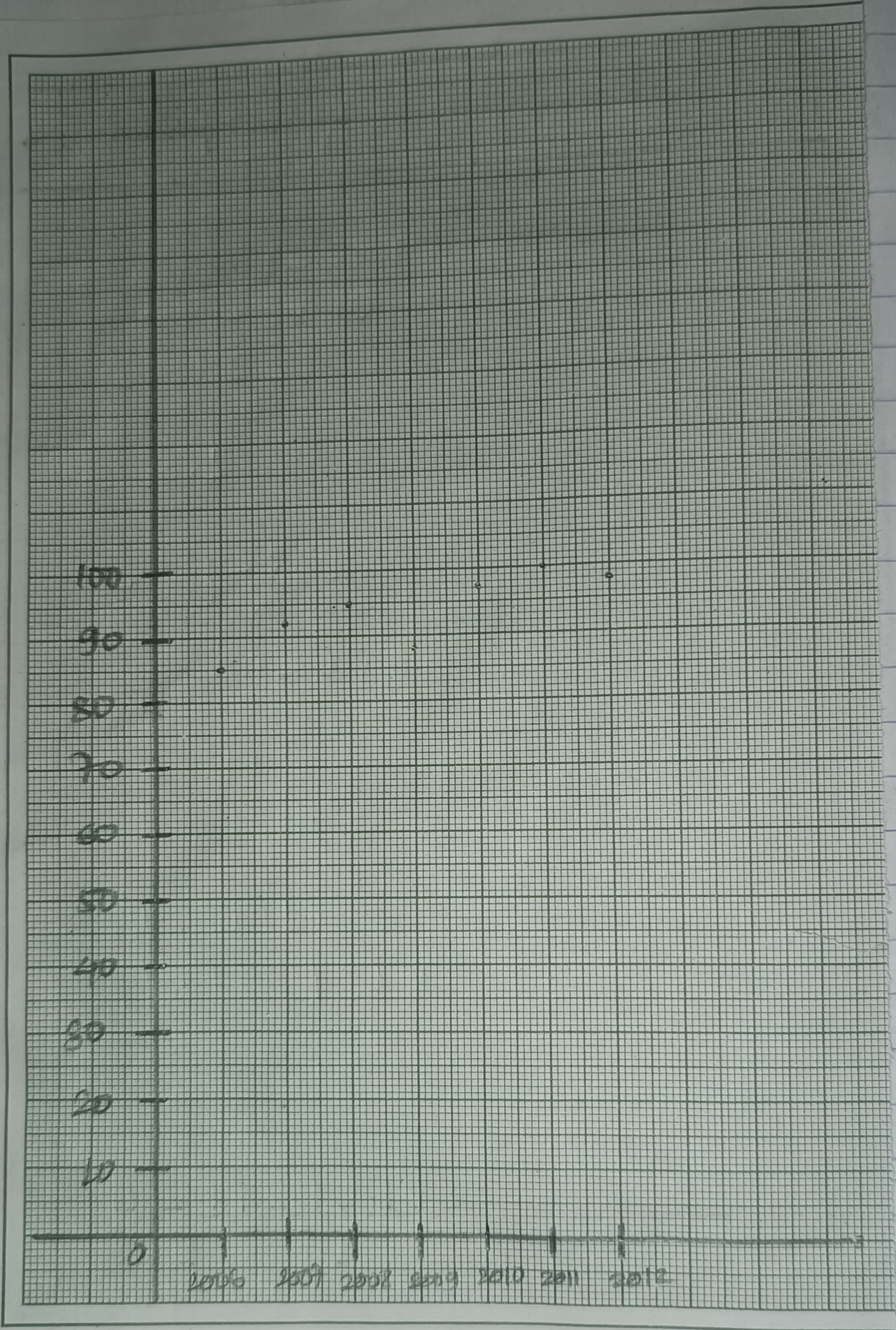
When $x = 5$,

$$y = 93.57 + 2.03 \times 5$$

$$\therefore y = 103.72$$

Therefore the estimated number of computers is 103.72 (000 units)

Scatter Diagram



5. The table presented below gives the number of salesmen working in a certain concern.

Year	2004	2005	2006	2007	2008
No. of salesman	12	15	17	20	25

a) Find the equation of the line using method of least square,

b) Estimate the number of the salesman in 2009.

c) Draw a scatter diagram.

Solution

Since the number of years n is 5. so, it is odd case.

Let the ~~line~~ trend line equation be $y = a + bx$ — (I)
where b and a are constants and can be calculated by the following formulae:

$$b = \frac{\Sigma x \Sigma y - n\Sigma xy}{(\Sigma x)^2 - n\Sigma x^2} \quad \text{———} \quad (II)$$

$$a = \frac{\Sigma y - b\Sigma x}{n} \quad \text{———} \quad (III)$$

CALCULATION TABLE

year (X)	No. of salesman (y)	$x = X -$ Mid. year $x = X - 2006$	x^2	xy
2004	12	-2	4	-24
2005	15	-1	1	-15
2006	17	0	0	0
2007	20	1	1	20
2008	25	2	4	50
n = 5	$\Sigma y = 89$	$\Sigma x = 0$	$\Sigma x^2 = 10$ ~~$\Sigma y = 10$~~	$\Sigma xy = 31$

Substituting the values of n, Σx, Σy, Σx^2 and Σxy in equations (ii) & (ii) we get,

$$b = \frac{\Sigma x \Sigma y - n \Sigma xy}{(\Sigma x)^2 - n \Sigma x^2}$$

$$= \frac{0 \times 89 - 5 \times 31}{(0)^2 - 5 \times 10}$$

$$= \frac{-155}{-50}$$

$$= 3.1$$

Also,

$$a = \frac{\Sigma y - b \Sigma x}{n}$$

$$= \frac{89 - 3.1 \times 0}{5}$$

$$= 17.8$$

Now, The trend line equation (i) becomes,

$$y = a + bx$$

$$\therefore y = 17.8 + 3.1x$$

When $X = 2009$, $x = 2009 - 2006 = 3$

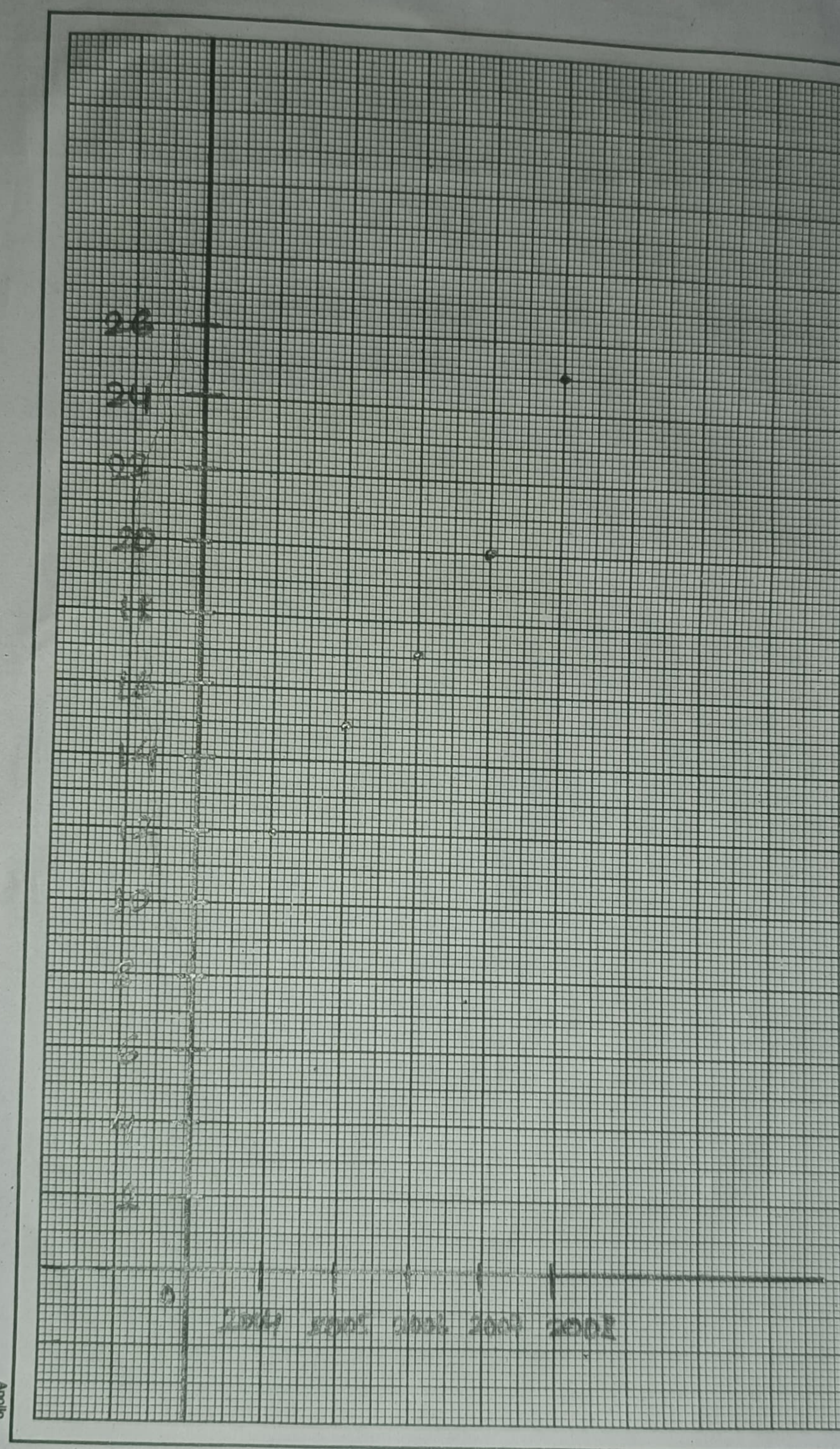
When $x = 3$,

$$y = 17.8 + 3.1 \times 3$$

$$\therefore y = 27.1$$

Therefore the number of salesmen in 2009 will be 27.1.

Scatter Diagram



6. The police department is studying the number of traffic fatalities in th country resulting from drunk driving for each of the last years.

Years	2008	2009	2010	2011	2012	2013	2014
Deaths	150	180	170	190	160	170	210

Determine the straight line by method of least squares. Also, estimate the number of traffic fatalities from drunk driving that the country can expect for 2015.
Also, draw a scatter diagram.

Solution
Since, the number of years n is 7. So, it is odd case.

Let the trend line equations be $y = a + bx$ ---(1)
where b and a are constants and can be calculated by the following formulae:

$$b = \frac{\Sigma x \Sigma y - n\Sigma xy}{(\Sigma x)^2 - n\Sigma x^2} \text{ ——— (II)}$$

$$a = \frac{\Sigma y - b\Sigma x}{n} \text{ ——— (III)}$$

CALCULATION TABLE

years (X)	No. of Death (y)	$x = X - \text{Mid Year}$ $x = X - 2011$	x^2	xy
2008	150	-3	9	-450
2009	180	-2	4	-360
2010	170	-1	1	-170
2011	190	0	0	0
2012	160	1	1	160
2013	170	2	4	340
2014	210	3	9	630
$n = 7$	$\Sigma y = 1230$	$\Sigma x = 0$	$\Sigma x^2 = 28$	$\Sigma xy = 150$

Substituting the values of n, Σy, Σx, Σx^2 & Σxy in equations (II) & (III) we get,

$$b = \frac{\Sigma x \Sigma y - n\Sigma xy}{(\Sigma x)^2 - n\Sigma x^2}$$

$$= \frac{0 \times 1230 - 7 \times 150}{(0)^2 - 7 \times 28}$$

$$= \frac{-1050}{-196}$$

$$= 5.35$$

Also,

$$a = \frac{\Sigma y - b\Sigma x}{n}$$

$$= \frac{1230 - 5.35 \times 0}{7}$$

$$= 175.71$$

Now the trend line equation (I) becomes,

$$y = a + bx$$

$$\therefore y = 175.71 + 5.35x$$

When $X = 2015$, $x = 2015 - 2011 = 4$

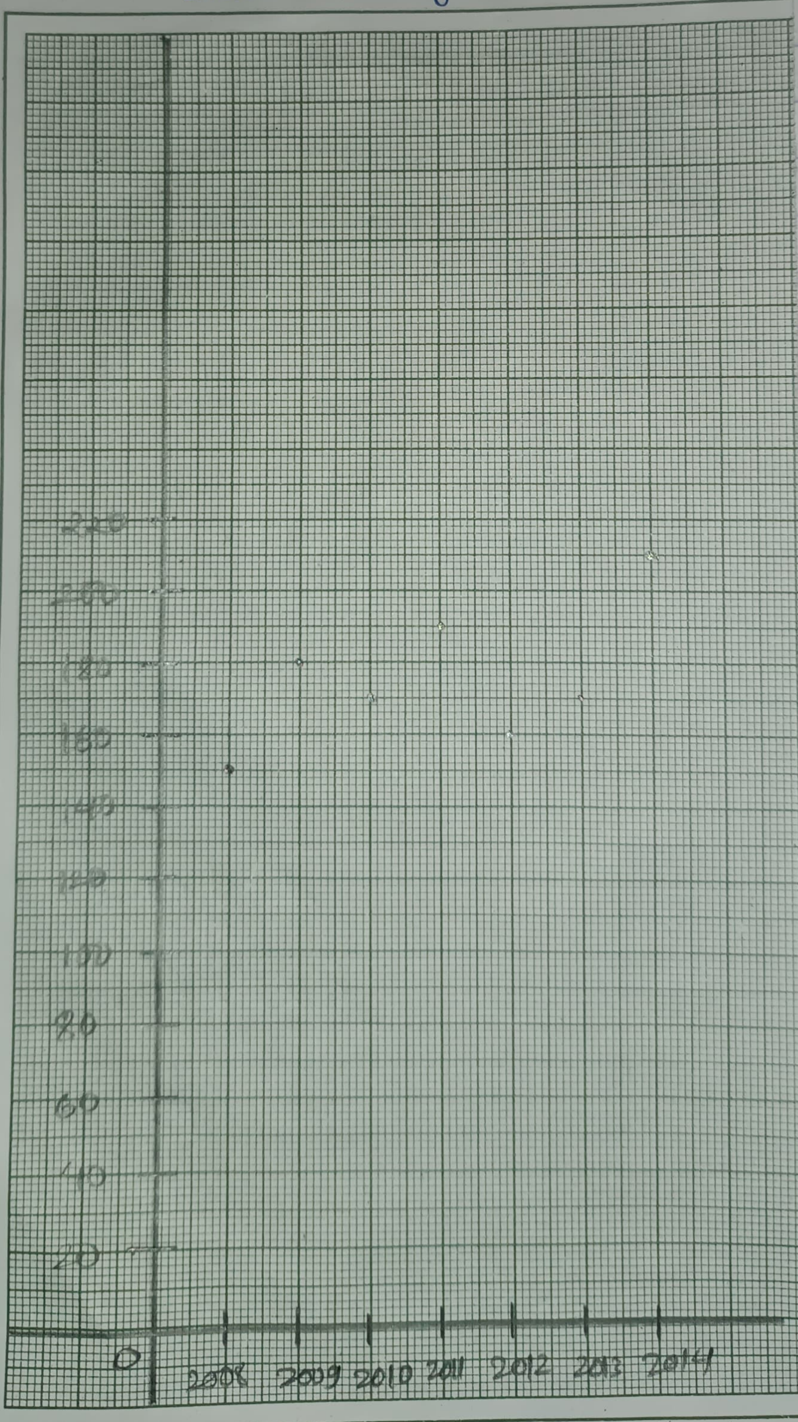
When $x = 4$,

$y = 175.71 + 5.35 \times 4$

$\therefore y = 197.11$

Therefore, the number of traffic fatalities from drunk driving that the country can expect for 2015 is 197.11 (lakhs)

Scatter Diagram



7. Given below are the figures of production in thousands of boxes of biscuits:

Year	1964	1965	1966	1967	1968	1969	1970
('000' boxes)	77	88	94	85	91	98	90

a) Fit a regression line of y on x by the method of least squares.

b) Estimate the production in 1975.

c) Draw a scatter diagram.

Solution

Since, the number of years n is 7. So, it is odd case.

Let the line of regression of y on x be $y = a + bx$ —— (I) where b and a are constants and can be calculated by the following formulae:

$$b = \frac{\Sigma x \Sigma y - n \Sigma xy}{(\Sigma x)^2 - n \Sigma x^2} \quad \text{—— (II)}$$

$$a = \frac{\Sigma y - b \Sigma x}{n} \quad \text{—— (III)}$$

CALCULATION TABLE

Year (X)	No. of boxes (y)	$x = X - \text{Mid year}$ $x = X - 1967$	x^2	xy
1964	77	-3	9	-231
1965	88	-2	4	-176
1966	94	-1	1	-94
1967	85	0	0	0
1968	91	1	1	91
1969	98	2	4	196
1970	90	3	9	270
$n = 7$	$\Sigma y = 623$	$\Sigma x = 0$	$\Sigma x^2 = 28$	$\Sigma xy = 56$

Substituting the value of n, Σy, Σx, Σx^2 & Σxy in equations (ii) & (iii) we get,

$$b = \frac{\Sigma x \Sigma y - n \Sigma xy}{(\Sigma x)^2 - n \Sigma x^2}$$

$$= \frac{0 \times 623 - 7 \times 56}{(0)^2 - 7 \times 28}$$

$$= \frac{-392}{-196}$$

$$= 2$$

Also,

$$a = \frac{\Sigma y - b \Sigma x}{n}$$

$$= \frac{623 - 2 \times 0}{7}$$

$$= 89$$

Now the regression line ① becomes,

$$y = a + bx$$

$$\therefore y = 89 + 2x$$

When $X = 1975$, $x = 1975 - 1967 = 8$

When $x = 8$,

$$y = 89 + 2 \times 8$$

$$\therefore y = 105$$

Therefore, the production will be 105 (000 boxes) in 1975.

<u>Scatter Diagram</u>

8. Fit a ~~so~~ straight line trend by using least square method for the following data:

Year	1975	1977	1978	1979	1980	1981	1984
Production (000quintal)	77	88	94	85	91	98	90

Solution

Since the number of years 'n' is 7, so it is odd case.

Let the trend line equation be $y = a + bx$ — (I)
Where b and a are constants and can be calculated by following formulae:

$$b = \frac{\Sigma x \Sigma y - n \Sigma xy}{(\Sigma x)^2 - n \Sigma x^2} \text{ ——— (II)}$$

$$a = \frac{\Sigma y - b \Sigma x}{n} \text{ ——— (III)}$$

CALCULATION TABLE

Year (X)	Production (y)	$x = X - \text{Mid}$ $x = X - 1979$	x^2	xy
1975	77	-4	16	-308
1977	88	-2	4	-176
1978	94	-1	1	-94
1979	85	0	0	0
1980	91	1	1	91
1981	98	2	4	196
1984	90	5	25	450
n=7	$\Sigma y = 623$	$\Sigma x = 1$	$\Sigma x^2 = 51$	$\Sigma xy = 159$

Substituting the value of n, Σx, Σy, Σx^2, Σxy in equations (ii) & (iii) we get,

$$b = \frac{\Sigma x \Sigma y - n \Sigma xy}{(\Sigma x)^2 - n \Sigma x^2}$$

$$= \frac{1 \times 623 - 7 \times 159}{(1)^2 - 7 \times 51}$$

$$= 1.37$$

$$a = \frac{\Sigma y - b \Sigma x}{n}$$

$$= \frac{623 - 1.37 \times 1}{7}$$

$$= 88.80$$

Now, The trend line equation becomes $y = a + bx$

$$y = 88.80 + 1.37x$$