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Exercise 5(D)

1. Find the general and particular solution for the following equations.

a) If $f'(x) = 2x^2+5x+6$ and $f(1) = 5$, find $f(x)$.

Solution

Given, $f'(x) = 2x^2+5x+6$

we know that,

$$f(x) = \int f'(x)\,dx$$

$$= \int (2x^2+5x+6)\,dx$$

$$= 2\int x^2\,dx + 5\int x\,dx + 6\int dx$$

$$= 2\times\frac{x^{2+1}}{2+1} + 5\times\frac{x^{1+1}}{1+1} + 6\times x + c$$

$$\left[\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + c,\ n\neq -1,\ \int dx = x+c\right]$$

$$\therefore f(x) = \frac{2x^3}{3} + \frac{5}{2}x^2 + 6x + c \text{ --- (1)}$$

when $x=1$,

$$f(1) = \frac{2}{3}\times(1)^3 + \frac{5}{2}\times(1)^2 + 6\times 1 + c$$

or, $5 = \frac{2}{3} + \frac{5}{2} + 6 + c \quad [\because f(1)=5]$

or, $5 = \frac{55}{6} + c$

$\therefore c = -\frac{25}{6}$

Now, (1) becomes,

$$f(x) = \frac{2}{3}x^3 + \frac{5}{2}x^2 + 6x - \frac{25}{6}$$

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b) If $f'(x) = x^3 + x^2 + x + 1$ and $f(1) = 2$, find $f(x)$.

Solution

Given, $f'(x) = x^3 + x^2 + x + 1$

We know that,

$$f(x) = \int f'(x)\,dx$$
$$= \int (x^3 + x^2 + x + 1)\,dx$$
$$= \int x^3 dx + \int x^2 dx + \int x\,dx + \int dx$$
$$= \frac{x^{3+1}}{3+1} + \frac{x^{2+1}}{2+1} + \frac{x^{1+1}}{1+1} + x + c$$

$$\therefore f(x) = \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + c \quad \text{—— (1)}$$

When $x = 1$,

$$f(1) = \frac{1^4}{4} + \frac{1^3}{3} + \frac{1^2}{2} + 1 + c$$

or, $2 = \frac{1}{4} + \frac{1}{3} + \frac{1}{2} + 1 + c$ $[\because f(1) = 2]$

or, $2 = \frac{25}{12} + c$

$\therefore c = -\frac{1}{12}$

Now, (1) becomes,

$$f(x) = \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x - \frac{1}{12}$$

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c) If $f'(x) = 2x+5$ and $f(0) = 12$, find $f(x)$.

Solution

Given, $f'(x) = 2x+5$

we know that,

$$f(x) = \int f'(x)\,dx$$
$$= \int (2x+5)\,dx$$
$$= 2\int x\,dx + 5\int dx$$
$$= 2 \times \frac{x^{1+1}}{1+1} + 5 \times x + c \quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right]$$
$$= 2 \times \frac{x^2}{2} + 5x + c$$
$$\therefore f(x) = x^2 + 5x + c \quad \text{---} \ (1)$$

when $x = 0$,

$$f(0) = 0^2 + 5 \times 0 + c$$
$$\text{or, } 12 = 0 + 0 + c \quad [\because f(0) = 12]$$
$$\therefore c = 12$$

Now, (1) becomes,

$$f(x) = x^2 + 5x + 12$$

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d) If $f'(x) = 4(2x^2+3)^2$ and $f(0) = 3$, find $f(x)$.

Solution

Given, $f'(x) = 4(2x^2+3)^2$

we know that,

$$f(x) = \int f'(x)\,dx$$
$$= \int 4(2x^2+3)^2\,dx$$
$$= 4\int (2x^2+3)^2\,dx$$

~~Put $2x^2+3 = t$~~

Differentiating both sides with respect to x,

$$\frac{d}{dx}(2x^2+3) = \frac{d}{dx}(t)$$

or, $4x+0 = \frac{dt}{dx}$

$\therefore 4x\,dx = dt$

$\therefore \cancel{x\,dx}$

Now,

$$\int 4(2x^2+3)^2 dx = \int \cancel{4x}(2x^2+3)^2 \cdot \frac{4x}{x}\,dx$$

$$= \int t^2 \cdot \frac{dt}{dx}$$

$$= 4\int\left[(2x^2)^2 + 2\cdot 2x^2\cdot 3 + 3^2\right]dx$$

$$= 4\int(4x^4+12x^2+9)\,dx$$

$$= 4\left(4\int x^4 dx + 12\int x^2 dx + 9\int dx\right)$$

$$= 4\left(4\times\frac{x^{4+1}}{4+1} + 12\times\frac{x^{2+1}}{2+1} + 9\times x\right) + c$$

$$= 4\left(\frac{4x^5}{5} + 4x^3 + 9x\right) + c$$

$$= \frac{16}{5}x^5 + 16x^3 + 36x + c$$

$\therefore f(x) = \frac{16}{5}x^5 + 16x^3 + 36x + c$ — (1)

when $x=0$,

$$f(0) = \frac{16}{5}\times(0)^5 + 16\times 0^3 + 36\times 0 + c$$

or, $3 = 0+0+0+c$

$\therefore c = 3$

Now, (1) becomes,

$\therefore f(x) = \frac{16}{5}x^5 + 16x^3 + 36x + 3$

2. If the marginal revenue functions for output Q is given by $MR = 3Q^2 + 2Q + 5$, show that the revenue function is $Q^3 + Q^2 + 5Q$.

Solution

Given, ~~MR~~ Marginal Revenue $= 3Q^2 + 2Q + 5$

i.e. $MR(Q) = 3Q^2 + 2Q + 5$

We know that,

$$R(Q) = \int MR(Q)\, dQ$$
$$= \int (3Q^2 + 2Q + 5)\, dQ$$
$$= 3\int Q^2\, dQ + 2\int Q\, dQ + 5\int dQ$$
$$= \frac{3 \times Q^{2+1}}{2+1} + \frac{2 \times Q^{1+1}}{1+1} + 5 \times Q + C$$

$\left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right] \left[\because \int dx = x + C\right]$

$\therefore R(Q) = Q^3 + Q^2 + 5Q + C$ ——— (1)

When $Q = 0$,

$R(0) = 0^3 + 0^2 + 5 \times 0 + C$

or, $0 = 0 + 0 + 0 + C$ $[\because R(0) = 0]$

$\therefore C = 0$

Now, (1) becomes,

$R(Q) = Q^3 + Q^2 + 5Q + 0$

i.e. $R(Q) = Q^3 + Q^2 + 5Q$ verified

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3\. a) If the marginal revenue function for output x is given by $MR = 3x^2 - 2x + 5$, find the total revenue function and find the demand function.

Solution

Given, $MR = 3x^2 - 2x + 5$

i.e. $MR(x) = 3x^2 - 2x + 5$

We know,

$$TR(x) = \int MR(x)\,dx$$
$$= \int (3x^2 - 2x + 5)\,dx$$
$$= 3\int x^2\,dx - 2\int x\,dx + 5\int dx$$
$$= 3 \times \frac{x^{2+1}}{2+1} - 2 \times \frac{x^{1+1}}{1+1} + 5 \times x + C$$

$$\therefore TR(x) = x^3 - x^2 + 5x + C \quad \text{--- (1)}$$

When $x = 0$,

$TR(0) = 0^3 - 0^2 + 5 \times 0 + C$

or, $0 = 0 - 0 + 0 + C \quad [\because TR(0) = 0]$

$\therefore C = 0$

Now, (1) becomes,

$TR(x) = x^3 - x^2 + 5x + 0$

i.e. $TR(x) = x^3 - x^2 + 5x$

Therefore the revenue function is $x^3 - x^2 + 5x$.

Again,

$$P(x) = \frac{TR(x)}{x}$$

$$\therefore P(x) = x^2 - x + 5$$

Therefore, the demand function is $x^2 - x + 5$.

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b) If for 2 units of production total revenue is 300, find the total revenue function given that $MR = 10 + 20x - 3x^2$. Also, find the demand function.

Solution

Given, Marginal Revenue $= 10 + 20x - 3x^2$

i.e. $MR(x) = 10 + 20x - 3x^2$

We know,

$$TR(x) = \int MR(x)\,dx$$
$$= \int (10 + 20x - 3x^2)\,dx$$
$$= 10\int dx + 20\int x\,dx - 3\int x^2\,dx$$
$$= 10.x + 20 \times \frac{x^{1+1}}{1+1} - 3 \times \frac{x^{2+1}}{2+1} + C$$

$$\left[\because \int dx = x + C,\ \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right]$$

$$\therefore TR(x) = 10x + 10x^2 - x^3 + C \quad \text{—— (1)}$$

when $x = 2$,

$TR(2) = 10 \times 2 + 10 \times 2^2 - 2^3 + C$

or, $300 = 20 + 40 - 8 + C$ $[\because TR(2) = 300]$

or, $C = 300 - \cancel{68}\ 52$

$\therefore C = 248$

Now, (1) becomes,

$TR(x) = 10x + 10x^2 - x^3 + 248$

Therefore, the total revenue function is $10x + 10x^2 - x^3 + 248$

Again,

$$P(x) = \frac{TR(x)}{x}$$

$$\therefore P(x) = 10 + 10x - x^2 + \frac{248}{x}$$

Therefore the demand function is $10 + 10x - x^2 + \frac{248}{x}$

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c) Suppose that the marginal revenue for a product is given by $MR = \frac{600}{\sqrt{3x+1}} + 2$. Find the total revenue function. Also, find the demand function.

Solution

Given, Marginal Revenue $= \frac{600}{\sqrt{3x+1}} + 2$

i.e. $MR(x) = \frac{600}{\sqrt{3x+1}} + 2$

we know;

$$TR(x) = \int MR(x)\,dx$$

$$= \int \left(\frac{600}{\sqrt{3x+1}} + 2 \right) dx$$

$$= \int \frac{600}{\sqrt{3x+1}}\,dx + \int 2\,dx$$

$$= 600 \int (3x+1)^{-1/2}\,dx + 2\int dx$$

$$= 600 \times \frac{(3x+1)^{-1/2+1}}{3(-\frac{1}{2}+1)} + 2 \times x + c$$

$\left[\because \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c,\ n \neq -1 \right]$

$\left[\because \int dx = x + c \right]$

$$= 600 \times \frac{(3x+1)^{1/2}}{\frac{3}{2}} + 2x + c$$

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$\therefore TR(x) = 400(3x+1)^{1/2} + 2x + C$ —— ①

when $x = 0$,

$TR(0) = 400\sqrt{3\times0+1} + 2\times0 + C$
or, $0 = 400\times1 + 0 + C$ $[\because TR(0) = 0]$
$\therefore C = -400$

Now, ① becomes
$TR(x) = 400\sqrt{3x+1} + 2x - 400$

Therefore, the total revenue function is $400\sqrt{3x+1} + 2x - 400$

Again,
$P(x) = \dfrac{TR(x)}{x}$

$\therefore P(x) = \dfrac{400\sqrt{3x+1}}{x} + 2 - \dfrac{400}{x}$

Therefore the demand function is $\dfrac{400\sqrt{3x+1}}{x} + 2 - \dfrac{400}{x}$

---

d) If the marginal revenue for a commodity is $MR = \dfrac{e^x}{100} + x + x^2$, find the revenue function.

Also, find the demand function.
Solution
Given, Marginal Revenue $= \dfrac{e^x}{100} + x + x^2$

i.e. $MR(x) = \dfrac{e^x}{100} + x + x^2$

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We know,

$$TR(x) = \int MR(x)\, dx$$

$$= \int \left(\frac{e^x}{100} + x + x^2\right) dx$$

$$= \int \frac{e^x}{100} dx + \int x\, dx + \int x^2 dx$$

$$= \frac{1}{100} \int e^x dx + \frac{x^{1+1}}{1+1} + \frac{x^{2+1}}{2+1} + c$$

$$\left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1\right]$$

$$= \frac{1}{100} \times e^x + \frac{x^2}{2} + \frac{x^3}{3} + c \quad \left[\because \int e^x dx = e^x + c\right]$$

$$\therefore TR(x) = \frac{e^x}{100} + \frac{x^2}{2} + \frac{x^3}{3} + c \text{ ——— } (1)$$

When $x = 0$,

$$TR(0) = \frac{e^0}{100} + \frac{0^2}{2} + \frac{0^3}{3} + c$$

$$\text{or, } 0 = \frac{1}{100} + 0 + 0 + c \quad [\because TR(0) = 0]$$

$$\therefore c = -\frac{1}{100}$$

Now, (1) becomes,

$$TR(x) = \frac{e^x}{100} + \frac{x^2}{2} + \frac{x^3}{3} - \frac{1}{100}$$

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Therefore, the revenue function is $\frac{e^x}{100}+\frac{x^2}{2}+\frac{x^3}{3}-\frac{1}{100}$.

Again,

Demand Function i.e. $P(x) = \frac{TR(x)}{x}$

$$= \frac{\frac{e^x}{100}+\frac{x^2}{2}+\frac{x^3}{3}-\frac{1}{100}}{x}$$

$$= \frac{e^x}{100x}+\frac{x}{2}+\frac{x^2}{3}-\frac{1}{100x}$$

Therefore, the demand function is $\frac{e^x}{100}+\frac{x}{2}+\frac{x^2}{3}-\frac{1}{100x}$

4. a) If the marginal cost of the product is given by $MC = 36-20x+6x^2$ and the initial cost is Rs. 20, find the cost and average cost function.

Solution

Given, Marginal cost $= 36-20x+6x^2$

i.e. $MC(x) = 36-20x+6x^2$

We know,

$$C(x) = \int MC(x)\,dx$$

$$= \int (36-20x+6x^2)\,dx$$

$$= \int 36\,dx - 20\int x\,dx + 6\int x^2\,dx$$

$$= 36\int dx - 20\int x\,dx + 6\int x^2\,dx$$

$$= 36\times x - 20\times\frac{x^{1+1}}{1+1} + 6\times\frac{x^{2+1}}{2+1} + K$$

$$\left[\because \int dx = x+c,\ \int x^n\,dx = \frac{x^{n+1}}{n+1}+c,\ n\neq -1\right]$$

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$\therefore C(x) = 36x - 10x^2 + 2x^3 + K$ — (1)

When $x = 0$,
$C(0) = 36\times0 - 10\times0^2 + 2\times0^3 + K$
or, $20 = 0 - 0 + 0 + K$ $[\because C(0) = 20]$
$\therefore K = 20$

Now, (1) becomes.

$$C(x) = 36x - 10x^2 + 2x^3 + 20$$

Again,
Average Cost $= \dfrac{\text{Total cost}}{\text{Total units}}$

i.e. $AC(x) = \dfrac{C(x)}{x}$

$= \dfrac{36x - 10x^2 + 2x^3 + 20}{x}$

$= 36 - 10x + 2x^2 + \dfrac{20}{x}$

Therefore the total cost function is $36x - 10x^2 + 2x^3 + 20$ and the average cost function is $36 - 10x + 2x^2 + \dfrac{20}{x}$.

b) The marginal cost function of a firm be $Q^2 + Q + 2$ where Q is the output. Find the total cost function and average cost function when the fixed cost is Rs. 50.

Solution
Given, Marginal cost $= Q^2 + Q + 2$

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i.e. $MC(Q) = Q^2 + Q + 2$

We know,

$C(Q) = \int MC(Q)\, dQ$

$= \int (Q^2 + Q + 2)\, dQ$

$= \int Q^2\, dQ + \int Q\, dQ + 2\int dQ$

$= \frac{Q^{2+1}}{2+1} + \frac{Q^{1+1}}{1+1} + 2 \times Q + K$

$\left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1\right] \left[\because \int dx = x + c\right]$

$\therefore C(Q) = \frac{Q^3}{3} + \frac{Q^2}{2} + 2Q + K$ —— (1)

When $Q = 0$,

$C(0) = \frac{0^3}{3} + \frac{0^2}{2} + 2 \times 0 + K$

or, $50 = 0 + 0 + 0 + K$ $[\because C(0) = 50]$

$\therefore K = 50$

Now, (1) becomes,

$C(Q) = \frac{Q^3}{3} + \frac{Q^2}{2} + 2Q + 50$

Again,

Average Cost $= \frac{\text{Total Cost}}{\text{Total Units}}$

i.e. $AC(Q) = \frac{C(Q)}{Q}$

$= \frac{\frac{Q^3}{3} + \frac{Q^2}{2} + 2Q + 50}{Q}$

$\therefore AC(Q) = \frac{Q^2}{3} + \frac{Q}{2} + 2 + \frac{50}{Q}$

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Therefore the total cost function is $\frac{Q^3}{3} + \frac{Q^2}{2} + 2Q + 50$
and the average cost function is $\frac{Q^2}{3} + \frac{Q}{2} + 2 + \frac{50}{Q}$.

c) The marginal cost function of manufacturing x units of a commodity in $3-2x-x^2$. If the fixed cost is 200, find the total cost and average cost functions.

Solution

Given, Marginal cost $= 3-2x-x^2$

i.e. $MC(x) = 3-2x-x^2$

we know,

$$TC(x) = \int MC(x)\,dx$$
$$= \int (3-2x-x^2)\,dx$$

$$= 3\int dx - 2\int x\,dx - \int x^2\,dx$$
$$= 3\times x - 2\times \frac{x^{1+1}}{1+1} - \frac{x^{2+1}}{2+1} + K$$

$$\left[\because \int dx = x + c,\ \int x^n dx = \frac{x^{n+1}}{n+c} + c,\ n \neq -1\right]$$

$$\therefore TC(x) = 3x - x^2 - \frac{x^3}{3} + K \quad \text{——— (1)}$$

when $x = 0$,

$$TC(0) = 3\times 0 - 0^2 - \frac{0^3}{3} + K$$

or, $200 = 0 - 0 - 0 + K \quad [\because TC(0) = 200]$

~~$\therefore K = 200$~~ $\therefore K = 200$

Now, (1) becomes,

$$TC(x) = 3x - x^2 - \frac{x^3}{3} + 200$$

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Again,

Average Cost = $\frac{\text{Total cost}}{\text{Total units}}$

i.e. $AC(x) = \frac{C(x)}{x}$

$= \frac{3x - x^2 - \frac{x^3}{3} + 200}{x}$

$\therefore AC(x) = 3 - x - \frac{x^2}{3} + \frac{200}{x}$

Therefore the total cost function is $3x - x^2 - \frac{x^3}{3} + 200$ and the average cost function is $3 - x - \frac{x^2}{3} + \frac{200}{x}$.

d) If the marginal cost for a product is $6x+4$ and the cost of producing 100 items is Rs 31,400. Find the total cost function.

Solution

Given, Marginal Cost $= 6x+4$

i.e. $MC(x) = 6x+4$

We know,

$C(x) = \int MC(x)\,dx$

$= \int (6x+4)\,dx$

$= \int 6x\,dx + \int 4\,dx$

$= 6\int x\,dx + 4\int dx$

$= 6 \times \frac{x^{1+1}}{1+1} + 4 \times x + k$

$\left[\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1\right]$

$\left[\because \int dx = x + c\right]$

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$\therefore C(x) = 3x^2 + 4x + K$ —— (1)

When $x = 100$,

$C(100) = 3\times(100)^2 + 4\times100 + K$

or, $31400 = 30000 + 400 + K$

~~or, 31400/30400 = K~~ or, $K = 31400 - 30400$

~~∴ K = 157/152~~ $\therefore K = 1000$

Now, (1) becomes

~~$C(x) = 3x^2 + 4x + \frac{157}{152}$~~ ~~1000~~

$C(x) = 3x^2 + 4x + 1000$

Therefore, the total cost function is $3x^2 + 4x + 1000$.

5. a) If the marginal revenue and the marginal cost for an output x of a commodity are given as $MR = 5 - 4x + 3x^2$ and $MC = 3 + 2x$ and if the fixed cost is zero, find the profit function and the profit when the output is $x = 4$.

Solution

Given, Marginal Revenue $=$ ~~$15 - 9x + 6x^2$~~ $5 - 4x + 3x^2$

i.e. $MR(x) =$ ~~$15 - 9x + 6x^2$~~ $5 - 4x + 3x^2$

Marginal Cost $= 3 + 2x$

We know,

~~$TR(x) = \int MR(x)\,dx$~~

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Marginal Profit = Marginal Revenue − Marginal Cost

i.e. $M[\pi(x)] = MR(x) - MC(x)$

$= 5 - 4x + 3x^2 - 3 - 2x$

$\therefore M[\pi(x)] = 2 - 6x + 3x^2$

Also, we know,

$\pi(x) = \int M[\pi(x)]\, dx$

$= \int (2 - 6x + 3x^2)\, dx$

$= 2\int dx - 6\int x\, dx + 3\int x^2\, dx$

$= 2x - 6 \times \frac{x^{1+1}}{1+1} + 3 \times \frac{x^{2+1}}{2+1} + C$

$\left[\because \int dx = x + C,\ \int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1\right]$

$\therefore \pi(x) = 2x - 3x^2 + x^3 + C$ —— (1)

when $x = 0$,

$\pi(0) = 2\times 0 - 3\times 0^2 + 0^3 + C$

or, $0 = 0 - 0 + 0 + C$ $[\because \pi(0) = 0]$

$\therefore C = 0$

Now, (1) becomes

$\pi(x) = 2x - 3x^2 + x^3 + 0$

i.e. $\pi(x) = 2x - 3x^2 + x^3$

Again, when $x = 4$

$\pi(4) = 2\times 4 - 3\times 4^2 + 4^3$

$= 8 - 48 + 64$

$\therefore \pi(4) = 24$

Therefore the profit function is $2x - 3x^2 + x^3$ and the profit is Rs 24 when the output is $x = 4$.

b) The marginal revenue function and the marginal cost function are given as $MR = 15-9x+6x^2$ and $MC = 10-24x-3x^2$, if the total cost of producing one unit is Rs. 25. Find the profit function. Also, find the total profit when $x=2$.

Solution

Given, Marginal Revenue $= 15-9x+6x^2$

Marginal Cost $= 10-24x-3x^2$

We know,

Total Cost $= \int MC(x)\,dx$

i.e. $C(x) = \int MC(x)\,dx$

$= \int (10-24x-3x^2)\,dx$

$= 10\int dx - 24\int x\,dx - 3\int x^2\,dx$

$= 10\times x - 24\times \frac{x^{1+1}}{1+1} - 3\times \frac{x^{2+1}}{2+1} + K$

$\left[\because \int dx = x+C,\ \int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n\neq -1\right]$

$\therefore C(x) = 10x - 12x^2 - x^3 + K$ — (1)

When $x=1$,

$C(1) = 10\times 1 - 12\times 1^2 - 1^3 + K$

or, $25 = 10-12-1+K$ $[\because C(1) = 25]$

or, $25 = -3+K$

$\therefore K = 28$

Now, (1) becomes,

$C(x) = 10x - 12x^2 - x^3 + 28$

Also,

$$R(x) = \int MR(x)\,dx$$
$$= \int (15 - 9x + 6x^2)\,dx$$
$$= 15\int dx - 9\int x\,dx + 6\int x^2\,dx$$
$$= 15 \times x - 9 \times \frac{x^{1+1}}{1+1} + 6 \times \frac{x^{2+1}}{2+1} + C$$
$$\therefore R(x) = 15x - \frac{9}{2}x^2 + 2x^3 + C \quad \text{— (A)}$$

When $x = 0$,

$$R(0) = 15 \times 0 - \frac{9}{2} \times 0^2 + 2 \times 0^3 + C$$
$$\text{or, } 0 = 0 - 0 + 0 + C \quad [\because R(0) = 0]$$
$$\therefore C = \cancel{20}\ 0$$

Now, (A) becomes

$$R(x) = 15x - \frac{9}{2}x^2 + 2x^3 + 0$$
$$\text{i.e. } R(x) = 15x - \frac{9}{2}x^2 + 2x^3$$

Now, we know,

Profit Function = Revenue Function − Cost Function

$$\text{i.e. } \pi(x) = R(x) - C(x)$$
$$= 15x - \frac{9}{2}x^2 + 2x^3 - 10x + 12x^2 + x^3 - 28$$

$$\cancel{\pi(x)} = 5x + \frac{15}{2}x^2 + 3x^3 - 28$$
$$\therefore \pi(x) = 3x^3 + \frac{15}{2}x^2 + 5x - 28$$

When $x = 2$,

$$\pi(2) = 3 \times (2)^3 + \frac{15}{2} \times (2)^2 + 5 \times 2 - 28$$
$$= 24 + 30 + 10 - 28$$
$$\therefore \pi(2) = 36$$

Therefore the profit function is $3x^3 + \frac{15}{2}x^2 + 5x - 28$ and the profit is Rs 36 at $x = 2$.

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Comprehensive Answer Questions

1. If the marginal revenue and marginal cost for an output x of a commodity are given by $MR = 5 - 4x + 3x^2$ and $MC = 3 + 2x$ and if the fixed cost is zero, find

a) Total revenue function
b) Demand function
c) Total cost function
d) Profit function
e) profit when output (x) is 4 units and 6 units.
f) Average cost at $x = 10$.

Solution

Given: Marginal Revenue $= 5 - 4x + 3x^2$.
Marginal cost $= 3 + 2x$

a) We know,

$TR(x) = \int MR(x)\,dx$

$= \int (5 - 4x + 3x^2)\,dx$

$= 5\int dx - 4\int x\,dx + 3\int x^2\,dx$

$= 5x - 4 \times \frac{x^{1+1}}{1+1} + 3 \times \frac{x^{2+1}}{2+1} + C$

$\left[\because \int dx = x + C, \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right]$

$\therefore TR(x) = 5x - 2x^2 + x^3 + C$ —— (1)

When $x = 0$,

$TR(0) = 5 \times 0 - 2 \times 0^2 + 0^3 + C$

or, $0 = 0 - 0 + 0 + C$ $[\because TR(0) = 0]$

$\therefore C = 0$

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Now, ① becomes

$TR(x) = 5x - 2x^2 + x^3 + 0$

i.e. $TR(x) = 5x - 2x^2 + x^3$

Therefore, the revenue function is $5x - 2x^2 + x^3$

b) We know,

Demand Function $= \dfrac{\text{Total Revenue}}{\text{Total Units}}$

i.e. $P(x) = \dfrac{TR(x)}{x}$

$= \dfrac{5x - 2x^2 + x^3}{x}$

$\therefore P(x) = 5 - 2x + x^2$

Therefore, the ~~re~~ demand function is $5 - 2x + x^2$

c) We know,

$TC(x) = \int MC(x)\,dx$

$= \int (3 + 2x)\,dx$

$= 3\int dx + 2\int x\,dx$

$= 3x + 2 \times \dfrac{x^{1+1}}{1+1} + K$

$\left[\because \int dx = x + c,\ \int x^n dx = \dfrac{x^{n+1}}{n+1} + c,\ n \neq -1\right]$

$\therefore TC(x) = 3x + x^2 + K$ —— Ⓐ

When $x = 0$,

$TC(0) = 3 \times 0 + 0^2 + K$

or, $0 = 0 + 0 + K$ $[\because TC(0) = 0]$

$\therefore K = 0$

Now, Ⓐ becomes,

$TC(x) = 3x + x^2 + 0$

i.e. $TC(x) = 3x + x^2$

Therefore the cost function is $3x + x^2$

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d) Profit Function = Revenue Function − Cost Function

i.e. $\pi(x) = TR(x) - TC(x)$

$\cancel{3x + x^2}$

$= 5x - 2x^2 + x^3 - 3x - x^2$

$\therefore \pi(x) = 2x - 3x^2 + x^3$

Therefore, the profit function is $2x - 3x^2 + x^3$.

e)

when $x = 4$,

$\pi(4) = 2\times4 - 3\times4^2 + 4^3$

$= 8 - 48 + 64$

$\therefore \pi(4) = 24$

When $x = 6$,

$\pi(6) = 2\times6 - 3\times6^2 + 6^3$

$= 12 - 108 + 216$

$\therefore \pi(6) = 120$

Therefore, the profit is Rs 24 when $x = 4$ units and the profit is Rs 120 at $x = 6$ units respectively.

f) When $x = 10$,

$TC(10) = 3\times10 + 10^2$

$= 30 + 100$

$\therefore TC(10) = 130$

$\therefore AC(10) = \frac{TC(10)}{10}$

$= \frac{130}{10}$

$\therefore AC(10) = 13$

Therefore, average cost is Rs 13 at $x = 10$.

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2. A company determines that the marginal cost of producing x units of a particular commodity during a one-day operation is $16x - 1591$, where the production cost ~~in~~ is in dollars. The selling price of commodity is fixed at dollars 9 per unit and the fixed cost is dollars 1800 per day. Find
a) the cost function
b) the revenue function
c) the profit function

Solution

Given, Marginal cost $= 16x - 1591$

i.e. $MC(x) = 16x - 1591$

a) We know,

$$C(x) = \int MC(x)\,dx$$
$$= \int (16x - 1591)\,dx$$
$$= 16\int x\,dx - 1591\int dx$$
$$= 16 \times \frac{x^{1+1}}{1+1} - 1591 \times x + K$$

$$\left[\because \int dx = x + C,\ \int x^n dx = \frac{x^{n+1}}{n+1} + C\right]$$

$$\therefore C(x) = 8x^2 - 1591x + K \quad \text{—— (1)}$$

when $x = 0$,

$C(0) = 8 \times 0^2 - 1591 \times 0 + K$

or, $1800 = 0 - 0 + K$ $[\because C(0) = 1800]$

$\therefore K = 1800$

Now, (1) becomes,

$C(x) = 8x^2 - 1591x + 1800$

Therefore the cost function is $8x^2 - 1591x + 1800$

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b) Also we know,

Total Revenue = Price x Quantity Sold

i.e. $R(x) = P \times Q$

$= 9 \times x$

$\therefore R(x) = 9x$

Therefore the revenue function is $9x$.

c) Profit Function = Revenue Function - Cost Function

i.e. $\pi(x) = R(x) - C(x)$

$= 9x - 8x^2 + 1591x - 1800$

$\therefore \pi(x) = -8x^2 + 1600x - 1800$

Exercise Questions

6. A television manufacturing company has developed service contract for the colour television it sells to customers. The rate of maintenance is estimated to be $\frac{dc}{dt}$ = Rs $40 + 24t$, where t is the number of years the contract remains in force.

a) Determine the amount ~~of~~ the company must charge per year for 12 years contract.

b) In how many years the total cost will be Rs. 5600.

Solution

Given, $\frac{dc}{dt} = 40 + 24t$

or, $c'(t) = 40 + 24t$

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a) We know,

$$C(t) = \int_0^{12} C'(t)\,dt$$

let $I = \int c'(t)\,dt$

$$= \int (40 + 24t)\,dt$$

$$= 40\int dt + 24\int t\,dt$$

$$= 40 \times t + 24 \times \frac{t^{1+1}}{1+1} \quad \left[\because \int dx = x + c,\ \int x^n dx = \frac{x^{n+1}}{n+1} + c\right]$$

$$= 40t + 12t^2$$

Now,

$$C(t) = \int_0^{12} \cancel{(40t + 12t^2)}\,(40 + 24t)\,dt$$

$$= (40t + 12t^2)\Big|_0^{12}$$

$$= (40 \times 12 + 12 \times 12^2) - (40 \times 0 + 12 \times 0^2)$$

$$= 480 + 1728 - 0$$

$$\therefore C(t) = 2208$$

Now,

~~Amou~~

Amount that the company must charge per year for 12 years contract $= \frac{C(t)}{12}$

$$= \frac{2208}{12}$$

$$= 184$$

Therefore the company must charge Rs 184 per year for 12 years contract.

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b) Let the number of years be t,

when $C(t) = 5,600$,

$$C(t) = \int_0^t C'(t)\,dt$$

or, $5600 = \int_0^t (40+24t)\,dt$

or, $5600 = 40t + 12t^2 \Big|_0^t$

or, $5600 = (40\times t + 12\times t^2) - (40\times 0 + 12\times 0^2)$

or, $5600 = 40t + 12t^2$

or, $12t^2 + 40t - 5600 = 0$

$\therefore t = 20, -23.33$ (Rejected)

$\therefore t = 20$

Therefore, the total cost will be Rs 5,600 in 20 years

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7. The rate of production of a book is given by $\frac{dy}{dt} = 200\left[1 + \frac{400}{(t+40)^2}\right]$, where y is the number of books and t is the number of weeks.

a) Assuming $y = 0$ when $t = 0$, find the equation that represents the total number of items produced as a function of time t.

b) How many books were produced in the five weeks?

Solution

Given, $\frac{dy}{dt} = 200\left[1 + \frac{400}{(t+40)^2}\right]$

or, $y'(t) = 200\left[1 + \frac{400}{(t+40)^2}\right]$

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a) We know,

$$y(t) = \int y'(t)\, dt$$

$$= \int 200\left[1 + \frac{400}{(t+40)^2}\right] dt$$

$$= 200\left[\int dt + 400 \int \frac{dt}{(t+40)^2}\right]$$

$$= 200\left[\int dt + 400 \int (t+40)^{-2} dt\right]$$

$$= 200\left[t + 400 \cdot \frac{(t+40)^{-2+1}}{1(-2+1)}\right] + K$$

$$\therefore y(t) = 200\left[t - \frac{400}{t+40}\right] + K \quad \text{——— (1)}$$

When $t=0$,

$$y(0) = 200\left[0 - \frac{400}{0+40}\right] + K$$

or, $0 = 200(0-10) + K \quad [\because y(0) = 0]$

or, $0 = -2000 + K$

$\therefore K = 2000$

Now, (1) becomes,

$$y(t) = 200\left[t - \frac{400}{t+40}\right] + 2000$$

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b) When $t=5$

$$y(5) = 200\left[5 - \frac{400}{45}\right] + 2000$$

$$= -\frac{7000}{9} + 2000$$

$$= 1222.22$$

Therefore 1222 books were produced in the five weeks.

8. If the marginal revenue function for output x is given by $\frac{6}{(x+2)^2} - 5$, show that the average revenue function is $\frac{3}{x+2} - 5$.

Solution

Given, Marginal Revenue $= \frac{6}{(x+2)^2} - 5$

i.e. $MR(x) = \frac{6}{(x+2)^2} - 5$

We know,

$$R(x) = \int MR(x)\, dx$$

$$= \int \left(\frac{6}{(x+2)^2} - 5\right) dx$$

$$= \int \frac{6}{(x+2)^2}\, dx - \int 5\, dx$$

$$= 6\int \frac{1}{(x+2)^2}\, dx - 5\int dx$$

$$= 6\int (x+2)^{-2}\, dx - 5\int dx$$

$$= 6 \times \frac{(x+2)^{-2+1}}{1(-2+1)} - 5x + C$$

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$$= -\frac{6}{(x+2)} - 5x + C$$

$$\therefore R(x) = -\frac{6}{x+2} - 5x + C \quad \text{—— (1)}$$

When $x = 0$,

$$R(0) = -\frac{6}{0+2} - 5\times0 + C$$

$$\text{or, } 0 = -\frac{6}{2} - 0 + C \quad [\because R(0) = 0]$$

$$\text{or, } 0 = -3 + C$$

$$\therefore C = 3$$

Now, (1) becomes,

$$R(x) = -\frac{6}{x+2} - 5x + 3$$

Now, we know,

$$AR(x) = \frac{R(x)}{x}$$

$$= \frac{-\frac{6}{x+2} - 5x + 3}{x}$$

$$= -\frac{6}{x(x+2)} - 5 + \frac{3}{x}$$

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$$= -\frac{6}{x(x+2)} + \frac{3}{x} - 5$$

$$= \frac{-6+3(x+2)}{x(x+2)} - 5$$

$$= \frac{-6+3x+6}{x(x+2)} - 5$$

$$= \frac{3x}{x(x+2)} - 5$$

$$= \frac{3}{x+2} - 5$$

$$\therefore AR(x) = \frac{3}{x+2} - 5$$ proved

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9. If the marginal revenue function is ~~MR(x) = mn~~ $MR(x) = \frac{mn}{(ax+b)^2} - c$, show that the revenue function and demand law are $\frac{mn}{a(ax+b)} - cx + \frac{mn}{ab}$ and $p = \frac{mn}{b(ax+b)} - c$ respectively.

Solution

Given,

$$MR(x) = \frac{mn}{(ax+b)^2} - c$$

We know,

$$R(x) = \int MR(x)\,dx$$

$$= \int \left(\frac{mn}{(ax+b)^2} - c\right) dx$$

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$$= \int \frac{mn}{(ax+b)^2}\,dx - \int c\,dx$$

$$= mn \int \frac{1}{(ax+b)^2}\,dx - c\int dx$$

$$= mn \int (ax+b)^{-2}\,dx - c\int dx$$

$$= mn \times \frac{(ax+b)^{-2+1}}{a(-2+1)} - c \times x + K$$

$$= mn \times \frac{1}{-a(ax+b)} - cx + K$$

$$\therefore R(x) = \frac{mn}{-a(ax+b)} - cx + K \quad \text{—— (1)}$$

When $x = 0$,

$$R(0) = \frac{mn}{-a(a\times 0+b)} - c\times 0 + K$$

$$\text{or } 0 = \frac{mn}{-ab} - 0 + K$$

$$\therefore K = \frac{mn}{ab}$$

Now, (1) becomes,

$$R(x) = \frac{mn}{-a(ax+b)} - cx + \frac{mn}{ab}$$

Therefore the revenue function is $\frac{mn}{-a(ax+b)} - cx + \frac{mn}{ab}$

Also,

Demand Function i.e. $P(x) = \frac{R(x)}{x}$

$$= \frac{-\frac{mn}{a(ax+b)} - Cx + \frac{mn}{ab}}{x}$$

$$= \frac{-\frac{mn}{a(ax+b)} + \frac{mn}{ab} - cx}{x}$$

$$= \frac{-\frac{mn}{a(ax+b)} + \frac{mn}{ab}}{x} - \frac{cx}{x}$$

$$= \frac{\frac{-mnb + mn(ax+b)}{ab(ax+b)}}{x} - c$$

$$= \frac{-mnb + mnax + mnb}{ab(ax+b)} \times \frac{1}{x} - c$$

$$= \frac{mnax}{ab(ax+b)} \times \frac{1}{x} - c$$

$$= \frac{mn}{b(ax+b)} - c$$ proved

10. Given $MR = \frac{101}{5+0.05x} - c$, rupees at x items per week. Find the increases in total revenue resulting from an increases in sales from $x=20$ to $x=40$ per week.

Solution

Given, $MR = \frac{101}{5+0.05x} - C$

We know,

$$TR(x) = \int MR(x)\,dx$$

$$= \int\left(\frac{101}{5+0.05x} - c\right)dx$$

$$= 101\int\frac{dx}{5+0.05x} - c\int dx$$

$$= 101 \times \frac{\ln(5+0.05x)}{0.05} - cx$$

$$\therefore TR(x) = 2020 \ln(5+0.05x) - cx \quad \text{—(1)}$$

When $x=0$,

$$TR(0) = 2020 \ln(5+0.05\times0) - c\times0$$

$\therefore$ ~~TR~~ or, 0

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Now,

~~we~~ from the question.

$$TR(x) = \int_{20}^{40} MR(x)\, dx$$

$$= 2020 \ln(5+0.05x) - cx \Big|_{20}^{40}$$

$$\cancel{= 2020 \ln(5+0.05\times 40) - c\times 40 - [2020 \ln}$$

$$= 2020 \left[\ln(5+0.05\times40) - c\times40 - \ln(5+0.05\times20) + c\times20\right]$$

$$= 2020 [\ln 7 - 40c - \ln 6 + 20c]$$

$$= 2020 [\ln 7 - \ln 6 - 20c]$$

$$= 2020 . \ln\left(\frac{7}{6}\right) - 20c$$