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Chapter 8: Matrices and Determinants.

Matrix

A rectangular array of numbers arranging into rows and columns and enclosed by two large brackets is called a matrix.

Matrices are denoted by capital letters whereas the elements of matrices are denoted by small letters.

For Example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Order of Matrix

The number of rows and the number of columns presented in a given matrix is called order of matrix or dimension of matrix.

Types of Matrices:

1) Row Matrix: A matrix having only one row is called a row matrix. For example: $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

2) Column Matrix: A matrix having only one column is called a column matrix.
For example:

$$A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

3) Square Matrix: A matrix in which the number of rows is equal to the number of columns is called square matrix.
For example:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

4) Diagonal Matrix: A matrix in which all the elements except the principal diagonal are zero, is called a diagonal matrix.
For example:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

5) Scalar Matrix: A diagonal matrix, in which all the principal diagonal elements are equal, is called scalar matrix.
For example:

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

6) Identity Matrix: A scalar matrix in which all the principal diagonal elements are equal to 1 is called identity matrix.
For example:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

7) Null Matrix: A matrix in which all the elements are zero is called a null matrix.

For example:

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

8) Triangular Matrix:

i) Upper Triangular Matrix: A square matrix is an upper triangular matrix if all the elements below the principal diagonal are zero.

For example:

$$A = \begin{bmatrix} 7 & 8 & 9 \\ 0 & 7 & 2 \\ 0 & 0 & 7 \end{bmatrix}$$

ii) Lower Triangular Matrix: A square matrix is said to be a lower triangular matrix if all the elements above the principal diagonal are zero.

For example:

$$A = \begin{bmatrix} 7 & 0 & 0 \\ 3 & 7 & 0 \\ 9 & 8 & 7 \end{bmatrix}$$

9) ~~Symmetrix~~ Symmetric Matrix: A square matrix $A = a_{ij}$ is said to be symmetric if $a_{ij} = a_{ji}$ for all i and j.

Example: $A = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

~~10)~~

10) Skew-symmetric Matrix: A square matrix $A = (a_{ij})$ is said to be an skew symmetric matrix if $a_{ij} = -a_{ji}$ for all i and j.

Example: $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & -7 & 5 \\ 7 & 0 & 2 \\ -5 & -2 & 0 \end{bmatrix}$

11) Sub-Matrix: A new matrix obtained by eliminating any number of rows ~~and~~ or columns or both from a given matrix is called a sub-matrix of that given matrix.

For example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$$

Here, B is the sub matrix of matrix A.

12) Equal Matrices: Two matrices having same order and corresponding elements equal are called equal matrix.

For example:

$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

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Exercise 8(A)

1. i) What do you mean by a matrix? What are the criteria for the validity of addition and subtraction of two matrices?

⇒ A rectangular Array of numbers arranging into rows and columns and enclosed by two large brackets is called a matrix.

For the addition and subtraction of two matrix, they must have same order.

ii) What is the condition for conformability of multiplication of two matrices?

⇒ Two matrices A and B are conformable for multiplication if the number of columns of A matrix and the number of rows of B matrix are equal.

iii) Define transpose of a matrix.

⇒ A new matrix obtained by interchanging the rows into columns and columns into row is called the transpose of matrix. It is denoted by A' or A^T.

For example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

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2. Write down a matrix of each of the following sizes.

a) 2 x 3

Let A be the 2x3 matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

For example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

b) 3x2

Let B be the 3x2 matrix.

$$B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

For example:

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

c) 1x6

Let A be the 1x6 matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix}$$

For example:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

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d) 2X1

let A be the 2X1 matrix.

$$A = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}$$

For example:

$$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

~~3. If~~

3. If $A = \begin{bmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \\ -7 & -8 & -9 \end{bmatrix}$, find $a_{11}, a_{13}, a_{22}, a_{23}, a_{31}, a_{33}$.

Given,

$$A = \begin{bmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \\ -7 & -8 & -9 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Then,

$a_{11} = -1$

$a_{13} = -3$

$a_{22} = -5$

$a_{23} = -6$

$a_{31} = -7$

$a_{33} = -9$

4. Construct a matrix of order 3×3 whose elements are given below :

a) $a_{ij} = i^j$

Solution

let the required matrix be $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Given condition,

$a_{ij} = i^j$

$a_{11} = 1^1 = 1$

$a_{12} = 1^2 = 1$

$a_{13} = 1^3 = 1$

$a_{21} = 2^1 = 2$

$a_{22} = 2^2 = 4$

$a_{23} = 2^3 = 8$

$a_{31} = 3^1 = 3$

$a_{32} = 3^2 = 9$

$a_{33} = 3^3 = 27$

Now,

The matrix becomes,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{bmatrix}$$

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b) $a_{ij} = 3i + 2j$

Solution

Let the required matrix be $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Given condition,

$a_{ij} = \cancel{ij}\ 3i + 2j$

$a_{11} = 3\times1 + 2\times1 = 5$

$a_{12} = 3\times1 + 2\times2 = 7$

$a_{13} = 3\times1 + 2\times3 = 9$

$a_{21} = 3\times2 + 2\times1 = 8$

$a_{22} = 3\times2 + 2\times2 = 10$

$a_{23} = 3\times2 + 2\times3 = 12$

$a_{31} = 3\times3 + 2\times1 = 11$

$a_{32} = 3\times3 + 2\times2 = 13$

$a_{33} = 3\times3 + 2\times3 = 15$

Now,

The matrix becomes,

$$A = \begin{bmatrix} 5 & 7 & 9 \\ 8 & 10 & 12 \\ 11 & 13 & 15 \end{bmatrix}$$

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c) $a_{ij} = 2i - j$

Solution

let the required matrix be $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Given condition,

$a_{ij} = 2i - j$

$a_{11} = 2 \times 1 - 1 = 1$

$a_{12} = 2 \times 1 - 2 = 0$

$a_{13} = 2 \times 1 - 3 = -1$

$a_{21} = 2 \times 2 - 1 = 3$

$a_{22} = 2 \times 2 - 2 = 2$

$a_{23} = 2 \times 2 - 3 = 1$

$a_{31} = 2 \times 3 - 1 = 5$

$a_{32} = 2 \times 3 - 2 = 4$

$a_{33} = 2 \times 3 - 3 = 3$

Now,

The matrix becomes,

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix}$$

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5. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$,

find $A^2 + B^2 - 2I$, where the symbols have their usual meanings.

Given,

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Now,

$$A^2 + B^2 - 2I = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^2 + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}^2 - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times0+1\times1 & 0\times1+1\times0 \\ 1\times0+0\times1 & 1\times1+0\times0 \end{bmatrix} + \begin{bmatrix} 1\times1+0\times0 & 1\times0+0\times(-1) \\ 0\times1+(-1)\times0 & 0\times0+(-1)\times(-1) \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 0+0 \\ 0+0 & 1+1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & 0-0 \\ 0-0 & 2-2 \end{bmatrix}$$

$$\therefore A^2 + B^2 - 2I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

6. If $A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ find the value of $A^2 - 2I$ where I is the 2×2 identity matrix.

Given,

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now,

$$A^2 - 2I = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}^2 - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times2 + 1\times4 & 2\times1 + 1\times3 \\ 4\times2 + 3\times4 & 4\times1 + 3\times3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ 20 & 13 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8-2 & 5-0 \\ 20-0 & 13-2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 5 \\ 20 & 11 \end{bmatrix}$$

$$\therefore A^2 - 2I = \begin{bmatrix} 6 & 5 \\ 20 & 11 \end{bmatrix}$$

~~7. a) If A~~

7. a) If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find $A^2 - 4A + 3I$.

Given,

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\cancel{I =}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now,

$$A^2 = A \times A$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1 + 2\times3 + 0\times2 & 1\times2 + 2\times1 + 0\times1 & 1\times0 + 2\times1 + 0\times3 \\ 3\times1 + 1\times3 + 1\times2 & 3\times2 + 1\times1 + 1\times1 & 3\times0 + 1\times1 + 1\times3 \\ 2\times1 + 1\times3 + 3\times2 & 2\times2 + 1\times1 + 3\times1 & 2\times0 + 1\times1 + 3\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+0 & 2+2+0 & 0+2+0 \\ 3+3+2 & 6+1+1 & 0+1+3 \\ 2+3+6 & 4+1+3 & 0+1+9 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 4 & 2 \\ 8 & 8 & 4 \\ 11 & 8 & 10 \end{bmatrix}$$

Now,

$$A^2 - 4A + 3I = \begin{bmatrix} 7 & 4 & 2 \\ 8 & 8 & 4 \\ 11 & 8 & 10 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 0 \\ 12 & 4 & 4 \\ 8 & 4 & 12 \end{bmatrix}$$

$$+ 3 \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7-4 & 4-8 & 2-0 \\ 8-12 & 8-4 & 4-4 \\ 11-8 & 8-4 & 10-12 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 & 2 \\ -4 & 4 & 0 \\ 3 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+3 & -4+0 & 2+0 \\ -4+0 & 4+3 & 0+0 \\ 3+0 & 4+0 & -2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -4 & 2 \\ -4 & 7 & 0 \\ 3 & 4 & 1 \end{bmatrix}$$

$$\therefore A^2 - 4A + 3I = \begin{bmatrix} 6 & -4 & 2 \\ -4 & 7 & 0 \\ 3 & 4 & 1 \end{bmatrix}$$

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b) If $A = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$, find $A^2 - 4I$.

Given,

$$A = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$$

Now,

$$A^2 - 4I = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times0 + (-2)\times(-2) & 0\times(-2) + (-2)\times0 \\ -2\times0 + 0\times(-2) & (-2)\times(-2) + 0\times0 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0+4 & 0+0 \\ 0+0 & 4+0 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 & 0-0 \\ 0-0 & 4-4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore A^2 - 4I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

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8. Show that,

$$\begin{bmatrix} 6 & 7 & 9 \\ 2 & 4 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 & 1 \\ 6 & 7 & 9 \\ 9 & 2 & 3 \end{bmatrix} \neq \begin{bmatrix} 2 & 4 & 1 \\ 6 & 7 & 9 \\ 9 & 2 & 3 \end{bmatrix} \begin{bmatrix} 6 & 7 & 9 \\ 2 & 4 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

Solution

Left Hand Side

$$\begin{bmatrix} 6 & 7 & 9 \\ 2 & 4 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 & 1 \\ 6 & 7 & 9 \\ 9 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6\times2+7\times6+9\times9 & 6\times4+7\times7+9\times2 & 6\times1+7\times9+9\times3 \\ 2\times2+4\times6+3\times9 & 2\times4+4\times7+3\times2 & 2\times1+4\times9+3\times3 \\ 1\times2+2\times6+4\times9 & 1\times4+2\times7+4\times2 & 1\times1+2\times9+4\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 12+42+81 & 24+49+18 & 6+63+27 \\ 4+24+27 & 8+28+6 & 2+\overset{36}{\cancel{28}}+9 \\ 2+12+36 & 4+14+8 & 1+18+12 \end{bmatrix}$$

$$= \begin{bmatrix} 135 & 91 & 96 \\ 55 & 42 & \cancel{39}\,47 \\ 50 & 26 & 31 \end{bmatrix}$$

Right Hand Side

$$\begin{bmatrix} 2 & 4 & 1 \\ 6 & 7 & 9 \\ 9 & 2 & 3 \end{bmatrix} \begin{bmatrix} 6 & 7 & 9 \\ 2 & 4 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times6+4\times2+1\times1 & 2\times7+4\times4+1\times2 & 2\times9+4\times3+1\times4 \\ 6\times6+7\times2+9\times1 & 6\times7+7\times4+9\times2 & 6\times9+7\times3+9\times4 \\ 9\times6+2\times2+3\times1 & 9\times7+2\times4+3\times2 & 9\times9+2\times3+3\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 12+8+1 & 14+16+2 & 18+12+4 \\ 36+14+9 & 42+28+18 & 54+21+36 \\ 54+4+3 & 63+8+6 & 81+6+12 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 32 & 34 \\ 59 & 88 & 111 \\ 61 & 77 & 99 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 6 & 7 & 9 \\ 2 & 4 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 & 1 \\ 6 & 7 & 9 \\ 9 & 2 & 3 \end{bmatrix} \neq \begin{bmatrix} 2 & 4 & 1 \\ 6 & 7 & 9 \\ 9 & 2 & 3 \end{bmatrix} \begin{bmatrix} 6 & 7 & 9 \\ 2 & 4 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

proved

9. If $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

find AB and BA. Hence comment the result.

Solution

Given,

$$A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Now,

$$A \times B = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1 + (-2)\times1 & 2\times1 + (-2)\times1 \\ (-2)\times1 + 2\times1 & (-2)\times1 + 2\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 - 2 & 2 - 2 \\ -2 + 2 & -2+2 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Also,

$$B \times A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times2 + 1\times(-2) & 1\times(-2) + 1\times2 \\ 1\times2 + 1\times(-2) & 1\times(-2) + 1\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & -2+2 \\ 2-2 & -2+2 \end{bmatrix}$$

$$\therefore BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, from the above result we can see that $AB = BA$.

10. If $A = \begin{bmatrix} 5 & 2 & -1 \\ 0 & 7 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 \\ 4 & 7 \\ 1 & -1 \end{bmatrix}$

verify that $(AB)^T = B^T A^T$

Given,

$$A = \begin{bmatrix} 5 & 2 & -1 \\ 0 & 7 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & 1 \\ 4 & 7 \\ 1 & -1 \end{bmatrix}$$

Now,

$$AB = \begin{bmatrix} 5 & 2 & -1 \\ 0 & 7 & 1 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 4 & 7 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 5\times(-3) + 2\times4 + (-1)\times1 & 5\times1 + 2\times7 + (-1)\times(-1) \\ 0\times(-3) + 7\times4 + 1\times1 & 0\times1 + 7\times7 + 1\times(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -15+8-1 & 5+14+1 \\ 0+28+1 & 0+49-1 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 20 \\ 29 & 48 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} -8 & 20 \\ 29 & 48 \end{bmatrix}$$

Now,

$$(AB)^T = \begin{bmatrix} -8 & 20 \\ 29 & 48 \end{bmatrix}^T$$

$$\therefore (AB)^T = \begin{bmatrix} -8 & 29 \\ 20 & 48 \end{bmatrix}$$

Also,

$$B^T A^T = \begin{bmatrix} -3 & 1 \\ 4 & 7 \\ 1 & -1 \end{bmatrix}^T \begin{bmatrix} 5 & 2 & -1 \\ 0 & 7 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 4 & 1 \\ 1 & 7 & -1 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 2 & 7 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3\times5 + 4\times2 + 1\times(-1) & -3\times0 + 4\times7 + 1\times1 \\ 1\times5 + 7\times2 + (-1)\times(-1) & 1\times0 + 7\times7 + (-1)\times1 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 29 \\ 20 & 48 \end{bmatrix}$$

$$\therefore B^T A^T = \begin{bmatrix} -8 & 29 \\ 20 & 48 \end{bmatrix}$$

From the result, we can see that $(AB)^T = B^T A^T$.

proved

~~11. If A =~~

11. If $A = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 0 \\ 8 & 2 & 5 \\ 4 & 0 & 6 \end{bmatrix}$

and $K = 2$, show that,

i) $AB \neq BA$.

Solution

$$AB = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 8 & 2 & 5 \\ 4 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1+2\times8+4\times4 & 2\times3+2\times2+4\times0 & 2\times0+2\times5+4\times6 \\ 1\times1+2\times8+3\times4 & 1\times3+2\times2+3\times0 & 1\times0+2\times5+3\times6 \\ 0\times1+1\times8+7\times4 & 0\times3+1\times2+7\times0 & 0\times0+1\times5+7\times6 \end{bmatrix}$$

$$= \begin{bmatrix} 34 & 10 & 34 \\ 29 & 7 & 28 \\ 36 & 2 & 47 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 34 & 10 & 34 \\ 29 & 7 & 28 \\ 36 & 2 & 47 \end{bmatrix}$$

Also,

$$BA = \begin{bmatrix} 1 & 3 & 0 \\ 8 & 2 & 5 \\ 4 & 0 & 6 \end{bmatrix} \begin{bmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times2+3\times1+0\times0 & 1\times2+3\times2+0\times1 & 1\times4+3\times3+0\times7 \\ 8\times2+2\times1+5\times0 & 8\times2+2\times2+5\times1 & 8\times4+2\times3+5\times7 \\ 4\times2+0\times1+6\times0 & 4\times2+0\times2+6\times1 & 4\times4+0\times3+6\times7 \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & 8 & 13 \\ 18 & 25 & 73 \\ 8 & 14 & 58 \end{bmatrix}$$

From the above result we can see that

$AB \neq BA$

proved.

(ii) $(A^T)^T = A$

Solution

$$(A^T)^T = \left[\begin{pmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{pmatrix}^T\right]^T$$

$$= \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \\ 4 & 3 & 7 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{bmatrix}$$

$= A$

$\therefore (A^T)^T = A$ proved

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(iii) $(A+B)^T = A^T + B^T$

Solution

~~Left Hand Side~~

$(A+B)^T = \left(\right.$

We have,

$$A = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 1 & 3 & 0 \\ 8 & 2 & 5 \\ 4 & 0 & 6 \end{bmatrix}$$

$$(A+B) = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 0 \\ 8 & 2 & 5 \\ 4 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 2+3 & 4+0 \\ 1+8 & 2+2 & 3+5 \\ 0+4 & 1+0 & 7+6 \end{bmatrix}$$

$$\therefore A+B = \begin{bmatrix} 3 & 5 & 4 \\ 9 & 4 & 8 \\ 4 & 1 & 13 \end{bmatrix}$$

Now,

$$(A+B)^T = \begin{bmatrix} 3 & 5 & 4 \\ 9 & 4 & 8 \\ 4 & 1 & 13 \end{bmatrix}^T$$

$$\therefore (A+B)^T = \begin{bmatrix} 3 & 9 & 4 \\ 5 & 4 & 1 \\ 4 & 8 & 13 \end{bmatrix}$$

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Also,

$$A^T + B^T = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{bmatrix}^T + \begin{bmatrix} 1 & 3 & 0 \\ 8 & 2 & 5 \\ 4 & 0 & 6 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \\ 4 & 3 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 8 & 4 \\ 3 & 2 & 0 \\ 0 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 1+8 & 0+4 \\ 2+3 & 2+2 & 1+0 \\ 4+0 & 3+5 & 7+6 \end{bmatrix}$$

$$\therefore A^T + B^T = \begin{bmatrix} 3 & 9 & 4 \\ 5 & 4 & 1 \\ 4 & 8 & 13 \end{bmatrix}$$

$\therefore (A+B)^T = A^T + B^T$ proved.

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iv) $(AB)^T = B^T A^T$

We have,

$$A = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 1 & 3 & 0 \\ 8 & 2 & 5 \\ 4 & 0 & 6 \end{bmatrix}$$

Then,

$$AB = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 8 & 2 & 5 \\ 4 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1+2\times8+4\times4 & 2\times3+2\times2+4\times0 & 2\times0+2\times5+4\times6 \\ 1\times1+2\times8+3\times4 & 1\times3+2\times2+3\times0 & 1\times0+2\times5+3\times6 \\ 0\times1+1\times8+7\times4 & 0\times3+1\times2+7\times0 & 0\times0+1\times5+7\times6 \end{bmatrix}$$

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$$= \begin{bmatrix} 34 & 10 & 34 \\ 29 & 7 & 28 \\ 36 & 2 & 47 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 34 & 10 & 34 \\ 29 & 7 & 28 \\ 36 & 2 & 47 \end{bmatrix}$$

Now,

$$(AB)^T = \begin{bmatrix} 34 & 10 & 34 \\ 29 & 7 & 28 \\ 36 & 2 & 47 \end{bmatrix}^T$$

$$\therefore (AB)^T = \begin{bmatrix} 34 & 29 & 36 \\ 10 & 7 & 2 \\ 34 & 28 & 47 \end{bmatrix}$$

Also,

$$B^T A^T = \begin{bmatrix} 1 & 3 & 0 \\ 8 & 2 & 5 \\ 4 & 0 & 6 \end{bmatrix}^T \begin{bmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 8 & 4 \\ 3 & 2 & 0 \\ 0 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \\ 4 & 3 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times2+8\times2+4\times4 & 1\times1+8\times2+4\times3 & 1\times0+8\times1+4\times7 \\ 3\times2+2\times2+0\times4 & 3\times1+2\times2+0\times3 & 3\times0+2\times1+0\times7 \\ 0\times2+5\times2+6\times4 & 0\times1+5\times2+6\times3 & 0\times0+5\times1+6\times7 \end{bmatrix}$$

$$\therefore B^T A^T = \begin{bmatrix} 34 & 29 & 36 \\ 10 & 7 & 2 \\ 34 & 28 & 47 \end{bmatrix}$$

$\therefore (AB)^T = B^T A^T$ proved

$\therefore (AB)^T = B^T A^T$ proved

v) $(KA)^T = KA^T$ where K is scalar.

Given,

$$A = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{bmatrix}$$

$K = 2.$

Then,

$$A^T = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{bmatrix}^T$$

$$\therefore A^T = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \\ 4 & 3 & 7 \end{bmatrix}$$

Now,

$$(KA)^T = \left\{ 2 \begin{bmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{bmatrix} \right\}^T$$

$$= \begin{bmatrix} 4 & 4 & 8 \\ 2 & 4 & 6 \\ 0 & 2 & 14 \end{bmatrix}^T$$

$$\therefore (KA)^T = \begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 8 & 6 & 14 \end{bmatrix}$$

Also,

$$KA^T = 2\begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \\ 4 & 3 & 7 \end{bmatrix}$$

$$\therefore KA^T = \begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 8 & 6 & 14 \end{bmatrix}$$

$\therefore (KA)^T = KA^T$ proved

12.

a) Prove that $(AB)^T = B^T A^T$

where $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 1 & 1 \end{bmatrix}$.

Solution

Given,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 1 & 1 \end{bmatrix}$$

Then,

$$(AB)^T = \left\{ \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 1 & 1 \end{bmatrix} \right\}^T$$

$$= \left\{ \begin{bmatrix} 1\times3 + 2\times4 + 3\times1 & 1\times1 + 2\times2 + 3\times1 \\ 3\times3 + 2\times4 + 4\times1 & 3\times1 + 2\times2 + 4\times1 \end{bmatrix} \right\}^T$$

$$= \begin{bmatrix} 14 & 8 \\ 21 & 11 \end{bmatrix}^T$$

$$= \begin{bmatrix} 14 & 21 \\ 8 & 11 \end{bmatrix}$$

$$\therefore (AB)^T = \begin{bmatrix} 14 & 21 \\ 8 & 11 \end{bmatrix}$$

Also,

$$B^T = \begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 1 & 1 \end{bmatrix}^T$$

$$\therefore B^T = \begin{bmatrix} 3 & 4 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 4 \end{bmatrix}^T$$

$$\therefore A^T = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 4 \end{bmatrix}$$

Then,

$$B^T A^T = \begin{bmatrix} 3 & 4 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times1 + 4\times2 + 1\times3 & 3\times3 + 4\times2 + 1\times4 \\ 1\times1 + 2\times2 + 1\times3 & 1\times3 + 2\times2 + 1\times4 \end{bmatrix}$$

$$\therefore B^T A^T = \begin{bmatrix} 14 & 21 \\ 8 & 11 \end{bmatrix}$$

$\therefore (AB)^T = B^T A^T$ proved.

b) Given that $A = \begin{bmatrix} 4 & 1 \\ 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 \\ 8 & 3 \end{bmatrix}$.

prove that $(AB)^T = B^T A^T$.

Solution

Given,

$$A = \begin{bmatrix} 4 & 1 \\ 4 & 7 \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 2 & 5 \\ 8 & 3 \end{bmatrix}$$

Then,

$$(AB)^T = \left\{ \begin{bmatrix} 4 & 1 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 8 & 3 \end{bmatrix} \right\}^T$$

$$= \begin{bmatrix} 4\times2 + 1\times8 & 4\times5 + 1\times3 \\ 4\times2 + 7\times8 & 4\times5 + 7\times3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 16 & 23 \\ 64 & 41 \end{bmatrix}^T$$

$$\therefore (AB)^T = \begin{bmatrix} 16 & 64 \\ 23 & 41 \end{bmatrix}$$

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Also,

$$B^T A^T = \begin{bmatrix} 2 & 5 \\ 8 & 3 \end{bmatrix}^T \begin{bmatrix} 4 & 1 \\ 4 & 7 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 8 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 4 & 4 \\ 1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times4+8\times1 & 2\times4+8\times7 \\ 5\times4+3\times1 & 5\times4+3\times7 \end{bmatrix}$$

$$\therefore B^T A^T = \begin{bmatrix} 16 & 64 \\ 23 & 41 \end{bmatrix}$$

$\therefore (AB)^T = B^T A^T$.

13. Let $f(x) = x^2-5x+6$. Find $f(A)$ if

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

Solution

Given,

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$f(x) = x^2-5x+6$$

Then,

when $x = A$,

$$f(A) = A^2 - 5A + 6I$$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}^2 - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6I$$

$$= \cancel{\begin{bmatrix} 2\times2+0\times2+1\times1 & 2\times0+0\times1+1\times(-1) \\ & \\ & \end{bmatrix}}$$

$$= \begin{bmatrix} 2\times2+0\times2+1\times1 & 2\times0+0\times1+1\times(-1) & 2\times1+0\times3+1\times0 \\ 2\times2+1\times2+3\times1 & 2\times0+1\times1+3\times(-1) & 2\times1+1\times3+3\times0 \\ 1\times2+(-1)\times2+0\times1 & 1\times0+(-1)\times1+0\times(-1) & 1\times1+(-1)\times3+0\times0 \end{bmatrix} - 5A + 6I$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6I$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + 6I$$

$$= \begin{bmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{bmatrix} + 6I$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5+6 & -1+0 & -3+0 \\ -1+0 & -7+6 & -10+0 \\ -5+0 & 4+0 & -2+6 \end{bmatrix}$$

$$\therefore f(A) = \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

14. Three types of newspapers The Himalayan Times, The Kantipur and The Rajadhani Dainik are available in three shops A, B, C. The matrices X and Y given below show the number of newspapers available at the beginning of the day in the shops and the sales of newspapers during a day respectively.

$$X = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} Hi & Ka & Ra \end{array} \\ \begin{bmatrix} 80 & 90 & 70 \\ 85 & 82 & 60 \\ 75 & 70 & 80 \end{bmatrix} \end{array} \qquad Y = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} Hi & Ka & Ra \end{array} \\ \begin{bmatrix} 76 & 85 & 60 \\ 82 & 80 & 58 \\ 75 & 63 & 73 \end{bmatrix} \end{array}$$

Using matrix algebra.

a) Find the number of each types of newspaper left at the end of the day in each shop.

⇒ Number of news paper left = Number of newspaper available at beginning − Number of sales.

$$= \begin{bmatrix} 80 & 90 & 70 \\ 85 & 82 & 60 \\ 75 & 70 & 80 \end{bmatrix} - \begin{bmatrix} 76 & 85 & 60 \\ 82 & 80 & 58 \\ 75 & 63 & 73 \end{bmatrix}$$

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$$= \begin{bmatrix} 80-76 & 90-85 & 70-60 \\ 85-82 & 82-80 & 60-58 \\ 75-75 & 70-63 & 80-73 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 & 10 \\ 3 & 2 & 2 \\ \cancel{8}0 & 7 & 7 \end{bmatrix}$$

b) Find the total cost of each type of newspaper available in each shop if the cost of a newspaper of each type is Rs 3.50.

$\Rightarrow$ Total Cost = Cost per each x Numbers of newspaper available

$$= 3.50 \times \begin{bmatrix} 80 & 90 & 70 \\ 85 & 82 & 60 \\ 75 & 70 & 80 \end{bmatrix}$$

$$= \begin{bmatrix} 280 & 315 & 245 \\ 297.5 & 287 & 210 \\ 262.5 & 245 & 280 \end{bmatrix}$$

c) Find the total revenue received by each shop from ~~the~~ the sell of newspapers if the price of each type of newspaper is Rs. 4.

Total Revenue in each shop

$$= \begin{bmatrix} 76 & 85 & 60 \\ 82 & 80 & 58 \\ 75 & 63 & 73 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 76\times4 + 85\times4 + 60\times4 \\ 82\times4 + 80\times4 + 58\times4 \\ 75\times4 + 63\times4 + 73\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 304 + 340 + 240 \\ 328 + 320 + 232 \\ 300 + 252 + 292 \end{bmatrix}$$

$$= \begin{bmatrix} 884 \\ 880 \\ 844 \end{bmatrix}$$

15. Bhanu-Memorial Trust fund has Rs. 15000 that must be invested in two different types of bonds. The first bond pays 5% interest per year and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs. 15000 among two types of bonds if the trust fund must obtain an annual total interest of Rs 1000.

Solution

Let Rs x be invested in bond of first type.
Then, Rs. $(15000 - x)$ will be invested in another bond.

Then,
Representing in matrix form

$$A = \begin{bmatrix} x & 15000 - x \end{bmatrix}$$

When the interest obtained is Rs. 1000,

$$\begin{bmatrix} x & 15000-x \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = \begin{bmatrix} 1000 \end{bmatrix}$$

or, $\begin{bmatrix} x & 15000-x \end{bmatrix} \begin{bmatrix} 0.05 \\ 0.07 \end{bmatrix} = \begin{bmatrix} 1000 \end{bmatrix}$

or, $\begin{bmatrix} x \times 0.05 + (15000-x) \times 0.07 \end{bmatrix} = \begin{bmatrix} 1000 \end{bmatrix}$

or, $0.05x + 1050 - 0.07x = 1000$

or, $-0.02x = -50$

$\therefore x = 2500$

Then,

Amount invested in first bond $= x = Rs\ 2500$

Amount invested in second bond $= 15000 - x$

$= 15000 - 2500$

$= Rs\ 12500$