Page No. :
Date :

Exercise 6(B)

1. Solve the following differential equation using variable separation method. (separation of variables)

i) $\frac{dy}{dx} = 2x^2 - 3x + 5$

The given differential equation is:

$$\frac{dy}{dx} = 2x^2 - 3x + 5$$

or, $dy = (2x^2 - 3x + 5)\,dx$

Integrating on both sides,

$$\int dy = \int (2x^2 - 3x + 5)\,dx$$

or, $\int dy = 2\int x^2\,dx - 3\int x\,dx + 5\int dx$

or, $y = 2 \times \frac{x^3}{3} - 3 \times \frac{x^2}{2} + 5 \times x + C$

$\left[\because \int dx = x + C,\ \int x^n\,dx = \frac{x^{n+1}}{n+1} + C\right]$

$\therefore$ $y = \frac{2}{3}x^3 - \frac{3}{2}x^2 + 5x + C$ is the required solution.

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ii) $\frac{dy}{dx} - 2x = e^{3x}$

The given differential equation is:

$$\frac{dy}{dx} - 2x = e^{3x}$$

Page No. :
Date :

or, $\frac{dy}{dx} = e^{3x} + 2x$

or, $dy = (e^{3x} + 2x)dx$

Integrating on both sides,

or, $\int dy = \int (e^{3x} + 2x)\, dx$

or, $\int dy = \int e^{3x} dx + 2\int x\, dx$

or, $y = \frac{e^{3x}}{3} + 2 \times \frac{x^{1+1}}{1+1} + c$

$\left[\because \int a\, dx = x + c,\ \int e^{ax} dx = \frac{e^{ax}}{a} + c,\ \int x^n dx = \frac{x^{n+1}}{n+1} + c\right]$

$\therefore y = \frac{e^{3x}}{3} + x^2 + c$ is the required solution.

---

iii) $(1+x^2)\, dy = xy\, dx$

The given differential equation is :

$(1+x^2)\, dy = xy\, dx$

or, $\frac{dy}{y} = \frac{x\, dx}{1+x^2}$

Integrating on both sides,

or, $\int \frac{dy}{y} = \int \frac{x}{1+x^2}\, dx$

or, $\int \frac{dy}{y} = \frac{1}{2} \int \frac{2x}{1+x^2}\, dx$

~~or~~ $\therefore \ln y = \frac{1}{2}\ln(1+x^2) + c$ is the required solution.

iv) $\frac{dy}{dx} = e^{x+y}$

The given differential equation is :

$\frac{dy}{dx} = e^x \times e^y$

~~or, $\frac{dy}{dx} =$~~

~~or, $dy = (e^x \times e^y)dx$~~

or, $\frac{dy}{e^y} = e^x . dx$

Integrating on both sides,

or, $\int \frac{dy}{e^y} = \int e^x . dx$

~~or, lny~~

or, $\int e^{-y} dy = \int e^x . dx$

or, $-e^{-y} = e^x + C$ ~~is the requ~~ is the required solution.

Page No.:
Date:

v) $(x^2 - yx^2)\,dy + (y^2 + x^2y^2)\,dx = 0$

The given differential equation is :

$$(x^2 - yx^2)\,dy + (y^2 + x^2y^2)\,dx = 0$$

or, $x^2(1-y)\,dy + y^2(1+x^2)\,dx = 0$

or, $x^2(1-y)\,dy = -y^2(1+x^2)\,dx$

or, $\frac{(1-y)}{y^2}\,dy = -\frac{(1+x^2)}{x^2}\,dx$

Integrating on both sides,

$$\int\left(\frac{1-y}{y^2}\right)dy = -\int\frac{1+x^2}{x^2}\,dx$$

or, $\int\left(\frac{1}{y^2} - \frac{y}{y^2}\right)dy = -\int\left(\frac{1}{x^2} + \frac{x^2}{x^2}\right)dx$

or, $\int\left(y^{-2} - \frac{1}{y}\right)dy = -\int\left(x^{-2} + \cancel{1}\right)dx$

or, $\frac{y^{-2+1}}{-2+1} - \ln y = -\frac{x^{-2+1}}{-2+1} - x + c$

or, $-\frac{1}{y} - \ln y = \frac{1}{x} - x + c$ is the required solution.

Page No. :
Date :

vi) $(x-1)\frac{dy}{dx} = 2xy$

The given differential equation is:

$$(x-1)\frac{dy}{dx} = 2xy$$

or, $\frac{dy}{dx} = \frac{2xy}{x-1}$

or, $\frac{dy}{y} = \left(\frac{2x}{x-1}\right)dx$

Integrating on both sides,

$$\int \frac{dy}{y} = \int \left(\frac{2x}{x-1}\right) dx$$

or, $\ln y = 2x\int\left(\frac{1}{x-1}\right) dx$

or, $\ln y = 2x \times \ln(x-1) + C$

$\therefore \ln y = 2x\ln(x-1) + C$ is the required solution.

---

vii) $(x+1)\sqrt{y}\,dx + (1+y)\sqrt{x}\,dy = 0$, $y(1)=0$

The given differential equation is:

$$(x+1)\sqrt{y}\,dx + (1+y)\sqrt{x}\,dy = 0$$

or, $(x+1)\sqrt{y}\,dx = -(1+y)\sqrt{x}\,dy$

or, $\left(\frac{x+1}{\sqrt{x}}\right)dx = -\left(\frac{1+y}{\sqrt{y}}\right)dy$

Page No. :
Date :

or, $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx = -\left(\frac{1}{\sqrt{y}} + \sqrt{y}\right) dy$

or, $\left(x^{1/2} + x^{-1/2}\right) dx = -\left(y^{-1/2} + y^{1/2}\right) dy$

~~or,~~ Integrating on both sides,

$\int (x^{1/2} + x^{-1/2}) \cdot dx = -\int (y^{-1/2} + y^{1/2}) dy$

or, $\int x^{1/2} dx + \int x^{-1/2} dx = -\int y^{-1/2} dy + \int y^{1/2} dy$

or, $\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = -\frac{y^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1}$

or, $\frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} = \cancel{y} - 2y^{1/2} + \frac{2}{3} y^{\frac{3}{2}} + C$

or, $\frac{2}{3} x^{3/2} + 2x^{1/2} + 2y^{1/2} - \frac{2}{3} y^{3/2} + C = 0$

is the required solution.

Date :

Exercise 6(B)

3. Solve the following differential equations.

i) $\frac{dy}{dx} + 3y = x$

The given differential equation is:

$$\frac{dy}{dx} + 3y = x \text{ ——— } (1)$$

Comparing (1) with $\frac{dy}{dx} + Py = Q$, we get,

$P = 3$, $Q = x$

we know,

Integrating Factor (I.F) $= e^{\int p\,dx}$

$= e^{\int 3\,dx}$

$= e^{3\int dx}$

$= e^{3x}$ [$\because \int dx = x + c$]

Now,

The required solution becomes,

$y \times IF = \int (Q \times IF)\,dx + C$

or, $y \times e^{3x} = \int (x \times e^{3x})\,dx + C$

or, $y \times e^{3x} = x \int e^{3x} - \int \left(\frac{d}{dx}(x) . \int e^{3x}\,dx\right) dx + C$

or, $y \times e^{3x} = x \int e^{3x} - \int \left(1 . \frac{e^{3x}}{3}\right) dx + C$

or, $y \times e^{3x} = x \int e^{3x} - \frac{1}{3} \int e^{3x}\,dx + C$

or, $y \times e^{3x} = x \times \frac{e^{3x}}{3} - \frac{1}{3} \times \frac{e^{3x}}{3} + C$

Page No.:
Date:

or, $y \times e^{3x} = x \times \frac{1}{3} \times e^{3x} - \frac{1}{3} \times e^{3x} \times \frac{1}{3} + c$

or, $y \times e^{3x} = \frac{1}{3} e^{3x} \left(x - \frac{1}{3} + \frac{c}{e^{3x}} \right)$

or, $\frac{y e^{3x}}{\frac{1}{3} e^{3x}} = x - \frac{1}{3} + c \cdot e^{-3x}$

or, $3 y e^{3x-3x} = \frac{3x-1}{3} + C \cdot e^{-3x}$

or, $3 \times 3y = 3x - 1 + 3C e^{-3x}$

i.e. ~~or,~~ $9y = 3x - 1 + e^{-3x}$ is the required solution.

ii) $\frac{dy}{dx} + \frac{y}{x} = x^2$

The given differential equation is

$$\frac{dy}{dx} + \frac{y}{x} = x^2 \quad \text{——— (1)}$$

Comparing (1) with $\frac{dy}{dx} + Py = Q$, we get,

$P = \frac{1}{x}$, $Q = x^2$

We know,

Integrating Factor (I.F) $= e^{\int P dx}$

$= e^{\int \frac{1}{x} dx}$

$= e^{\log x}$

$= x \quad [\because e^{\log x} = x]$

Date :

Now, the required solution becomes,

$y \times IF = \int (Q \times IF)\, dx + C$

or, $\cancel{x \times}\ y \times x = \int (x^2 \times x)\, dx + c$

or, $y \times x = \int x^3\, dx + c$

~~or~~ i.e. $xy = \frac{x^4}{4} + c$ is the required solution.

---

iii) $(x+1)\frac{dy}{dx} + 2y = \frac{e^x}{x+1}$

The ~~required~~ given differential equation is:

$$(x+1)\frac{dy}{dx} + 2y = \frac{e^x}{x+1}$$

or, $$\frac{dy}{dx} + \frac{2y}{(x+1)} = \frac{e^x}{(x+1)^2} \quad \text{---} \quad (1)$$

Comparing (1) with $\frac{dy}{dx} + Py = Q$, we get

$$P = \frac{2}{x+1}, \quad Q = \frac{e^x}{(x+1)^2}$$

We know,

~~Integrating Factor (I.F) = $\int e^{pdx}$~~

~~$= \int e^{\frac{2}{x+1}} dx$~~

~~$= \int e^{2(x+1)^{-1}} dx$~~

=

Page No.:
Date:

Integrating Factor (IF) $= e^{\int P dx \cancel{+c}}$

$= e^{\int \frac{2}{x+1} dx} \cancel{+c}$

$= e^{2 \cdot \int \frac{dx}{x+1}} \cancel{+c}$

$= e^{2 \cdot \log(x+1)} \cancel{+c}$

$= e^{\log(x+1)^2} \cancel{+c}$

$[\because \log m^n = n \log m]$

$= (x+1)^2 \cancel{+c}$

Now, the required solution becomes,

$y \times IF = \int (Q \times IF)\, dx + c$

or, $y \times (x+1)^2 = \int \left(\frac{e^x}{(x+1)^2} \times (x+1)^2 \right) dx + c$

or, $y \times (x+1)^2 = \int e^x\, dx + C$

~~or~~, i.e. $y \times (x+1)^2 = e^x + C$ is the required solution.

iv) $x \frac{dy}{dx} + y = x^3$

The given differential equation is

$x \frac{dy}{dx} + y = x^3$

or, $\frac{dy}{dx} + \frac{y}{x} = x^2$ ——— ①

Comparing ① with $\frac{dy}{dx} + Py = Q$,

$P = \frac{1}{x}, \ Q = x^2$

Now,

Integrating Factor (IF) $= e^{\int P dx}$

$= e^{\int \frac{1}{x} dx}$

$= e^{\ln x}$

$= x \quad [\because e^{\ln x} = x]$

Now, the required solution becomes,

$\cancel{y}$ $y \times IF = \int (Q \times IF)\, dx + C$

or, $y \times x = \int (x^2 \times x)\, dx + C$

or, $y \times x = \int x^3\, dx + C$

or, $y \times x = \frac{x^4}{4} + C$ is the required solution.

---

v) $(x^2 - 1)\frac{dy}{dx} + 2xy = 4x^3$

The given differential equation is

$(x^2 - 1)\frac{dy}{dx} + 2xy = 4x^3$

or, $\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = \frac{4x^3}{x^2 - 1}$ ——— (1)

Comparing (1) with $\frac{dy}{dx} + Py = Q$, we get

Date :

$\cancel{\frac{dy}{}}$ $P = \cancel{2xy}\, \frac{2x}{x^2-1}$, $Q = \frac{4x^3}{x^2-1}$

Integrating Factor (I.F) $= e^{\int P dx}$

$= e^{\int \frac{2x}{x^2-1} dx}$

$= e^{\log(x^2-1)}$

$= (x^2-1)$ $[\because e^{\log x} = x]$

Now, the required solution becomes,

$y \times IF = \int (Q \times IF) dx + C$

or, $y \times (x^2-1) = \int \frac{4x^3}{x^2-1} \times (x^2-1)\, dx + C$

or, $y \times (x^2-1) = \int 4x^3\, dx + C$

or, $y \times (x^2-1) = x^4 + C$ is the required solution.

vi) $\frac{dy}{dx} + ay = e^{mx}$

The given differential equation is:

$\frac{dy}{dx} + ay = e^{mx}$ ——— (1)

Comparing (1) with $\frac{dy}{dx} + Py = Q$, we get,

$P = a$, $Q = e^{mx}$

Integrating Factor (IF) $= e^{\int p\,dx}$
$= e^{\int a\,dx}$
$= e^{a\int dx}$
$= e^{ax}$

Now,
The required solution becomes,

$y \times IF = \int (Q \times IF)\,dx + c$

or, $y \times e^{ax} = \int (e^{mx} \times e^{ax})\,dx + c$

or, $y \times e^{ax} = \int (e^{mx+ax})\,dx + c$

or, $y \times e^{ax} = \int e^{(m+a)x}\,dx + c$

i.e. $y \times e^{ax} = \dfrac{e^{(m+a)x}}{m+a} + c$ is the required solution.

2. Solve the following homogeneous differential equations:

i) $x\,dy = (x+y)\,dx$

The given differential equation is :

$x\,dy = (x+y)\,dx$

or, $\dfrac{dy}{dx} = \dfrac{x+y}{x}$

or, $\dfrac{dy}{dx} = 1 + \dfrac{y}{x}$ ——— (1)

Put $y = vx$,

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}(v.x)$$

or, $\frac{dy}{dx} = v.\frac{d}{dx}(x) + x.\frac{d}{dx}(v)$

$\therefore \frac{dy}{dx} = v + x.\frac{dv}{dx}$ ——— (11)

Substituting the value of $\frac{dy}{dx}$ in equation (1),

or, $v + x.\frac{dv}{dx} = 1 + \frac{vx}{x}$ $[\because y = vx]$

or, $v + x.\frac{dv}{dx} = 1 + v$

or, $x.\frac{dv}{dx} = 1 + v - v$

or, $\frac{dv}{dx} = \frac{1}{x}$

or, $dv = \frac{dx}{x}$

Integrating on both sides,

or, $\int dv = \int \frac{dx}{x}$

or, $v = \ln x + \cancel{\ln c}\ c$ $[\because \int \frac{dx}{x} = \ln x + c]$

or, $\frac{vx}{x} = \ln x + c$

or, $vx = x(\ln x + c)$

i.e. $y = x(\ln x + c)$ $[\because y = vx]$ is the required solution.

Date :

~~ii) $\frac{dy}{dx}$~~

ii) $(x^2-y^2)\,dx - xy\,dy = 0$

The given differential equation is :

$(x^2-y^2)\,dx - xy\,dy = 0$

or, $(x^2-y^2)\,dx = xy\,dy$

or, $\frac{dy}{dx} = \frac{x^2-y^2}{xy}$ ——— (i)

Put $y = vx$

Differentiating both sides with respect to x,

$\frac{d}{dx}(y) = \frac{d}{dx}(vx)$

or, $\frac{dy}{dx} = v\cdot\frac{d}{dx}(x) + x\cdot\frac{d}{dx}(v)$

$\therefore \frac{dy}{dx} = v + x\cdot\frac{dv}{dx}$ ——— (ii)

Substituting the value of $\frac{dy}{dx}$ in equation (i)

or, $v + x\cdot\frac{dv}{dx} = \frac{x^2 - v^2x^2}{x\cdot vx}$ $[\because y = vx]$

or, $v + x\cdot\frac{dv}{dx} = \frac{x^2(1-v^2)}{x^2 v}$

or, $v + x\cdot\frac{dv}{dx} = \frac{1-v^2}{v}$

Page No. :
Date :

or, $x \cdot \frac{dv}{dx} = \frac{1-v^2}{v} - v$

or, $x \cdot \frac{dv}{dx} = \frac{1-v^2-v^2}{v}$

or, $x \cdot \frac{dv}{dx} = \frac{1-2v^2}{v}$

or, $\frac{v\,dv}{1-2v^2} = \frac{dx}{x}$

or, $\frac{v\,dv}{2v^2-1} = -\frac{dx}{x}$

Integrating on both sides,

$$\int \frac{v}{2v^2-1}\,dv = -\int \frac{dx}{x}$$

or, $\int \frac{4v}{2v^2-1}\,dv = -4\int \frac{dx}{x}$

or, $\log(2v^2-1) = -4\log x + \log C$

or $\log(2v^2-1) = -\log x^4 + \log C$ $[\because \log m^n = n\log m]$

or, $\log(2v^2-1) + \log x^4 = \log C$

or, $\log\{(2v^2-1)x^4\} = \log C$

or, $\log(2v^2x^4 - x^4) = \log C$

or, $2v^2x^4 - x^4 = C$

or, $2v^2x^2 - x^4 = C$

or, $2y^2 - x^4 = C$ $[\because y = vx]$

$\therefore$ $2y^2 - x^4 = C$ is the required solution.

iii) $x\frac{dy}{dx} = x+y,\ y(1) = 4$

The given differential equation is:

$$x \cdot \frac{dy}{dx} = x+y$$

$$\text{or, } \frac{dy}{dx} = \frac{x+y}{x}$$

$$\text{or, } \frac{dy}{dx} = 1 + \frac{y}{x} \quad \text{—— (1)}$$

Put $y = vx$,

Differentiating both sides with respect to x,

$$\frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\text{or, } \frac{dy}{dx} = v \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(v)$$

$$\therefore \frac{dy}{dx} = v + x \cdot \frac{dv}{dx} \quad \text{—— (II)}$$

Substituting the value of $\frac{dy}{dx}$ in equation (1)

$$\text{or, } v + x \cdot \frac{dv}{dx} = 1 + \frac{y}{x}$$

Date :

or, $v + x \cdot \frac{dv}{dx} = 1 + \frac{vx}{x}$ $[\because y = vx]$

or, $x \cdot \frac{dv}{dx} = 1 + v - v$

or, $x \cdot \frac{dv}{dx} = 1$

or, $\frac{dv}{dx} = \frac{1}{x}$

or, $x \cdot dv = 1\, dx$

or, $dv = \frac{dx}{x}$

Integrating on both sides,

$$\int dv = \int \frac{dx}{x}$$

or, $v = \log x + c$

or, $vx = x(\log x + c)$

$\therefore y = x(\log x + c)$ - (iii) is the general solution.

when $x = 1$, $y = 4$

or, $4 = 1(\log 1 + c)$

or, $4 = 1(0 + c)$

$\therefore c = 4$

Now, (iii) becomes,

$y = x(\log x + 4)$ is the required particular solution.