Exercise 5(F)

1. The demand function for a commodity is $p = 19 - x$. Find the consumer's surplus when $x = 6$.

Solution

Given, Demand Function : $P = 19 - x$

i.e. $P_d(x) = 19 - x$

when $x = 6$,

$$P = 19 - 6 = 13$$

Now, we know that,

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Consumer's Surplus (C.S) $= \int_0^x Pd(x)\,dx - P \times x$

$= \int_0^6 (19-x)\,dx - 13 \times 6$

$= \int_0^6 19\,dx - \int_0^6 x\,dx - 78$

$= 19 \times x \Big|_0^6 - \frac{x^2}{2} \Big|_0^6 - 78$

$= 19 \times (6-0) - \frac{1}{2} \times (6^2 - 0^2) - 78$

$= 114 - 18 - 78$

$= 18$

Therefore, the consumer's surplus is 18 when $x = 6$.

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2. If the supply function for a commodity is $p = 40 + 30(x+1)^2$, find the producer's surplus at $x = 4$.

Solution

Given, $p = 40 + 30(x+1)^2$

When $x = 4$,

$p = 40 + 30(4+1)^2$

$= 40 + 750$

$\therefore p = 790$

We know that,

Producer's Surplus $= p \times x - \int_0^x P_s(x)\,dx$

$= 790 \times 4 - \int_0^4 [40 + 30(x+1)^2]\,dx$

$= 3160 - 40\int_0^4 dx - 30\int_0^4 (x+1)^2\,dx$

$= 3160 - 40 \times x \Big|_0^4 - 30 \times \frac{(x+1)^{2+1}}{1(2+1)} \Big|_0^4$

$\left[\because \int dx = x + c,\ \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c,\ n \neq -1\right]$

$= 3160 - 40 \times (4-0) - 10 \times \left[(4+1)^3 - (0+1)^3\right]$

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$= 3160 - 160 - 1240$
$= 1760$
Therefore, the producer's surplus is 1760 at $x=4$.

3. If the market curve is given as $p = 30-2q$ where p and q denote price and quantity, calculate the consumer's surplus if the equilibrium price is 6.

Solution
Given, $p = 30-2q$
or, $6 = 30-2q$
or, $2q = 24$
$\therefore q = 12$

We know that,

Consumer's surplus (C.S) $= \int_0^q Pd(q)\,dq - P \times q$

$= \int_0^q (30-2q)\,dq - 6 \times 12$

$= \int_0^{12} (30-2q)\,dq - 72$

$= 30\int_0^{12} dq - 2\int_0^{12} q\,dq - 72$

$= 30 \times q\Big|_0^{12} - 2 \times \frac{q^{1+1}}{1+1}\Big|_0^{12} - 72$

$\left[\because \int dx = x + c, \int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1\right]$

$= 30 \times (12-0) - (12^2 - 0^2) - 72$
$= 360 - 144 - 72$
$= 144$
Therefore, the consumer's surplus is 144.

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4. The demand and supply function under pure competition are $p_d = 16 - x^2$ and $p_s = 2x^2 + 4$. Find the consumer's surplus, and producer's surplus and total surplus.

Solution

Given, $P_d = 16 - x^2$

i.e. $P_d(x) = 16 - x^2$ ——— (1)

$P_s = 2x^2 + 4$ i.e. $P_s(x) = 2x^2 + 4$

We know,

~~Consumer's surplus (C.S) = $\int_0^x P_d(x)\,dx - P \times x$~~

At Equilibrium,

$P_d = P_s$

or, $16 - x^2 = 2x^2 + 4$

or, $0 = 2x^2 + 4 - 16 + x^2$

or, $3x^2 - 12 = 0$

or, $3(x^2 - 4) = 0$

or, $(x+2)(x-2) = 0$

Either $x + 2 = 0$ ∴ $x = -2$ (Rejected)

or, $x - 2 = 0$ ∴ $x = 2$

From (1), $P = 16 - 2^2 = 16 - 4 = 12$

We know,

Consumer's Surplus (C.S) $= \int_0^x P_d(x)\,dx - P \times x$

$= \int_0^2 (16 - x^2)\,dx - 12 \times 2$

$= 16 \times \int_0^2 dx - \int_0^2 x^2\,dx - 24$

$= 16 \times x \Big|_0^2 - \frac{x^3}{3}\Big|_0^2 - 24$

$= 16 \times (2-0) - \frac{1}{3} \times (2^3 - 0^3) - 24$

$= 32 - \frac{8}{3} - 24$

$= \frac{16}{3}$

Therefore, the ~~producers~~ consumer's surplus is $\frac{16}{3}$

Also,

Producer's Surplus $= P \times x - \int_0^x P_s(x)\,dx$

$= 12 \times 2 - \int_0^2 (2x^2 + 4)\,dx$

$= 24 - \int_0^2 2x^2\,dx - \int_0^2 4\,dx$

$= 24 - 2 \cdot \int_0^2 x^2\,dx - 4\int_0^2 dx$

$= 24 - 2 \times \frac{x^{2+1}}{2+1}\Big|_0^2 - 4 \times x\Big|_0^2$

$[\because \int dx = x + c, \int x^n\,dx = \frac{x^{n+1}}{n+1} + c, n \neq -1]$

$= 24 - \frac{2}{3} \times x^3\Big|_0^2 - 4 \times x\Big|_0^2$

$= 24 - \frac{2}{3} \times (2^3 - 0^3) - 4 \times (2 - 0)$

$= 24 - \frac{2}{3} \times 8 - 8$

$= 16 - \frac{16}{3}$

$= \frac{32}{3}$

Therefore, the producer's surplus is $\frac{32}{3}$

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Finally,

Total surplus = Consumer's Surplus + Producer's Surplus

$$= \frac{16}{3} + \frac{32}{3}$$

$$= \frac{48}{3}$$

$$= 16$$

Therefore, the total surplus is 16.

5. In a perfect competition, the demand and supply curves of a commodity are given by $p_d = 20 - 3x - x^2$ and $p_s = x - 1$. Find the point of equilibrium, the consumer's surplus, the producer's surplus and the total surplus.

Solution

Given, $P_d = 20 - 3x - x^2$ —— (1)

$P_s = x - 1$

We know, At equilibrium $P_d = P_s$

or, $20 - 3x - x^2 = x - 1$

or, $20 - 3x - x^2 - x + 1 = 0$

or, $-x^2 - 4x + 21 = 0$

or, $x^2 + 4x - 21 = 0$

$\therefore x = -7$ (Rejected), 3

Therefore the point of equilibrium is at $x = 3$.

Now, when $x = 3$, $P = 20 - 3 \times 3 - 3^2 = 20 - 18 = 2$

Consumer's surplus (C.S) $= \int_0^x P_d(x)\,dx - P \times x$

$$= \int_0^3 (20 - 3x - x^2)\,dx - 2 \times 3$$

$$= 20\int_0^3 dx - 3\int_0^3 x\,dx - \int_0^3 x^2\,dx - 6$$

$$= 20 \times x\Big|_0^3 - 3 \times \frac{x^2}{2}\Big|_0^3 - \frac{x^3}{3}\Big|_0^3 - 6$$

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$= 20 \times (3-0) - \frac{3}{2} \times (3^2 - 0^2) - \frac{1}{3} \times (3^3 - 0^3) - 6$

$= 60 - \frac{27}{2} - 9 - 6$

$= \frac{63}{2}$

Therefore, the consumer's surplus is $\frac{63}{2}$.

Also,

Producer's Surplus (P.S) $= P \times x - \int_0^x P_s(x)\,dx$

$= 2 \times 3 - \int_0^3 (x-1)\,dx$

$= 6 - \int_0^3 x\,dx + 1 \int_0^3 dx$

$= 6 - \frac{x^2}{2}\Big|_0^3 + 1 . x\Big|_0^3$

$= 6 - \frac{1}{2} \times (3^2 - 0^2) + 1 \times (3-0)$

$= 6 - \frac{1}{2} \times 9 + 1 \times 3$

$= 6 - \frac{9}{2} + 3$

$= \frac{9}{2}$

Therefore, the producer's surplus is $\frac{9}{2}$

Also,

Total Surplus (T.S) = Consumer's Surplus + Producer's Surplus

$= \frac{63}{2} + \frac{9}{2}$

$= \frac{72}{2}$

$= 36$

Therefore, the total surplus is 36.

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6. If the demand and supply functions are $20-5x$ and $4x+8$ respectively. Determine the consumers surplus, producers surplus and total surplus under pure-competition.

Solution

Given, Demand Function : $P_d(x) = 20-5x$ —— (I)

Supply Function : $P_s(x) = 4x+8$ —— (II)

In the perfect competition market, $P_d = P_s$

or, $20-5x = 4x+8$

or, $20-8 = 4x+5x$

or, $9x = 12$

$\therefore x = \frac{4}{3}$

Now, ~~we kn~~

Substituting value of x in (I),

$P = 20 - 5 \times \frac{4}{3}$

$= 20 - \frac{20}{3}$

$\therefore P = \frac{40}{3}$

we know,

Consumer's surplus (C.S) $= \int_0^x P_d(x)dx - P \times x$

$= \int_0^{4/3} (20-5x)\,dx - \frac{40}{3} \times \frac{4}{3}$

$= \int_0^{4/3} 20\,dx - \int_0^{4/3} 5x\,dx - \frac{160}{9}$

$= 20 \times x \Big|_0^{4/3} - 5 \times \frac{x^2}{2} \Big|_0^{4/3} - \frac{160}{9}$

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$$= 20\times\left(\frac{4}{3}-0\right) - \frac{5}{2}\times\left[\left(\frac{4}{3}\right)^2 - (0)^2\right] - \frac{160}{9}$$

$$= \frac{80}{3} - \frac{40}{9} - \frac{160}{9}$$

$$= \frac{40}{9}$$

Therefore, the consumer's surplus is $\frac{40}{9}$.

Also,

Producer's Surplus (P.S) $= P\times x - \int_0^x P_s(x)\,dx$

$$= \frac{40}{3}\times\frac{4}{3} - \int_0^{40/3}(4x+8)\,dx$$

$$= \frac{160}{9} - \int_0^{40/3} 4x\,dx - 8\int_0^{40/3} dx$$

$$= \frac{160}{9} - 4\times\frac{x^2}{2}\Big|_0^{40/3} - 8\times x\Big|_0^{40/3}$$

$$= \frac{160}{9} - 2\times\left[\left(\frac{40}{3}\right)^2 - 0^2\right] - 8\times\left(\frac{40}{3}-0\right)$$

$$= \frac{160}{9} - \frac{3200}{9} - \frac{320}{3}$$

$$= \frac{160}{9} - \int_0^{40/3}(4x+8)\,dx$$

$$= \frac{160}{9} - \left[4\int_0^{40/3} x\,dx + 8\int_0^{40/3} dx\right]$$

$$= \frac{160}{9} - \left[4\times\frac{x^2}{2}\Big|_0^{40/3} + 8\times x\Big|_0^{4/3}\right]$$

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$$= \frac{160}{9} - \left[2 \times \left\{\left(\frac{4}{3}\right)^2 - 0^2\right\} + 8 \times \left(\frac{4}{3} - 0\right)\right]$$

$$= \frac{160}{9} - \left[\frac{32}{9} + \frac{32}{3}\right]$$

$$= \frac{160}{9} - \frac{128}{9}$$

$$= \frac{32}{9}$$

Therefore, the producer's surplus is $\frac{32}{9}$

Finally,
Total surplus = Consumer's Surplus + Producer's Surplus

$$= \frac{40}{9} + \frac{32}{9}$$

$$= \frac{72}{9}$$

$$= 8$$

Therefore the total surplus is 8.

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7. The market demand function for a commodity is given as $p = 120 - 4q$ and the supply function $p = -60 + 5q$, where p and q denote price and quantity. Calculate the consumer's surplus, producer's surplus and total surplus under competition.

Solution
Given, $p_d(q) = 120 - 4q$ —— (I)
$p_s(q) = -60 + 5q$ —— (II)

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Under perfect competition, $P_d = P_s$.

or, $120 - 4q = -60 + 5q$
or, $120 + 60 = 5q + 4q$
or, $9q = 180$
$\therefore q = 20$

Substituting the value of q in (1) we get,

$P = 120 - 4 \times 20$
$= 120 - 80$
$\therefore P = 40$

We know,

Consumer's Surplus (C.S) $= \int_0^q P_d(q)\,dq - P \times q$

$= \int_0^{20} (120 - 4q)\,dq - 40 \times 20$
$= 120 \int_0^{20} dq - 4 \int_0^{20} q - 800$

$= 120 \times q \Big|_0^{20} - 4 \times \frac{q^2}{2} \Big|_0^{20} - 800$

$\left[\because \int dx = x + c,\ \int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1\right]$

$= 120 \times (20 - 0) - 2 \times (20^2 - 0^2) - 800$

$= 2400 - 800 - 800$
$= 800$

Therefore, the consumer's surplus is 800.

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Also,

Producer's surplus (P.S) = $P \times q - \int_0^q P_s(q)\,dq$

$= 40 \times 20 - \int_0^{20} (-60 + 5q)\,dq$

$= 800 + 60\int_0^{20} dq - 5\int_0^{20} q\,dq$

$= 800 + \cancel{20}\,60 \times q\Big|_0^{20} - 5 \times \frac{q^2}{2}\Big|_0^{20}$

$\left[\because \int dx = x + c,\ \int x^n\,dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1\right]$

$= 800 + 60 \times (20-0) - \frac{5}{2} \times (20^2 - 0^2)$

$= 800 + 1200 - 1000$

$= 1000$

Therefore, the producer's surplus is 1000.

Finally,

Total Surplus = Consumer's surplus + Producer's surplus

$= 800 + 1000$

$= 1800$

Therefore, the total surplus is 1800.

8. The demand and supply function for a goods are given by the equations

$P_d = 300e^{-0.2Q}$ and $P_s = 2e^{0.8Q}$

i) Find the equilibrium price and quantity and
ii) Calculate consumer's surplus and the producer's surplus at equilibrium point.

Solution

Given, Demand Function : $P_d = 300e^{-0.2Q}$ — (I)

Supply Function : $P_s = 2e^{0.8Q}$ — (II)

i) At equilibrium,

$P_d = P_s$

or, $300e^{-0.2Q} = 2e^{0.8Q}$

or, $300e^{-0.2Q} - 2e^{0.8Q} = 0$

or, $2(150e^{-0.2Q} - e^{0.8Q}) = 0$

or, $150e^{-0.2Q} - e^{0.8Q} = 0$

or, $e^{-0.2Q}(150 - e^{4}) = 0$

or, $e^{-0.2Q}(150 - 54.5981) = 0$

or, $e^{-0.2Q} = 95.4019$

or, $300e^{-0.2Q} = 2e^{0.8Q}$

or, $300 = \dfrac{2e^{0.8Q}}{e^{-0.2Q}}$

or, $300 = 2 \cdot e^{0.8Q + 0.2Q}$

or, $300 = 2 \cdot e^{Q}$

or, $e^{Q} = 150$

Taking ln on both sides

$\ln(e^{Q}) = \ln(150)$

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or, $Q = \ln(150)$ $[\because \ln e^x = x]$

$\therefore Q = 5.0106$

Substituting the value of Q in (11)

$P = 2 \times e^{0.8 \times 5.0106}$

$= 2 \times e^{4.00848}$

$= \cancel{2 \times 54.59}$ 2×55.063

$\therefore P = \cancel{109.19}$ 110.12

Therefore the equilibrium price is ~~₹~~ $P = 110.12$ and the equilibrium quantity is $Q = 5.0106$.

ii) We know,

Consumer's Surplus (CS) $= \int_0^Q P_d(Q)\, dQ - P \times Q$

$= \int_0^{5.0106} (300 e^{-0.2Q})\, dQ - 110.12 \times 5.0106$

$= 300 \times \int_0^{5.0106} e^{-0.2Q}\, dQ - 551.7672$

$= 300 \times \frac{e^{-0.2Q}}{-0.2} \Big|_0^{5.0106} - 551.7672$

$[\because \int e^{ax} dx = \frac{e^{ax}}{a} + c]$

$= \cancel{-1500 \times e^{(-0.2 \times 5.0106 + 0.2 \times 0)} - 551.7672}$

$= \cancel{-1500 \times e^{-1.00212} - 551.7672}$

$= \cancel{-1500 \times 0.3671 - 551.7672}$

$= \cancel{-550.650 - 551.7672}$

$=$

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$= -1500 \times \left[e^{-0.2 \times 5.0106} - e^{-0.2 \times 0} \right] - 551.7672$

$= -1500 \times \left(e^{-1.00212} - e^{0} \right) - 551.7672$

$= -1500 \times (0.3671 - 1) - 551.7672$

$= 949.34 - 551.7672$

$= 397.58$

Therefore the consumer's surplus is 397.58.

Also,

Producer's Surplus (P.S) $= \int P \times Q - \int_0^Q P_s(Q)\, dQ$

$= 110.12 \times 5.0106 - \int_0^{5.0106} 2e^{0.8Q}\, dQ$

$= 551.7672 - 2 \int_0^{5.0106} e^{0.8Q}\, dQ$

$= 551.7672 - 2 \times \frac{e^{0.8Q}}{0.8} \Big|_0^{5.0106}$

$\left[\because \int e^{ax} dx = \frac{e^{ax}}{a} \right]$

$= 551.7672 - \frac{2}{0.8} \times \left(e^{0.8 \times 5.0106} - e^{0.8 \times 0} \right)$

$= 551.7672 - 2.5 \left(e^{4.00848} - e^{0} \right)$

$= 551.7672 - 2.5 (55.06311 - 1)$

$= 551.7672 - 135.1577$

$= 416.6094$

Therefore the producer's surplus is 416.6094

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9. If the demand function $p = 25 - x - 0.3x^2$, by how much does the consumer's ~~tha~~ surplus change if x increases from 5 to 6 units?

Solution

Given, ~~p=~~ Demand Function: $p = 25 - x - 0.3x^2$

when $x = 5$,

$$P = 25 - 5 - 0.3 \times 5^2$$
$$= 25 - 5 - 7.5$$
$$\therefore P = 12.5$$

Now, we know,

$$\text{Consumer's Surplus (CS)} = \int_0^x P\,d(x)\,dx - P \times x$$
$$= \int_0^5 (25 - x - 0.3x^2)\,dx - 12.5 \times 5$$
$$= 25\int_0^5 dx - 1\int_0^5 x\,dx - 0.3\int_0^5 x^2\,dx - 62.5$$
$$= 25 \times x \Big|_0^5 - 1 \times \frac{x^2}{2}\Big|_0^5 - 0.3 \times \frac{x^3}{3}\Big|_0^5 - 62.5$$

$$\left[\because \int dx = x + c,\ \int x^n\,dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1\right]$$

$$= 25 \times (5 - 0) - \frac{1}{2} \times (5^2 - 0^2) - \frac{0.3}{3} \times (5^3 - 0^3) - 62.5$$
$$= 125 - 12.5 - 0.1 \times 125 - 62.5$$
$$= 125 - 12.5 - 12.5 - 62.5$$
$$= 37.5$$

Again,

~~when $x = 9$,~~

$$\cancel{P = 25 - 9 - 0.3 \times 9^2}$$
$$\cancel{= 25 - 9 - 24.3}$$
$$\cancel{\therefore P =}$$

when $x = 6$,

$$P = 25 - 6 - 0.3 \times 6^2$$
$$= 25 - 6 - 10.8$$
$$\therefore p = 8.2$$

Now, we know,

Consumers Surplus (C.S) $= \int_0^x P\,d(x)\,dx - P \times x$

$$= \int_0^6 (25 - x - 0.3x^2)\,dx - 8.2 \times 6$$

$$= 25\int_0^6 dx - 1\int_0^6 x\,dx - 0.3\int_0^6 x^2\,dx - 49.2$$

$$= 25 \times x\Big|_0^6 - 1 \times \frac{x^2}{2}\Big|_0^6 - 0.3 \times \frac{x^3}{3}\Big|_0^6 - 49.2$$

$$= 25 \times (6 - 0) - \frac{1}{2} \times (6^2 - 0^2) - \frac{0.3}{3} \times (6^3 - 0^3) - 49.2$$

$$= 150 - 18 - 21.6 - 49.2$$
$$= 61.2$$

Then,

Change in consumer surplus $= 61.2 - 37.5$
$= 23.7$

Therefore, the consumer's surplus change by 23.7 if x increases from 5 to 6 units.

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10. The demand and supply law under a pure competition are given by $p_d = 23 - x^2$ and $p_s = 2x^2 - 4$. Find the consumer's surplus, producer's surplus and the total surplus at the market equilibrium price.

Solution

Given, $P_d = 23 - x^2$

i.e. $P_d(x) = 23 - x^2$ —— (i)

$P_s = 2x^2 - 4$

i.e. $P_s(x) = 2x^2 - 4$ —— (ii)

Now, At Equilibrium, $P_d = P_s$,

or, $23 - x^2 = 2x^2 - 4$

or, $23 + 4 = 2x^2 + x^2$

or, $27 = 3x^2$

or, $x^2 = 9$

Either $x = 3$

or, $x = -3$ (Rejected)

$\therefore x = 3$

when $x = 3$, From (i)

$P = \cancel{2 \times 3^2 - 4}\ 23 - 3^2$

$= 23 - 9$

$\therefore P = 14$

Now,

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we know that,

Consumer's Surplus (C.S) $= \int_0^a Pd(x)dx - P \times x$

$= \int_0^3 (23-x^2)dx - 14 \times 3$

$= 23\int_0^3 dx - \int_0^3 x^2 - 42$

$= 23 \times x\Big|_0^3 - \frac{x^{2+1}}{2+1}\Big|_0^3 - 42$

$\left[\because \int dx = x+C, \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right]$

$= 23 \times (3-0) - \frac{1}{3} \times (3^3 - 0^3) - 42$

$= 69 - 9 - 42$
$= 18$

Therefore, the consumer's surplus is 18.

Also,

~~Producer Surplus (P.S) = $\int_0^a Ps(x)dx$~~

Producer Surplus (P.S) $= P \times x - \int_0^a Ps(x)dx$

$= 14 \times 3 - \int_0^3 (2x^2-4)dx$

$= 42 - \int_0^3 2x^2 dx + \int_0^3 4 dx$

$= 42 - 2\int_0^3 x^2 dx + 4\int_0^3 dx$

$= 42 - 2 \times \frac{x^3}{3}\Big|_0^3 + 4 \times x\Big|_0^3$ $\left[\because \int dx = x+C, \int x^n dx = \frac{x^{n+1}}{n+1} + C\right]$

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$$= 42 - \frac{2}{3} \times (3^3 - 0^3) + 4 \times (3-0)$$

$$= 42 - \frac{2}{3} \times 27 + 12$$

$$= 42 - 18 + 12$$
$$= 36$$

Therefore, the producer's surplus is 36.

Again,
Total Surplus = ~~Prod~~ Consumer's surplus + Producer's surplus
= 18 + 36
= 54

Therefore, the total surplus is 54.

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11. In a perfect competition, the demand and supply curve of a commodity are given by $P_d = 40 - x^2$ and $P_s = 3x^2 + 8x + 8$. Find the consumer's surplus, producers surplus and total surplus at the market equilibrium price.

Solution

Given, $P_d = 40 - x^2$

i.e. $P_d(x) = 40 - x^2$ —— (i)

$P_s = 3x^2 + 8x + 8$

i.e. $P_s(x) = 3x^2 + 8x + 8$ —— (ii)

At Equilibrium, $P_d = P_s$

or, $40 - x^2 = 3x^2 + 8x + 8$

or, $40 - x^2 - 3x^2 - 8x - 8 = 0$

or, $-4x^2 - 8x + 32 = 0$

or, $-4(x^2 + 2x - 8) = 0$

or, $x^2 + 2x - 8 = 0$

or, $x^2 + 4x - 2x - 8 = 0$

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or, $x(x+4) - 2(x+4) = 0$
or, $(x+4)(x-2) = 0$

Either
$x+4=0$
$\therefore x = -4$ [Rejected]

or,
$x-2=0$
$\therefore x = 2$

$\therefore x = 2$

Substituting value of x in ①
$P = 40 - 2^2$
$= 40 - 4$
$\therefore P = 36$

Now, we know that,

Consumer's Surplus (C.S) $= \int_0^x P_d(x)\,dx - P \times x$

$= \int_0^2 (40 - x^2)\,dx - 36 \times 2$

$= 40\int_0^2 dx - \int_0^2 x^2\,dx - 72$

$= 40 \times x\Big|_0^2 - \frac{x^{2+1}}{2+1}\Big|_0^2 - 72$

$\left[\because \int dx = x + c,\ \int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1\right]$

$= 40 \times (2-0) - \frac{1}{3} \times (2^3 - 0^3) - 72$

$= 80 - \frac{8}{3} - 72$

$= \frac{16}{3}$

Therefore consumer's surplus is $\frac{16}{3}$

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Also,

Producer's Surplus (PS) $= P \times x - \int_0^x P_s(x)\,dx$

$= 36 \times 2 - \int_0^2 (3x^2 + 8x + 8)\,dx$

$= 72 - 3\int_0^2 x^2 - 8\int_0^2 x - 8\int_0^2 dx$

$= 72 - 3 \times \frac{x^3}{3}\Big|_0^2 - 8 \times \frac{x^2}{2}\Big|_0^2 - 8 \times x\Big|_0^2$

$= 72 - x^3\Big|_0^2 - 4x^2\Big|_0^2 - 8x\Big|_0^2$

$= 72 - (2^3 - 0^3) - 4(2^2 - 0^2) - 8 \times (2-0)$

$= 72 - 8 - 4 \times 4 - 16$

$= 72 - 8 - 16 - 16$

$= 32$

Therefore, the producer's surplus is 32.

Again,

Total surplus = Consumer's Surplus + Producer's surplus

$= \frac{16}{3} + 32$

$= \frac{16 + 96}{3}$

$= \frac{112}{3}$

Therefore, the total surplus is $\frac{112}{3}$.

12. The demand and supply curves are given by $P_d = \frac{16}{x+4}$ and $P_s = \frac{x}{2}$. Find the consumer's surplus, producer's surplus and total surplus at the market equilibrium price.

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Solution

Given,

$$pd = \frac{16}{x+4} \quad \text{——— (I)}$$

$$ps = \frac{x}{2} \quad \text{——— (II)}$$

At Equilibrium, $pd = ps$

or, $\frac{16}{x+4} = \frac{x}{2}$

or, $x^2 + 4x = 32$

or, $x^2 + 4x - 32 = 0$

or, $x^2 + 8x - 4x - 32 = 0$

or, $x(x+8) - 4(x+8) = 0$

or, $(x+8)(x-4) = 0$

Either $x+8 = 0$, $\therefore x = -8$ (Rejected)

or, $x-4 = 0$, $\therefore x = 4$

Substituting the value of x in (II)

$P = \frac{4}{2}$

$\therefore P = 2$

Now, we know,

Consumer's Surplus (C.S) $= \int_0^x P(x)dx - P \times x$

$= \int_0^4 \frac{16}{x+4} dx - 2 \times 4$

$= 16 \times \int_0^4 \frac{1}{x+4} dx - 8$

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$= \cancel{16 \times \int_0^4 (x+4)^{-1} dx - 8}$

$= 16 \times \ln(x+4) |_0^4 - 8$

$= 16 \times (\ln(4+4) - \ln(0+4)) - 8$

$= 16 \times (\ln 8 - \ln 4) - 8$

$= 16 \times \ln\left(\frac{8}{4}\right) - 8$

$= 16 \times \ln 2 - 8$

$= 16 \ln 2 - 8$

Therefore, the consumer's surplus is $16 \ln 2 - 8$

Also,

Producers Surplus (P.S) $= P \times x - \int_0^a P_s(x)\,dx$

$= 2 \times 4 - \int_0^4 \frac{x}{2}\,dx$

$= 8 - \frac{1}{2} \int_0^4 x\,dx$

$= 8 - \frac{1}{2} \times \frac{x^2}{2} |_0^4$

$= 8 - \frac{1}{4} \times (4^2 - 0^2)$

$= 8 - \frac{1}{4} \times 16$

$= 8 - 4$

$= 4$

Therefore, the producers surplus is 4.

Again,

Total surplus = Consumer's Surplus + Producer's Surplus

$= 16 \ln 2 - 8 + 4$

$= 16 \ln 2 - 4$

Therefore, the total surplus is $16 \ln 2 - 4$