Exercise 5(E)

1. Find the area bounded by x-axis, the following curves and ordinates.

a) The curve $y = 2x - x^2$, $x = 0$, $x = 2$.

Solution

Given, $y = 2x - x^2$, $x = 0$, $x = 2$

Required area $= \int_a^b y \, dx$

$= \int_0^2 (2x - x^2) \, dx$

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$$= \int_0^2 2x\,dx + \int_0^2 -x^2\,dx$$

$$= 2\int_0^2 x\,dx + (-1)\int_0^2 x^2\,dx$$

$$= 2 \times \frac{x^{1+1}}{1+1}\Big|_0^2 + (-1)\cdot\frac{x^{2+1}}{2+1}\Big|_0^2$$

$$= x^2\Big|_0^2 - \frac{x^3}{3}\Big|_0^2$$

$$= [(2)^2 - (0)^2] - \left[\frac{2^3}{3} - \frac{0^3}{3}\right]$$

$$= 4 - \frac{8}{3}$$

$$= \frac{12-8}{3}$$

$$= \frac{4}{3} \text{ sq. units}$$

Therefore, the required area is $\frac{4}{3}$ sq. units

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b) The curve $y^2 = 4ax$, $x=0$, $x=a$

Solution

Given, $y^2 = 4ax$, $x=0$, $x=a$

$\therefore y = \pm\sqrt{4ax} = \pm 2\sqrt{ax}$ [$\because$ Negative sign is neglected]

Required Area $= \int_a^b y\,dx$

$$= \int_0^a \cancel{2\sqrt{a}\sqrt{x}\,dx}\ 2\sqrt{ax}\,dx$$

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$$= 2 \cdot \int_0^a \sqrt{ax}\, dx$$

$$= 2 \cdot \int_0^a (ax)^{1/2}\, dx$$

$$= 2 \cdot \int_0^a (ax+0)^{1/2}\, dx$$

$$= 2 \cdot \frac{(ax+0)^{\frac{1}{2}+1}}{a\left(\frac{1}{2}+1\right)} \Bigg|_0^a \quad \left[\because \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c,\ n \neq -1\right]$$

$$= 2 \cdot \frac{(ax)^{3/2}}{a \times \frac{3}{2}} \Bigg|_0^a$$

$$= \frac{4}{3} \cdot \frac{(ax)^{3/2}}{a} \Bigg|_0^a$$

$$= \frac{4}{3} \cdot \frac{1}{a} \left[(ax)^{3/2} \Big|_0^a \right]$$

$$= \frac{4}{3a} \left[(a.a)^{3/2} - (a.0)^{3/2} \right]$$

$$= \frac{4}{3a} \left(a^3 - 0 \right)$$

$$= \frac{4}{3a} \times a^3$$

$$= \frac{4}{3} a^2 \text{ sq. units}$$

Therefore, the required area is $\frac{4}{3} a^2$ sq. units.

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c) The curve $y = 4 - x^2$, $x = \pm 2$.

Solution

Given, $y = 4 - x^2$, $x = 2$, $x = -2$

Required Area $= \int_a^b y \, dx$

$= \int_{-2}^{2} (4 - x^2) \, dx$

$= \int_{-2}^{2} (4) \, dx - \int_{-2}^{2} x^2 \, dx$

$= 4 \int_{-2}^{2} dx - \int_{-2}^{2} x^2 \, dx$

$= 4 \times x \Big|_{-2}^{2} - \frac{x^{2+1}}{2+1} \Big|_{-2}^{2}$

$= 4 \times x \Big|_{-2}^{2} - \frac{x^3}{3} \Big|_{-2}^{2}$

$= 4 \times x \Big|_{-2}^{2} - \frac{1}{3} \times x^3 \Big|_{-2}^{2}$

$= 4 \times (2 - (-2)) - \frac{1}{3} \times (2^3 - (-2)^3)$

$= 4 \times (2 + 2) - \frac{1}{3} \times (8 + 8)$

$= 16 - \frac{1}{3} \times 16$

$= 16 - \frac{16}{3}$

$= \frac{32}{3}$ sq. units

Therefore the required area is $\frac{32}{3}$ sq. units

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d) The curve $y = 2-3x^2$, $x=1$, $x=2$.

Solution

Given, $y = 2-3x^2$, $x=1$, $x=2$

Required area $= \int_a^b y \, dx$

$$= \int_1^2 (2-3x^2)\, dx$$

$$= \int_1^2 2\,dx - \int_1^2 3x^2\, dx$$

$$= 2\int_1^2 dx - 3\int_1^2 x^2\, dx$$

$$= 2 \times x \Big|_1^2 - 3 \times \frac{x^{2+1}}{2+1}\Big|_1^2 \quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right]$$

$$= 2\times x\Big|_1^2 - \cancel{\frac{3}{3}}\, x^3 \Big|_1^2$$

$$= 2\times(2-1) - (2^3 - 1^3)$$

$$= 2\times 1 - (8-1)$$

$$= 2 - 7$$

$$= -5$$

$= 5$ square units [$\because$ Area must be +ve]

Therefore, the required area is 5 sq. units.

e) The curve $y = 2x^2$, $x=0$, $x=3$

Solution

Given, $y = 2x^2$, $x=0$, $x=3$

Required area $= \int_a^b y\, dx$

$$= \int_0^3 (2x^2)\, dx$$

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$= 2\int_0^3 x^2\,dx$

$= 2\left[\frac{x^3}{3}\right]\Big|_0^3$ $\left[\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1\right]$

$= \frac{2}{3} \times x^3\Big|_0^3$

$= \frac{2}{3} \times (3^3 - 0^3)$

$= \frac{2}{3} \times 27$

$= 18$ sq.units

Therefore, the required area is 18 sq.units

f) The curve $y = x^2 - 2x + 3,\ x = 0,\ x = 3$

Given,

$y = x^2 - 2x + 3,\ x = 0,\ x = 3$

Required area $= \int_a^b y\,dx$

$= \int_0^3 (x^2 - 2x + 3)\,dx$

$= \int_0^3 x^2\,dx - \int_0^3 2x\,dx + \int_0^3 3\,dx$

$= \cancel{\int_0^3} \frac{x^{2+1}}{2+1}\Big|_0^3 - 2.\frac{x^{1+1}}{1+1}\Big|_0^3 + 3.x\Big|_0^3$

$= \frac{x^3}{3}\Big|_0^3 - x^2\Big|_0^3 + 3.x\Big|_0^3$

$= \frac{1}{3}(3^3 - 0^3) - (3^2 - 0^2) + 3.(3-0)$

$= \frac{1}{3} \times 27 - 9 + 9$

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$= 9$ sq. units

Therefore, the required area is 9 sq units.

g) The curve $x^2 - 3y + 5 = 0$, $x = 1$, $x = 3$.

Solution

Given, $x^2 - 3y + 5 = 0$

or, $3y = x^2 + 5$

or, $y = \frac{x^2}{3} + \frac{5}{3}$

or, $y = \frac{x^2+5}{3}$

Required area $= \int_a^b y\,dx$

$= \int_1^3 \frac{x^2+5}{3}\,dx$

$= \frac{1}{3}\int_1^3 (x^2+5)\,dx$

$= \frac{1}{3}\int_1^3 x^2\,dx + \int_1^3 5\,dx$

$= \frac{1}{3}\left[\frac{x^{2+1}}{2+1}\Big|_1^3 + 5.x\Big|_1^3\right]$

$= \frac{1}{3}\left[\frac{x^3}{3}\Big|_1^3 + 5.x\Big|_1^3\right]$

$= \frac{1}{3}\left[\frac{(3^3-1^3)}{3} + 5.(3-1)\right]$

$= \frac{1}{3}\left[\frac{26}{3} + 10\right]$

$= \frac{1}{3}(9+10) = \frac{1}{3} \times \frac{56}{3}$

$= \frac{56}{9}$ sq. units

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h) The curve $3x^2 = 4by$, $x = 0$, $x = b$

Solution

Given, $3x^2 = 4by$

or, $y = \frac{3x^2}{4b}$

Required area $= \int_a^b y\,dx$

$= \int_0^b \frac{3x^2}{4b}\,dx$

$= \frac{3}{4b}\int_0^b x^2\,dx$

$= \frac{3}{4b} \times \frac{x^{2+1}}{2+1}\Big|_0^b$ $\left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right]$

$= \frac{1}{4b} \times x^3 \Big|_0^b$

$= \frac{1}{4b} \times (b^3 - 0^3)$

$= \frac{1}{4b} \times b^3$

$= \frac{b^2}{4}$ sq units

Therefore, the required area is $\frac{b^2}{4}$ sq. units.

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i) The curve $y = x^2 - 2x + 3$, $x = 0$, $x = 3$

Solution

Given, $y = x^2 - 2x + 3$

Required area $= \int_a^b y\,dx$

$= \int_0^3 (x^2 - 2x + 3)\, dx$

$= \int_0^3 x^2 dx - 2\int_0^3 x\, dx + 3\int_0^3 dx$

$= \frac{x^{2+1}}{2+1}\Big|_0^3 - 2 \times \frac{x^{1+1}}{1+1}\Big|_0^3 + 3 \times x\Big|_0^3$

$\left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right]$

$= \frac{x^3}{3}\Big|_0^3 - x^2\Big|_0^3 + 3x\Big|_0^3$

$= \frac{1}{3}(3^3 - 0^3) - (3^2 - 0^2) + 3 \times (3-0)$

$= \frac{1}{3} \times 27 - 9 + 9$

$=$ 9 sq.units.

Therefore, the required area is 9 sq.units.

2. Find the area bounded by the x-axis and following curves:

a) $f(x) = -x^2 - 2x$

Solution

Given, $f(x) = -x^2 - 2x$

~~In x-axis, the value of y is 0.~~

~~Required area $= \int_a^b f(x)\, dx$~~

Let $y = f(x)$

$y = -x^2 - 2x$

In x-axis, the value of y is '0'.

or $-x^2 - 2x = 0$

or $x^2 + 2x = 0$

or $x(x+2) = 0$

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$\therefore x = 0, -2$

We know,

Required area $= \int_a^b f(x)\,dx$

$= \int_{-2}^{0} (-x^2 - 2x)\,dx$

$= -1\int_{-2}^{0} x^2\,dx - 2\int_{-2}^{0} x\,dx$

$= -1 \times \frac{x^{2+1}}{2+1}\Big|_{-2}^{0} - 2 \times \frac{x^{1+1}}{1+1}\Big|_{-2}^{0}$

$= -\frac{1}{3} \times x^3\Big|_{-2}^{0} - x^2\Big|_{-2}^{0}$

$= -\frac{1}{3}\left[0^3 - (-2)^3\right] - \left[0^2 - (-2)^2\right]$

$= -\frac{1}{3}(0+8) - (0-4)$

$= -\frac{8}{3} + 4$

$= \frac{4}{3}$ sq units

Therefore, the required area is $\frac{4}{3}$ sq. units

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b) $f(x) = x^2 - 3x + 2$

Solution

Let $y = f(x)$. Then,

$y = x^2 - 3x + 2$

In x-axis, value of y is 0.

$x^2 - 3x + 2 = 0$

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or, $x^2 - 2x - x + 2 = 0$
or, $x(x-2) - 1(x-2) = 0$
or, $(x-2)(x-1) = 0$
$\therefore x = 2, 1$

We know,

Required area $= \int_a^b f(x)\,dx$

$= \int_1^2 (x^2 - 3x + 2)\,dx$

$= \int_1^2 x^2\,dx - 3\int_1^2 x\,dx + 2\int_1^2 dx$

$= \frac{x^{2+1}}{2+1}\Big|_1^2 - 3 \times \frac{x^{1+1}}{1+1}\Big|_1^2 + 2 \times x\Big|_1^2$

$= \frac{1}{3} \times x^3\Big|_1^2 - \frac{3}{2} \times x^2\Big|_1^2 + 2 \times x\Big|_1^2$

$= \frac{1}{3} \times (2^3 - 1^3) - \frac{3}{2} \times (2^2 - 1^2) + 2 \times (2-1)$

$= \frac{1}{3} \times 7 - \frac{3}{2} \times 3 + 2 \times 1$

$= \frac{7}{3} - \frac{9}{2} + 2$

$= -\frac{1}{6}$

$= \frac{1}{6}$ sq. units [$\because$ Area must be +ve]

Therefore, the required area is $\frac{1}{6}$ sq. units

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2. Find the area bounded by the x-axis and following curves.

c) $f(x) = (x+1)(x-2)(x-3)$

Solution

Given,

$f(x) = (x+1)(x-2)(x-3)$

$= x^2 - 2x + x - 2\ (x-3)$

$= x^2 - x - 2\ (x-3)$

$= x^3 - 3x^2 - x^2 + 3x - 2x + 6$

$\therefore f(x) = x^3 - 4x^2 + x + 6$

Let $y = f(x)$. Then,

$y = x^3 - 4x^2 + x + 6$

In x-axis, $y = 0$

or, $0 = x^3 - 4x^2 + x + 6$

$\therefore x = -1, 3, 2$

Now,

$A_1 = \int_{-1}^{2} (x^3 - 4x^2 + x + 6)\, dx$

$= \int_{-1}^{2} x^3 dx - 4\int_{-1}^{2} x^2 dx + \int_{-1}^{2} x\, dx + 6\int_{-1}^{2} dx$

$= \frac{x^{3+1}}{3+1}\Big|_{-1}^{2} - 4 \times \frac{x^{2+1}}{2+1}\Big|_{-1}^{2} + \frac{x^2}{2}\Big|_{-1}^{2} + 6 \times x\Big|_{-1}^{2}$

$\left[\because \int dx = x + c,\ \int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1\right]$

$$= \frac{1}{4} \times [(2)^4 - (-1)^4] - \frac{4}{3} \times [2^3 - (-1)^3] + \frac{1}{2} \times [2^2 - (-1)^2] + 6 \times [2 \times (-1)]$$

$$= \frac{1}{4} \times 15 - \frac{4}{3} \times 9 + \frac{1}{2} \times 3 + 6 \times -2$$

$$= \frac{15}{4} - 12 + \frac{3}{2} - 12$$

$$= -\frac{75}{4}$$

let $I = \int (x^3 - 4x^2 + x + 6)\, dx$

$$= \int x^3 dx - 4\int x^2 dx + \int x\, dx + 6\int dx$$

$$= \frac{x^{3+1}}{3+1} - 4 \times \frac{x^{2+1}}{2+1} + \frac{x^{1+1}}{1+1} + 6 \times x$$

$$= \frac{x^4}{4} - \frac{4}{3} \times x^3 + \frac{x^2}{2} + 6x$$

$$= \frac{3x^4 - 16x^3 + 6x^2 + 72x}{12}$$

$$= \frac{1}{12} \times (3x^4 - 16x^3 + 6x^2 + 72x)$$

Now,

$$A_1 = \int_{-1}^{2} (x^3 - 4x^2 + x + 6)\, dx$$

$$= \frac{1}{12} \cancel{\int_{-1}} \left[(3x^4 - 16x^3 + 6x^2 + 72x)\right]\Big|_{-1}^{2}$$

$$\cancel{= \frac{1}{12} \left[3 \times \int_{-1}^{2} x^4 \right.}$$

$$= \frac{1}{12} \left[3 \times (2^4 - (-1)^4) - 16 \times (2^3 - (-1)^3) + 6 \times (2^2 - (-1)^2) + 72 \times (2 - (-1)) \right]$$

$$= \frac{1}{12} (45 - 144 + 18 + 216)$$

$= \frac{1}{12} \times 135$

$= \frac{45}{4}$ sq units

Also,

$A_2 = \int_2^3 (x^3 - 4x^2 + x + 6)\, dx$

$= \frac{1}{12} \times \left[3x^4 - 16x^3 + 6x^2 + 72x\right]\Big|_2^3$

$= \frac{1}{12} \times \left[3 \times (3^4 - 2^4) - 16 \times (3^3 - 2^3) + 6 \times (3^2 - 2^2) + 72 \times (3-2)\right]$

$= \frac{1}{12} \times (195 - 304 + 30 + 72)$

$= \frac{1}{12} \times (-7)$

$= -\frac{7}{12}$ ~~sq units~~

$= \frac{7}{12}$ sq units [∵ Area must be +ve]

Now,

Total Area $= A_1 + A_2$

$= \left[\frac{45}{4} + \left(+\frac{7}{12}\right)\right]$ sq units

$= \left(\frac{45}{4} + \frac{7}{12}\right)$ sq units

$= \cancel{\frac{32}{3}}$ ~~sq. units~~ $= \frac{71}{6}$ sq units

Therefore the required area is $\cancel{\frac{32}{3}}$ $\frac{71}{6}$ sq units

$\frac{71}{6}$ sq. units

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d) $y = x(x-1)(x-2)$

Solution

Given,

$$y = x(x-1)(x-2)$$
$$= x(x^2-2x-x+2)$$
$$= x(x^2-3x+2)$$
$$\therefore y = x^3-3x^2+2x$$

In x-axis, $y=0$

or $x^3-3x^2+2x=0$

$\therefore x = 0, 1, 2$

Now,

$$\text{Let } I = \int (x^3-3x^2+2x)\,dx$$
$$= \int x^3\,dx - 3\int x^2\,dx + 2\int x\,dx$$
$$= \frac{x^{3+1}}{3+1} - 3\times\frac{x^{2+1}}{2+1} + 2\times\frac{x^2}{2}$$
$$= \frac{x^4}{4} - x^3 + x^2$$

Then,

$$A_1 = \int_0^1 (x^3-3x^2+2x)\,dx$$
$$= \frac{1}{4}\times(x^4-4x^3+4x^2)\Big|_0^1$$
$$= \frac{1}{4}\times\left[(1)^4-0^4) - 4\times(1^3-0^3) + 4\times(1^2-0^2)\right]$$
$$= \frac{1}{4}\times[1-4+4]$$
$$= \frac{1}{4} \text{ sq. units}$$

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Also,

$$A_2 = \int_1^2 (x^3 - 3x^2 + 2x)\,dx$$

$$= \frac{1}{4} \times (x^4 - 4x^3 + 4x^2) \Big|_1^2$$

$$= \frac{1}{4} \times \left[(2^4 - 1^4) - 4 \times (2^3 - 1^3) + 4 \times (2^2 - 1^2)\right]$$

$$= \frac{1}{4} \times (15 - 28 + 12)$$

$$= \frac{1}{4} \times -1$$

$$= -\frac{1}{4}$$

$= \frac{1}{4}$ sq units. [$\because$ Area must be positive]

Now,

Total Area $= A_1 + A_2$

$= \left(\frac{1}{4} + \frac{1}{4}\right)$ sq units

$= \frac{2}{4}$ sq units

$= \frac{1}{2}$ sq units

Therefore the required area is $\frac{1}{2}$ sq. units

e) $f(x) = x^2 - 3x + 2$

Solution

Given,

$f(x) = x^2 - 3x + 2$

$\cancel{= x^2 - 2x - x + 2}$

$\cancel{= x(x-2) - 1(x-2)}$

$\cancel{= (x-2)(x-1)}$

Let $y = f(x)$. Then,

$y = x^2 - 3x + 2$

In x-axis, $y = 0$

or, $0 = x^2 - 3x + 2$

$\cancel{= x =}$ or, $x^2 - 2x - x + 2 = 0$

or, $x(x-2) - 1(x-2) = 0$

or, $(x-2)(x-1) = 0$

$\therefore x = 1, 2$

We know,

Required area $= \int_a^b f(x)\,dx$

$= \int_1^2 (x^2 - 3x + 2)\,dx$

$= \int_1^2 x^2\,dx - 3\int_1^2 x\,dx + 2\int_1^2 dx$

$= \frac{1}{3} \times x^3 \Big|_1^2 - 3 \times \frac{1}{2} x^2 \Big|_1^2 + 2 \times x \Big|_1^2$

$= \frac{1}{3} x^3 \Big|_1^2 - \frac{3}{2} x^2 \Big|_1^2 + 2x \Big|_1^2$

$= \frac{1}{3}(2^3 - 1^3) - \frac{3}{2} \times (2^2 - 1^2) + 2 \times (2-1)$

$= \frac{1}{3} \times 7 - \frac{3}{2} \times 3 + 2$

$= \frac{7}{3} - \frac{9}{2} + 2$

$= -\frac{1}{6}$

$= \frac{1}{6}$ [∵ Area must be +ve]

let $I = \int (x^2 - 3x + 2)\, dx$

$= \int x^2\, dx - 3\int x + 2\int dx$

$= \frac{x^3}{3} - 3 \cdot \frac{x^2}{2} + 2 \times x$

$= \frac{x^3}{3} - \frac{3}{2}x^2 + 2x$

$= \frac{2x^3 - 9x^2 + 12x}{6}$

$= \frac{1}{6}\left[2x^3 - 9x^2 + 12x\right]$

We know,

Required area $= \int_a^b f(x)\, dx$

$= \int_1^2 (x^2 - 3x + 2)\, dx$

$= \frac{1}{6} \times \left[2x^3 - 9x^2 + 12x\right]\Big|_1^2$

$= \frac{1}{6} \times \left[2 \times (2^3 - 1^3) - 9 \times (2^2 - 1^2) + 12 \times (2-1)\right]$

$= \frac{1}{6} \times \left[14 - 27 + 12\right]$

$= -\frac{1}{6}$

$= \frac{1}{6}$ [∵ Area must be +ve]

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Therefore, the required area is $\frac{1}{6}$ sq units.

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f) $f(x) = x^2 - 9$

Solution

Given,

$f(x) = x^2 - 9$

Let $y = f(x)$. Then,

$y = x^2 - 9$

In x-axis, $y = 0$

or, $x^2 - 9 = 0$

or, $x^2 = 9$

$\therefore x = \pm 3 = 3, -3$

We know,

Required area $= \int_a^b f(x)\, dx$

$$= \int_{-3}^{3} (x^2 - 9)\, dx$$

$$= \int_{-3}^{3} x^2\, dx - 9 \int_{-3}^{3} dx$$

$$= \frac{x^{2+1}}{2+1}\Big|_{-3}^{3} - 9 \times x\Big|_{-3}^{3}$$

$$= \frac{1}{3} x^3\Big|_{-3}^{3} - 9 \times x\Big|_{-3}^{3}$$

$$= \frac{1}{3} \times [3^3 - (-3)^3] - 9 \times [3 - (-3)]$$

$$= \frac{1}{3} \times (27 + 27) - 9 \times (3 + 3)$$

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$= \frac{54}{3} - 54$

$= -36$

$= 36$ sq. units [$\because$ Area must be +ve]

Therefore, the required area is 36 sq. units

g) $f(x) = x^2 - 25$

Solution

Given,

$f(x) = x^2 - 25$

Let $y = f(x)$

$y = x^2 - 25$

In x-axis, $y = 0$

or, $x^2 - 25 = 0$

Or, $x^2 = 25$

or, $x = \pm 5$

$\therefore x = 5, -5$

we know,

Required Area $= \int_a^b f(x)\,dx$

$= \int_{-5}^{5} (x^2 - 25)\,dx$

$= \int_{-5}^{5} x^2\,dx - 25 \int_{-5}^{5} dx$

$= \frac{x^{2+1}}{2+1}\Big|_{-5}^{5} - 25 \times x\Big|_{-5}^{5}$

[$\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$] [$\because \int dx = x + c$]

$= \frac{1}{3} \times x^3 \Big|_{-5}^{5} - 25 \times x \Big|_{-5}^{5}$

$= \frac{1}{3} \times [(5)^3 - (-5)^3] - 25 \times [5-(-5)]$

$= \frac{1}{3}(125+125) - 25 \times (5+5)$

$= \frac{250}{3} - 250$

$= -\frac{500}{3}$

$= \frac{500}{3}$ sq. units [∵ Area is always +ve]

Therefore, the required area is $\frac{500}{3}$ sq. units.

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h) $y = x^2 - 8x + 15$

Solution

Given,

$y = x^2 - 8x + 15$

~~In the~~ At the x-axis, the value of y is 0,

or, $x^2 - 8x + 15 = 0$

or, $x^2 - 5x - 3x + 15 = 0$

or, $\cancel{x^2}\ x(x-5) - 3(x-5) = 0$

or, $(x-5)(x-3) = 0$

Either, $x - 5 = 0$ ∴ $x = 5$

Or, $x - 3 = 0$ ∴ $x = 3$

∴ $x = 3, 5$

Now,

Required area $= \int_a^b y\,dx$

$= \int_3^5 (x^2 - 8x + 15)\,dx$

$= \int_3^5 x^2\,dx - 8\int_3^5 x\,dx + 15\int_3^5 dx$

$= \frac{x^{2+1}}{2+1}\Big|_3^5 - 8\times\frac{x^{1+1}}{1+1}\Big|_3^5 + 15\times x\Big|_3^5$

$= \frac{1}{3}\times x^3\Big|_3^5 - \cancel{8}4\times x^2\Big|_3^5 + 15\times x\Big|_3^5$

$= \frac{1}{3}\times(5^3 - 3^3) - 4\times(5^2 - 3^2) + 15\times(5-3)$

$= \frac{1}{3}\times 98 - 4\times 16 + 15\times 2$

$= \frac{98}{3} - 64 + 30$

$= -\frac{4}{3}$

$= \frac{4}{3}$ sq.unit [∵ Area is always +ve]

Therefore, the ~~area bounded by the x-axis~~ required area is $\frac{4}{3}$ sq. units

i) $y = x^3 - x^2 - 3x$

Solution

Given,

$y = x^3 - x^2 - 3x$

At the x-axis value of y is 0,

or, $x^3 - x^2 - 3x = 0$

or, $x(x^2 - x - 3) = 0$

~~or, $x[x^2$~~

$\therefore x = 0,\ \cancel{2.30}\ 2.30,\ -1.3027$

Now,

~~Requir~~ $A_1 = \int_{-1.30}^{0} (x^3 - x^2 - 3x)\, dx$

$= \int_{-1.30}^{0} x^3\, dx - \int_{-1.30}^{0} x^2\, dx - 3 \times \int_{-1.30}^{0} x\, dx$

$= \frac{x^{3+1}}{3+1}\Big|_{-1.30}^{0} - \frac{x^{2+1}}{2+1}\Big|_{-1.30}^{0} - 3 \times \frac{x^{1+1}}{1+1}\Big|_{-1.30}^{0}$

$= \frac{1}{4} \times x^4 \Big|_{-1.30}^{0} - \frac{1}{3} \times x^3 \Big|_{-1.30}^{0} - \frac{3}{2} \times x^2 \Big|_{-1.30}^{0}$

$= \frac{1}{4} \times [0^4 - (-1.30)^4] - \frac{1}{3} \times [(0)^3 - (-1.30)^3] - \frac{3}{2} \times (0^2 - (-1.30)^2)$

$= \frac{1}{4} \times (-2.8561) - \frac{1}{3} \times 2.197 - \frac{3}{2} \times -1.69$

$= -0.7140 - 0.7323 + 2.535$

$= 1.0887$ sq units

Also,

$$A_2 = \int_0^{2.30} (x^3 - x^2 - 3x)\,dx$$

$$= \int_0^{2.30} x^3\,dx - \int_0^{2.30} x^2\,dx - 3\int_0^{2.30} x\,dx$$

$$= \frac{x^{3+1}}{3+1}\Big|_0^{2.30} - \frac{x^{2+1}}{2+1}\Big|_0^{2.30} - 3 \times \frac{x^{1+1}}{1+1}\Big|_0^{2.30}$$

$$\left[\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1\right]$$

$$= \frac{1}{4}x^4\Big|_0^{2.30} - \frac{1}{3}x^3\Big|_0^{2.30} - \frac{3}{2} \times x^2\Big|_0^{2.30}$$

$$= \frac{1}{4} \times \left[(2.30)^4 - 0^4\right] - \frac{1}{3} \times \left[(2.30)^3 - 0^3\right] - \frac{3}{2} \times \left[(2.30)^2 - 0^2\right]$$

$$= \frac{1}{4} \times 27.9841 - \frac{1}{3} \times 12.167 - \frac{3}{2} \times 5.29$$

$$= 6.9960 - 4.0556 - 7.935$$

$$= -4.9946$$

$= 4.9946$ sq units [$\because$ Area must be +ve]

Now,

Required area $= A_1 + A_2$

$= (1.0887 + 4.9946)$ sq. units

$= 6.0833$ sq units

Therefore, the required area is 6.0833 sq units.