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$$= -2x \cdot \frac{(a-bx)^{1/2}}{b} - \int \frac{(a-bx)^{1/2}}{-\frac{b}{2}} dx$$

$$= -2x \cdot \frac{(a-bx)^{1/2}}{b} + \frac{2}{b} \int (a-bx)^{1/2} dx$$

$$= -2x \cdot \frac{(a-bx)^{1/2}}{b} + \frac{2}{b} \times \frac{(a-bx)^{\frac{1}{2}+1}}{-b(\frac{1}{2}+1)} + c$$

$$= -2x \cdot \frac{(a-bx)^{1/2}}{b} + \frac{2}{b} \times \frac{(a-bx)^{\frac{3}{2}}}{-b \times \frac{3}{2}} + c$$

$$= -\frac{2x}{b}(a-bx)^{1/2} - \frac{4}{3b^2}(a-bx)^{3/2} + c$$

Exercise 5(c)

1. Evaluate the following integrals.

a) $\int_2^3 x^2 dx$

let the indefinite integral be I i.e. $I = \int x^2 dx$

$$\int x^2 dx = \frac{x^{2+1}}{2+1} \quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1\right]$$

$$\therefore \int x^2 dx = \frac{x^3}{3}$$

So, $\int_2^3 x^2 dx = \left[\frac{x^3}{3}\right]_2^3 = \frac{1}{3}\left[(3)^3 - (2)^3\right]$

$$= \frac{1}{3}(27-8)$$

$$= \frac{19}{3}$$

$$\therefore \int_2^3 x^2 dx = \frac{19}{3}$$

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b) $\int_{2}^{3} \frac{1}{x} dx$

Let the indefinite integral be I i.e. $I = \int \frac{1}{x} dx$

$\int \frac{1}{x} dx = \int \frac{dx}{x}$

$\therefore \int \frac{1}{x} dx = \ln x$

So,

$\int_{2}^{3} \frac{1}{x} dx = [\ln x]_{2}^{3} = \ln(3) - \ln(2)$

$= \ln\left(\frac{3}{2}\right)$ $[\because \ln a - \ln b = \ln(\frac{a}{b})]$

$\therefore \int_{2}^{3} \frac{1}{x} dx = \ln\left(\frac{3}{2}\right)$

~~2. Evaluate the following integrals.~~

~~a) $\int_{0}^{1} \frac{1-x}{1+x} dx$~~

c) $\int_{-1}^{1} (x+1) dx$

Let the indefinite integral be I i.e. $I = \int (x+1) dx$

$\int (x+1) dx = \frac{(x+1)^{1+1}}{1+1}$ $[\because \int (ax+b) dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c]$

$= \frac{(x+1)^2}{2}$

So,

$\int_{-1}^{1} (x+1) dx = \frac{(x+1)^2}{2}\Big|_{-1}^{1}$

$= \frac{(1+1)^2}{2} - \frac{(-1+1)^2}{\cancel{8}\,2}$

$= \frac{(2)^2}{2} - \frac{(0)^2}{2}$

$= 2 - 0$

$= 2$

d) $\int_1^2 (4x^3 - 5x^2 + 6x + 9)\,dx$

Let the indefinite integral be I i.e. $I = \int (4x^3 - 5x^2 + 6x + 9)\,dx$

$\int (4x^3 - 5x^2 + 6x + 9)\,dx = \int 4x^3\,dx - \int 5x^2\,dx + \int 6x\,dx + \int 9\,dx$

$= 4\int x^3\,dx - 5\int x^2\,dx + 6\int x\,dx + \int 9\,dx$

$= 4 \times \frac{x^{3+1}}{3+1} - 5 \times \frac{x^{2+1}}{2+1} + 6 \times \frac{x^{1+1}}{1+1} + 9x$

$\left[\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1,\ \int dx = x + C\right]$

$= 4 \times \frac{x^4}{4} - 5 \times \frac{x^3}{3} + 6 \times \frac{x^2}{2} + 9x$

$= x^4 - \frac{5x^3}{3} + 3x^2 + 9x$

~~Then~~ Now,

$\int_1^2 (4x^3 - 5x^2 + 6x + 9)\,dx\ \cancel{=}\ \cancel{\left[(2)^4 - \frac{5\times(2)^3}{3} + 3\times(2)^2 + 9\times 2\right]}$

$= \left[(2)^4 - \frac{5\times(2)^3}{3} + 3\times(2)^2 + 9\times 2\right] - \left[(1)^4 - \frac{5\times(1)^3}{3} + 3\times(1)^2 + 9\right]$

$= \left(16 - \frac{40}{3} + 12 + 18\right) - \left(1 - \frac{5}{3} + 3 + 9\right)$

$= \frac{98}{3} - \frac{34}{3}$

$= \frac{64}{3}$

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e) $\int_0^1 \frac{1}{\sqrt{5x+3}} dx$

let the indefinite integral be I i.e. $I = \int \frac{1}{\sqrt{5x+3}} dx$

$$I = \int \frac{1}{\sqrt{5x+3}} dx$$
$$= \int (5x+3)^{-1/2} dx$$
$$= \frac{(5x+3)^{-\frac{1}{2}+1}}{5(-\frac{1}{2}+1)} \quad \left[\because \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)}, n \neq -1\right]$$
$$= \frac{2(5x+3)^{1/2}}{5} = \frac{2}{5}\sqrt{5x+3}$$

Now,

$$\int_0^1 \frac{1}{\sqrt{5x+3}} dx = \frac{2(5\times 1+3)^{1/2}}{5} - \frac{2(5\times 0+3)^{1/2}}{5}$$
$$= \frac{2}{5}\sqrt{5+3} - \frac{2}{5}\sqrt{0+3}$$
$$= \frac{2}{5}\sqrt{8} - \frac{2}{5}\sqrt{3}$$
$$= \frac{2}{5}(\sqrt{8} - \sqrt{3})$$

---

f) $\int_0^{1/2} \frac{dx}{\sqrt{1-x}}$

Let the indefinite integral be I i.e. $I = \int \frac{dx}{\sqrt{1-x}}$

$$I = \int \frac{1}{\sqrt{1-x}} dx$$
$$= \int \frac{1}{(1-x)^{1/2}} dx$$
$$= \int (1-x)^{-1/2} dx$$

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$$= \frac{(1-x)^{-\frac{1}{2}+1}}{-1(-\frac{1}{2}+1)} \quad \left[\because \int (a-bx)^n = \frac{(a-bx)^{n+1}}{-b(n+1)} + c,\ n \neq -1\right]$$

$$= \frac{(1-x)^{1/2}}{-\frac{1}{2}}$$

$$= -2(1-x)^{1/2}$$
$$= -2\sqrt{1-x}$$

Now,

$$\int_0^{1/2} \frac{dx}{\sqrt{1-x}} = -2\sqrt{1-\frac{1}{2}} - (-2\sqrt{1-0})$$
$$= -\frac{2}{\sqrt{2}} + 2$$
$$= 2-\sqrt{2}$$

g) $\int_{0.2}^{1} \left(x^2 + \frac{1}{x}\right) dx$

Let $I = \int \left(x^2 + \frac{1}{x}\right) dx$

$$= \int x^2\, dx + \int \frac{1}{x}\, dx$$

$$= \frac{x^{2+1}}{2+1} + \ln x \quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1, \quad \because \int \frac{1}{x} dx = \ln x + c\right]$$

$$= \frac{x^3}{3} + \ln x$$

Now,

$$\int_{0.2}^{1} \left(x^2 + \frac{1}{x}\right) dx = \left(\frac{1^3}{3} + \ln 1\right) - \left(\frac{(0.2)^3}{3} + \ln(0.2)\right)$$
$$= \frac{1}{3} + 1.60677$$
$$= \cancel{1.27}\ 1.9401$$

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h) $\int_{5}^{9} \left(\frac{10}{6+x}\right) dx$

Let $I = \int \left(\frac{10}{6+x}\right) dx$

$= 10. \int \left(\frac{1}{6+x}\right) dx$

put $6+x = t$

Differentiating both sides with respect to x,

$\frac{d}{dx}(6+x) = \frac{d(t)}{dx}$

or, $0 + 1 = \frac{dt}{dx}$

$\therefore 1\,dx = dt$

Now,

$\int \left(\frac{10}{6+x}\right) dx = 10. \int \frac{1}{t}\, dt$

$= 10. \ln(t) \quad \left[\because \int \frac{1}{x} dx = \ln(x) + C\right]$

$= 10. \ln(6+x)$

Now,

$\int_{5}^{9} \left(\frac{10}{6+x}\right) dx = 10\left[\ln(6+9) - \ln(6+5)\right]$

$= 10\left[\ln 15 - \ln 11\right]$

$= 10 \times 0.3101$

$= 3.1015$

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i) $\int_1^{12} \left(\frac{Q^2+20Q+12}{Q}\right) dQ$

let $I = \int \left(\frac{Q^2+20Q+12}{Q}\right) dQ$

$= \int Q\, dQ + 20\int 1\, dQ + 12\int \frac{1}{Q}\, dQ$

$= \frac{Q^{1+1}}{1+1} + 20 \times Q + 12.\ln(Q)$

$= \frac{Q^2}{2} + 20Q + 12.\ln(Q)$

Now,

$\int_1^{12} \left(\frac{Q^2+20Q+12}{Q}\right) dQ = \frac{Q^2}{2} + 20Q + 12.\ln(Q) \Big|_1^{12}$

$= \left(\frac{12^2}{2} + 20\times12 + 12.\ln(12)\right) - \left(\frac{1^2}{2} + 20\times1 + 12.\ln(1)\right)$

$= (72+240+29.8188) - (\frac{1}{2}+20+0)$

$= 321.32$

j) $\int_0^{8} \left(10 + \frac{1}{Q+1}\right) dQ$

let $I = \int \left(10 + \frac{1}{Q+1}\right) dQ$

$= 10.\int 1\, dQ + \int \frac{1}{Q+1}\, dQ$

$= 10Q + \int \frac{1}{Q+1}\, dQ$

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$= 10Q + \ln(Q+1)\, dQ \quad \left[\because \int \frac{f'(x)}{f(x)} = \ln[f(x)] + C\right]$

Now,

$$\int_0^5 \left(10 + \frac{1}{Q+1}\right) dQ = 10Q + \ln(Q+1)\Big|_0^5$$

$= 10\times5 + \ln(5+1) - [10\times0 + \ln(0+1)]$

$= 50 + \ln(6) - 0 - \ln(1)$

$\equiv 51.79$

---

k) $\int_0^1 \left(\frac{e^{2Q}+4}{e^{2Q}}\right) dQ$

let $I = \int \left(\frac{e^{2Q}+4}{e^{2Q}}\right) dQ$

$$= \int\left(1 + \frac{4}{e^{2Q}}\right) dQ$$

$$= \int 1\, dQ + 4.\int \frac{1}{e^{2Q}} \cdot dQ$$

$$= Q + 4. \int e^{-2Q}\, dQ$$

$$= Q + 4. \frac{e^{-2Q}}{-2}$$

$$= Q - 2e^{-2Q}$$

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Now,

$$\int_0^1 \left(\frac{e^{2Q}+4}{e^{2Q}}\right) dQ = Q - 2e^{-2Q}\Big|_0^1$$

$= 1-2e^{-2\times1} - (0-2e^{-2\times0})$
$= 1-2e^{-2} - 0 + 2e^{0}$
$= 1-2e^{-2}+2$
$= 3-2e^{-2}$
$= 2.7293$

l) $\int_1^3 4(1-e^{-0.9x})\,dx$

Let $I = \int 4(1-e^{-0.9x})\,dx$

$= 4\int(1-e^{-0.9x})\,dx$

$= 4\left(\int 1\,dx - \int e^{-0.9x}\,dx\right)$

$= 4\left(x - \dfrac{e^{-0.9x}}{-0.9}\right)$

$= 4\left(\dfrac{-0.9x - e^{-0.9x}}{-0.9}\right)$

Now,

$$\int_1^3 4(1-e^{-0.9x})\,dx = 4\left(\frac{-0.9x - e^{-0.9x}}{-0.9}\right)\Big|_1^3$$

$$= 4\left[\left(\frac{-0.9\times3 - e^{-0.9\times3}}{-0.9}\right) - \left(\frac{-0.9\times1 - e^{-0.9\times1}}{-0.9}\right)\right]$$

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$= 4\left[\cancel{3.4517 + 1.4517}\right]$

$= 4[3.07 - 1.45]$
$= 6.49$

2. Evaluate the following integrals:

a) $\int_0^1 \frac{\cancel{1+x}\,1-x}{1+x}\,dx$

Let $I = \int \frac{1-x}{1+x}\,dx$

$= \int \left(\frac{1+x-2x}{1+x}\right) dx$

$= \int \left(1 - \frac{2x}{1+x}\right) dx$

$= \int 1\,dx - \int \frac{2x}{1+x}\,dx$

$= x - 2\int \frac{x}{1+x}\,dx$

$= x - 2\int \frac{1+x-1}{1+x}\,dx$

$= x - 2\int 1 - \frac{1}{1+x}\,dx$

$= x - 2\left(\int 1\,dx - \int \frac{1}{1+x}\,dx\right)$
$= x - 2\left(x - \ln(1+x)\,\cancel{dx}\right)$ $\left[\because \int \frac{1}{x}\,dx = \ln(x)+C\right]$
$= x - 2x + 2\ln(1+x)\,\cancel{dx}$
$= -x + 2\ln(1+x)$

Now,

$$\int_0^1 \frac{1-x}{1+x}\,dx = -x + 2\ln(1+x)\Big|_0^1$$

$$= -1 + 2\ln(1+1) - (-0 + 2\ln(1+0))$$
$$= -1 + 2\ln(2) - 2\ln(1)$$
$$= \cancel{0.3862 - 0}$$
$$= -1 + 2\ln 2 - 0$$
$$= 2\ln 2 - 1$$

b) $\displaystyle\int_0^1 \frac{1}{\sqrt{x+1}-\sqrt{x}}\,dx$

Let $\displaystyle I = \int \frac{1}{\sqrt{x+1}-\sqrt{x}}\,dx$

$$= \int\left[\frac{1}{\sqrt{x+1}-\sqrt{x}} \times \frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}\right]dx$$

$$= \int \frac{\sqrt{x+1}+\sqrt{x}}{x+1-x}\,dx$$

$$= \int(\sqrt{x+1}+\sqrt{x})\,dx$$

$$= \int(\sqrt{x+1})\,dx + \int(\sqrt{x})\,dx$$

$$= \int(x+1)^{1/2}\,dx + \int(x)^{1/2}\,dx$$

$$= \frac{(x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{(x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} \quad \left[\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right]$$

$$= \frac{2(x+1)^{\frac{3}{2}}}{3} + \frac{2(x)^{3/2}}{3}$$

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$$= \frac{2}{3}(x+1)^{3/2} + \frac{2}{3}(x)^{3/2}$$

$$= \frac{2}{3}\left[(x+1)^{3/2} + (x)^{3/2}\right]$$

$$\therefore \int_0^1 \frac{1}{\sqrt{x+1} - \sqrt{x}}\, dx = \frac{2}{3}\left[(x+1)^{3/2} + (x)^{3/2}\right]\Big|_0^1$$

$$= \frac{2}{3}\left[(1+1)^{3/2} + (1)^{3/2} - (0+1)^{3/2} - (0)^{3/2}\right]$$

$$= \frac{2}{3}\left[2.828 + 1 - 1 - 0\right]$$

$$= 1.88$$

---

c) $\displaystyle\int_{-1}^{0} \frac{dx}{\sqrt{3x+4} - \sqrt{3x+9}}$

Let $I = \displaystyle\int \frac{1}{\sqrt{3x+4} - \sqrt{3x+9}}\, dx$

$$= \int \left(\frac{1}{\sqrt{3x+4} - \sqrt{3x+9}} \times \frac{\sqrt{3x+4} + \sqrt{3x+9}}{\sqrt{3x+4} + \sqrt{3x+9}}\right) dx$$

$$= \int \frac{\sqrt{3x+4} + \sqrt{3x+9}}{3x+4-3x-9}\, dx$$

$$= \int \frac{\sqrt{3x+4} + \sqrt{3x+9}}{-5}\, dx$$

$$= -\frac{1}{5}\int \left(\sqrt{3x+4} + \sqrt{3x+9}\right) dx$$

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$$= -\frac{1}{5}\left[\int (3x+4)^{1/2}dx + \int (3x+9)^{1/2}dx\right]$$

$$= -\frac{1}{5}\left[\frac{(3x+4)^{\frac{1}{2}+1}}{3(\frac{1}{2}+1)} + \frac{(3x+9)^{\frac{1}{2}+1}}{3(\frac{1}{2}+1)}\right]$$

$$\left[\because \int (ax+b)^n = \frac{(ax+b)^{n+1}}{a(n+1)}, n \neq -1\right]$$

$$= -\frac{1}{5}\left[\frac{(3x+4)^{3/2}}{\frac{9}{2}} + \frac{(3x+9)^{3/2}}{\frac{9}{2}}\right]$$

$$= -\frac{1}{5}\left[\frac{2}{9}(3x+4)^{3/2} + \frac{2}{9}(3x+9)^{3/2}\right]$$

$$= -\frac{1}{5} \times \frac{2}{9}\left[(3x+4)^{3/2} + (3x+9)^{3/2}\right]$$

$$= -\frac{2}{45}\left[(3x+4)^{3/2} + (3x+9)^{3/2}\right]$$

$$\therefore \int_{-1}^{0} \frac{dx}{\sqrt{3x+4} - \sqrt{3x+9}} = -\frac{2}{45}\left[(3x+4)^{3/2} + (3x+9)^{3/2}\right]\Big|_{-1}^{0}$$

$$= -\frac{2}{45}\left[(3\times0+4)^{3/2} + (3\times0+9)^{3/2} - (3\times-1+4)^{3/2} - (3\times-1+9)^{3/2}\right]$$

$$= -\frac{2}{45}\left[8 + 27 - 1 - 14.696\right]$$

$$= -\frac{2}{45}\left[19.304\right]$$

$$= -0.8579$$

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d) $\int_{2}^{4} \frac{6x^2-1}{\sqrt{2x^3-x-2}}\,dx$

Let $I = \int \frac{6x^2-1}{\sqrt{2x^3-x-2}}\,dx$

Put $2x^3-x-2=t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(2x^3-x-2) = \frac{d}{dx}(t)$$

or, $6x^2-1 = \frac{dt}{dx}$

$\therefore$ ~~$(6x^2-1)\,dx$~~

$\therefore dt = (6x^2-1)\,dx$

$\therefore dx = \frac{dt}{6x^2-1}$

Now,

$$\int \frac{6x^2-1}{\sqrt{2x^3-x-2}}\,dx = \int\left(\frac{6x^2-1}{\sqrt{2x^3-x-2}} \times \frac{dt}{6x^2-1}\right)$$

$$= \int \frac{1}{\sqrt{2x^3-x-2}} \times dt$$

$$= \int \frac{1}{\sqrt{t}} \times dt$$

$$= \int t^{-1/2}\,dt$$

$$= \frac{t^{-1/2+1}}{-1/2+1} \quad \left[\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1\right]$$

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$= 2t^{1/2}$
$= 2\sqrt{2x^3 - x - 2}$

Now,

$$\int_2^4 \frac{6x^2-1}{\sqrt{2x^3-x-2}}\,dx = 2\sqrt{2x^3-x-2}\,\Big|_2^4$$

$= 2\left(\sqrt{2\times(4)^3-4-2} - \sqrt{2\times(2)^3-2-2}\right)$

$= 2\left(\sqrt{128-6} - \sqrt{16-4}\right)$

$= 2\left(\sqrt{122} - \sqrt{12}\right)$

e) $\displaystyle\int_0^1 \frac{6x-5}{(3x^2-5x+7)^2}\,dx$

Let $I = \displaystyle\int \frac{6x-5}{(3x^2-5x+7)^2}\,dx$

~~$= \ln(3x^2-5x+7)$~~

Put $3x^2-5x+7 = t$

Differentiating both sides with respect to x,

$\frac{d}{dx}(3x^2-5x+7) = \frac{d}{dx}(t)$

or $6x - 5 + 0 = \frac{dt}{dx}$

$\therefore (6x-5)\,dx = dt$

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Now,

$$\int \frac{6x-5}{(3x^2-5x+7)^2}\,dx = \int \frac{1}{(3x^2-5x+7)^2}\cdot(6x-5)\,dx$$

$$= \int \frac{1}{t^2}\cdot dt$$

$$= \int t^{-2}\,dt$$

$$= \frac{t^{-2+1}}{-2+1} \quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1\right]$$

$$= -t^{-1}$$

$$= -\frac{1}{t}$$

$$= -\frac{1}{3x^2-5x+7}$$

Therefore,

$$\int_0^1 \frac{6x-5}{(3x^2-5x+7)^2} = -\frac{1}{3x^2-5x+7}\Bigg|_0^1$$

$$= -\left[\frac{1}{3\times(1)^2-5\times1+7} - \frac{1}{3\times(0)^2-5\times0+7}\right]$$

$$= -\left[\frac{1}{3-5+7} - \frac{1}{0-0+7}\right]$$

$$= -\left[\frac{1}{5} - \frac{1}{7}\right]$$

$$= -\frac{2}{35}$$

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t) $\int_1^2 \frac{x}{\sqrt{2x^2+1}}\,dx$

Let $I = \int \frac{x}{\sqrt{2x^2+1}}\,dx$

Put $2x^2+1 = t$

Differentiating both sides with respect to x,

$\frac{d}{dx}(2x^2+1) = \frac{d}{dx}(t)$

or $4x + 0 = \frac{dt}{dx}$

or, $4x\,dx = dt$

$\therefore$ ~~or,~~ $dx = \frac{dt}{4x}$

Now,

$$\int \frac{x}{\sqrt{2x^2+1}}\,dx = \int \frac{x}{\sqrt{2x^2+1}} \times \frac{dt}{4x}$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{t}}\,dt$$

$$= \frac{1}{4}\int t^{-1/2}\,dt$$

$$= \frac{1}{4} \times \frac{t^{-1/2+1}}{-1/2+1} \quad \left[\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + c\right]$$

$$= \frac{1}{4} \times \frac{t^{1/2}}{1/2}$$

$$= \frac{1}{4} \times 2t^{1/2}$$

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$$= \frac{1}{2}\sqrt{t}$$

$$= \frac{\sqrt{2x^2+1}}{2}$$

Therefore,

$$\int_1^2 \frac{x}{\sqrt{2x^2+1}}\,dx = \frac{1}{2} \times \sqrt{2x^2+1}\Big|_1^2$$

$$= \frac{1}{2}\left[\sqrt{2\times(2)^2+1} - \sqrt{2\times(1)^2+1}\right]$$

$$= \frac{1}{2}\left[3 - \sqrt{3}\right]$$

$$= \frac{3-\sqrt{3}}{2}$$

---

g) $\int_0^2 x\sqrt{x+2}\,dx$

Let $I = \int x\sqrt{x+2}\,dx$

put $x+2 = t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(x+2) = \frac{dt}{dx}$$

Or, $1+0 = \frac{dt}{dx}$

$\therefore 1\,dx = dt$

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g) $\int_0^2 x\sqrt{x+2}\,dx$

let $I = \int x\sqrt{x+2}$

$= x.\int (x+2)^{1/2} dx - \int\left(\frac{d}{dx}(x).\int (x+2)^{1/2} dx\right) dx$

$\left[\because \int uv\,dx = u.\int v\,dx - \int\left(\frac{d}{dx}(u).\int v\,dx\right) dx\right]$

$= x.\frac{(x+2)^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \int\left(1.\frac{(x+2)^{1/2+1}}{\frac{1}{2}+1}\right) dx$

$= x.\frac{2}{3}(x+2)^{3/2} - \frac{2}{3}\int (x+2)^{3/2} dx$

$= \frac{2x}{3}(x+2)^{3/2} - \frac{2}{3}.\frac{(x+2)^{3/2+1}}{3/2+1}$

$= \frac{2x}{3}(x+2)^{3/2} - \frac{2}{3}.\frac{2}{5}(x+2)^{5/2}$

$= \frac{2x}{3}(x+2)^{3/2} - \frac{4}{15}(x+2)^{5/2}$

Now,

$\int_0^2 x\sqrt{x+2}\,dx = \frac{2x}{3}(x+2)^{3/2} - \frac{4}{15}(x+2)^{5/2}\Big|_0^2$

$= \left[\frac{4}{3}\times(4)^{3/2} - \frac{4}{15}(4)^{5/2}\right] - \left[\frac{0}{3}(2)^{3/2} - \frac{4}{15}(2)^{5/2}\right]$

$= \left[\frac{32}{3} - \frac{128}{15}\right] - \left[0 - 1.5084\right]$

$= \frac{32}{15} + 1.5084$

$= 3.6417$

---

h) $\int_0^1 x^3 \sqrt{1+3x^4}\, dx$

Let $I = \int x^3 \sqrt{1+3x^4}\, dx$

$\cancel{= x^3 \cdot \int (1+3x^4)^{1/2} dx - \int \left(\frac{d}{dx}(x^3) \cdot \int (1+3x^4)^{1/2} dx\right) dx}$

$\cancel{\left[\because \int uv\, dx = u \cdot \int v\, dx - \int \left(\frac{d}{dx}(u) \cdot \int v\, dx\right) dx\right]}$

$= \int x^3 (1+3x^4)^{1/2}\, dx$

Put $1+3x^4 = t$

Differentiating both sides with respect to x,

$\frac{d}{dx}(1+3x^4) = \frac{d(t)}{dx}$

or, $0 + 12x^3 = \frac{dt}{dx}$

or, $12x^3\, dx = dt$

$\therefore dx = \frac{dt}{12x^3}$

Now,

$\int x^3 \sqrt{1+3x^4}\, dx = \int x^3 \cdot (1+3x^4)^{1/2} \cdot \frac{dt}{12x^3}$

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$= \frac{1}{12} \int \sqrt{t}\, dt$

$= \frac{1}{12} \int t^{\frac{1}{2}}\, dt$

$= \frac{1}{12} \left(\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right)$ $\left[\because \int x^n\, dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1 \right]$

$= \frac{1}{12} \left(\frac{t^{3/2}}{3/2} \right)$

$= \frac{1}{12} \times \frac{2}{3}\, t^{3/2}$

$= \frac{1}{18}\, t^{3/2}$

$= \frac{1}{18} (1+3x^4)^{3/2}$

Finally,

$\int_0^1 x^3 \sqrt{1+3x^4}\, dx = \frac{1}{18} (1+3x^4)^{3/2} \Big|_0^1$

$= \frac{1}{18} \left[(1+3 \times (1)^4)^{3/2} - (1+3 \times (0)^4)^{3/2} \right]$

$= \frac{1}{18} [8-1]$

$= \frac{7}{18}$

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i) $\int_0^a \frac{x\, dx}{\sqrt{x^2+a^2}}$

Let $I = \int \frac{x\, dx}{\sqrt{x^2+a^2}}$

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Put $x^2+a^2=t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(x^2+a^2)=\frac{d}{dx}(t)$$

or, $2x+0=\frac{dt}{dx}$

or, $2x\,dx=dt$

$\therefore x\,dx=\frac{dt}{2}$

Now,

$$\int\frac{x\,dx}{\sqrt{x^2+a^2}}=\int\frac{1}{\sqrt{x^2+a^2}}\cdot x\,dx$$

$$=\int\frac{1}{\sqrt{t}}\cdot\frac{dt}{2}$$

$$=\frac{1}{2}\int t^{-1/2}\,dt$$

$$=\frac{1}{2}\cdot\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\quad\left[\because\int x^n\,dx=\frac{x^{n+1}}{n+1}+C,\ n\neq-1\right]$$

$$=\frac{1}{2}\cdot\frac{t^{1/2}}{\frac{1}{2}}$$

$$=t^{1/2}$$

$$=\sqrt{t}$$

$$\therefore\int\frac{x\,dx}{\sqrt{x^2+a^2}}=\sqrt{x^2+a^2}\quad\left[\because t=x^2+a^2\right]$$

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Therefore,

$$\int_0^a \frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}\Big|_0^a$$

$$= \sqrt{a^2+a^2} - \sqrt{0^2+a^2}$$

$$= \sqrt{2a^2} - \sqrt{a^2}$$

$$\cancel{= \sqrt{a^2} + \sqrt{a^2}}$$

$$= a\sqrt{2} - a$$
$$= a(\sqrt{2}-1)$$

3. Evaluate the following integrals.

a) $\displaystyle\int_0^2 \frac{x-\frac{5}{6}}{(3x^2-5x+4)}\,dx$

let $I = \displaystyle\int \frac{x-\frac{5}{6}}{(3x^2-5x+4)}\,dx$

$$= \int \frac{6x-5}{6(3x^2-5x+4)}\,dx$$

$$= \frac{1}{6}\int \frac{6x-5}{3x^2-5x+4}\,dx$$

Put $3x^2-5x+4 = t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(3x^2-5x+4) = \frac{d}{dx}(t)$$

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or, $6x - 5 + 0 = \frac{dt}{dx}$

$\therefore (6x-5)\,dx = dt$

Now,

$$\int \frac{x - \frac{5}{6}}{(3x^2-5x+4)}\,dx = \frac{1}{6}\int \frac{1}{(3x^2-5x+4)}(6x-5)\,dx$$

$$= \frac{1}{6}\int \frac{1}{t}\,dt$$

$$= \frac{1}{6}\ln(t) \quad \left[\because \int \frac{1}{x}\,dx = \ln(x)+c\right]$$

$$\therefore \int \frac{x-\frac{5}{6}}{(3x^2-5x+4)}\,dx = \frac{1}{6}\ln(3x^2-5x+4)$$

Therefore,

$$\int_0^2 \frac{x-\frac{5}{6}}{(3x^2-5x+4)}\,dx = \frac{1}{6}\ln(3x^2-5x+4)\Big|_0^2$$

$$= \frac{1}{6}\left[\ln(3\times2^2-5\times2+4) - \ln(3\times0^2-5\times0+4)\right]$$

$$= \frac{1}{6}[\ln 6 - \ln 4]$$

$$\cancel{= \frac{1}{6}\times 0.4054}$$

$$= \frac{1}{6}\ln\left(\frac{6}{4}\right) \quad \left[\because \ln a - \ln b = \ln\left(\frac{a}{b}\right)\right]$$

$$= \frac{1}{6}\ln\left(\frac{3}{2}\right)$$

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b) $\int_0^1 xe^{x^2} dx$

let $I = \int x e^{x^2} dx$

$\cancel{= x \int e}$ put $x^2 = t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(x^2) = \frac{d}{dx}(t)$$

or, $2x = \frac{dt}{dx}$

or, $2x . dx = dt$

$\therefore$ ~~or,~~ $x . dx = \frac{dt}{2}$

Then,

$$\int x e^{x^2} dx = \int e^{x^2} . x \, dx$$

$$= \int e^t . \frac{dt}{2}$$

$$= \frac{1}{2} \int e^t . dt$$

$$= \frac{1}{2} . e^t \quad [\because \int e^x dx = e^x + c]$$

$$= \frac{1}{2} . e^{x^2} \quad [\because t = x^2]$$

Therefore,

$$\int_0^1 xe^{x^2} dx = \frac{1}{2} . e^{x^2} \Big|_0^1$$

$$= \frac{1}{2} [e^1 - e^0]$$

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$= \frac{1}{2}(e^1 - 1)$

$= \frac{e-1}{2}$

c) $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}\, dx$

Let $I = \int \frac{e^{\frac{1}{x}}}{x^2}$

put $\frac{1}{x} = t$

Differentiating both sides with respect to x,

$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(t)$

Or, $\cancel{-\frac{1}{2\sqrt{x}}}\ -x^{-2} = \frac{dt}{dx}$

or, $-x^{-2}\, dx = \cancel{\sqrt{x}}\ dt$

$\therefore dx = \cancel{-\sqrt{x}\, dt}\ \frac{dt}{x^{-2}}$

Then,

$\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}\, dx = \int \frac{e^{\frac{1}{x}}}{x^2} \cdot -\sqrt{x}\, dt$

$= -1 \int \frac{e^{\frac{1}{x}}}{x^2} \cdot \sqrt{x}\, dt$

$= -1 \int \frac{e^{\frac{1}{x}}}{x \cdot \sqrt{x}}\, dt$

$= -1 \int e^t \cdot t$

Then,

$$\int \frac{e^{1/x}}{x^2}\,dx =$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(t)$$

or, $-x^{-1-1} = \frac{dt}{dx}$

or, $-\frac{1}{x^2} = \frac{dt}{dx}$

or, $-dx = dt \cdot x^2$

$\therefore -\frac{1}{x^2}\,dx = dt$

Then,

$$\int \frac{e^{1/x}}{x^2}\,dx = -1 \int e^{\frac{1}{x}} \cdot -\frac{1}{x^2}\,dx$$

$$= -1 \int e^t \cdot dt$$

$$= \cancel{-1 \cdot \frac{e^{t+1}}{t+1}} \quad [\because \int e^n = n+$$

$$= -1 \cdot e^t \quad [\because \int e^x dx = e^x + c]$$

$$= -1 \cdot e^{\frac{1}{t}}$$

$$= -1 \cdot e^x$$

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$\Rightarrow$ Therefore,

$$\int_1^2 \frac{e^{4x}}{x^2}\,dx = -1\; e^{1/x}\Big|_1^2$$

$$= -1\left[e^{1/2} - e^{1}\right]$$

$$= -\sqrt{e} + e$$

$$= e - \sqrt{e}$$

d) $\displaystyle\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

Let $I = \displaystyle\int \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Put $e^x + e^{-x} = t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(e^x + e^{-x}) = \frac{d(t)}{dx}$$

or $e^x + (-e^{-x}) = \dfrac{dt}{dx}$

or $e^x - e^{-x} = \dfrac{dt}{dx}$

$\therefore (e^x - e^{-x})\,dx = dt$

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Now,

$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx = \int \frac{1}{e^x + e^{-x}} \cdot (e^x - e^{-x})\, dx$$

$$= \int \frac{1}{t} \cdot dt$$

$$= \ln(t) \quad \left[\because \int \frac{1}{x}\, dx = x + C\right]$$

$$= \ln(e^x + e^{-x})$$

Therefore,

$$\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx = \ln(e^1 + e^{-1}) - \ln(e^0 + e^{-0})$$

$$= \ln\left(e + \frac{1}{e}\right) - \ln(1+1)$$

$$= \ln\left(\frac{e^2+1}{e}\right) - \ln 2$$

$$= \ln(e^2+1) - \ln e - \ln 2$$

$$= \ln(e^2+1) - 1 - \ln 2$$

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e) $\int_0^3 \frac{e^x}{1+e^x}\, dx$

Let $I = \int \frac{e^x}{1+e^x}$

~~Put $1+e^x$~~

$$= \ln(1+e^x) \quad \left[\because \int \frac{f'(x)}{f(x)} = \ln[f(x)] + c\right]$$

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Therefore,

$$\int_0^3 \frac{e^x}{1+e^x}\,dx = \ln(1+e^x)\Big|_0^3$$

$$= \ln(1+e^3) - \ln(1+e^0)$$

$$= \ln(1+e^3) - \ln(1+1)$$

$$= \ln(1+e^3) - \ln 2$$

f) $\displaystyle\int_1^{e^2} \frac{dx}{x(1+\ln x)^2}$

Let $I = \displaystyle\int \frac{dx}{x(1+\ln x)^2}$

Put $1+\ln x = t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(1+\ln x) = \frac{d}{dx}(t)$$

or, $0 + \dfrac{1}{x} = \dfrac{dt}{dx}$

or, $\dfrac{1}{x} = \dfrac{dt}{dx}$

~~or,~~ $dx = x\,dt$

~~or,~~ $\therefore \dfrac{dx}{x} = dt$

Now,

$$\int \frac{dx}{x(1+\ln x)^2} = \int \frac{1}{(1+\ln x)^2} \cdot \frac{dx}{x}$$

$$= \int \frac{1}{t^2} \cdot dt$$

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$= \int t^{-2} dt$

$= \frac{t^{-2+1}}{-2+1}$ $\left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right]$

$= \frac{t^{-1}}{-1}$

$= -t^{-1}$

$= -\frac{1}{t}$

$= -\frac{1}{1+\ln x}$

$\therefore \int \frac{dx}{x(1+\ln x)^2} = -\frac{1}{1+\ln x}$

Therefore,

$\int_1^{e^2} \frac{dx}{x(1+\ln x)^2} = -\frac{1}{1+\ln x}\Big|_1^{e^2}$

$= -\left[\frac{1}{1+\ln e^2} - \frac{1}{1+\ln 1}\right]$

$= -\left[\frac{1}{1+2\ln e} - \frac{1}{1+0}\right]$ $[\because \ln m^n = n \ln m]$

$= -\left[\frac{1}{1+2\times 1} - 1\right]$

$= -\left[\frac{1}{3} - 1\right]$

$= \frac{2}{3}$

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g) $\int_0^1 2^{2x+3}\,dx$

Let $I = \int 2^{2x+3}\,dx$

Put $2x+3 = t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(2x+3) = \frac{d}{dx}(t)$$

or, $$2+0 = \frac{dt}{dx}$$

$$\therefore 2\,dx = dt$$

$$\therefore dx = \frac{dt}{2}$$

Then,

$$\int 2^{2x+3}\,dx = \int 2^t\,dx$$

$$= \int 2^t \cdot \frac{dt}{2}$$

$$= \frac{1}{2}\int 2^t \cdot dt$$

$$= \frac{1}{2}\cdot\frac{2^t}{\ln 2} \quad \left[\because \int a^x\,dx = \frac{a^x}{\ln a} + c, a>0\right]$$

$$= \frac{1}{2}\cdot\frac{2^{2x+3}}{\ln 2} \quad [\because t = 2x+3]$$

$$\therefore \int 2^{2x+3}\,dx = \frac{1}{2\ln 2}\cdot \cancel{2x}\, 2^{2x+3}$$

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Therefore,

$$\int_0^1 2^{2x+3}\,dx = \frac{1}{2\ln 2}\cdot 2^{2x+3}\Big|_0^1$$

$$= \frac{1}{2\ln 2}\left[2^{2\times1+3} - 2^{2\times0+3}\right]$$

$$= \frac{1}{2\ln 2}\left[2^5 - 2^3\right]$$

$$= \frac{1}{2\ln 2}(32-8)$$

$$= \frac{24}{2\ln 2}$$

$$= \frac{12}{\ln 2}$$