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Exercise 5(B)

1. Evaluate the following integrals by substitution method.

a) $\int (2x+3)\sqrt{x^2+3x}\,dx$

Solution

put $x^2+3x = t$

Differentiating both sides with respect to 'x'

$\frac{d}{dx}(x^2+3x) = \frac{d}{dx}(t)$

or, $\frac{d}{dx}(x^2) + \frac{d}{dx}(3x) = \frac{dt}{dx}$

or, $2x+3 = \frac{dt}{dx}$

~~or, $dt = (2x+3)dx$~~

~~or,~~ $\therefore (2x+3)\,dx = dt$

So,

$\cancel{\int (2x+3)\sqrt{x^2+3x}\,dx} = \cancel{\int (2x+3)\,dx\,\sqrt{x^2+3x}}$

$= \cancel{\int dt\,(\sqrt{x^2+3x})}$

$\int (2x+3)\sqrt{x^2+3x}\,dx = \int (2x+3)\,dx\,\sqrt{x^2+3x}$

$= \int dt.\sqrt{t}$

$= \cancel{\int dt}\ \int t^{1/2}\,dt$

$= \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C \quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1}\right]$

$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{2t^{3/2}}{3} + C$

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$\therefore \int (2x+3)\sqrt{x^2+3x}\, dx = \frac{2}{3}(x^2+3x)^{3/2} + C$

b) $\int (4x+2)\sqrt{x^2+x+1}\, dx$

Solution

put $x^2+x+1 = t$

Differentiating both sides with respect to 'x',

$\frac{d(x^2+x+1)}{dx} = \frac{d(t)}{dx}$

or $\frac{d}{dx}(x^2) + \frac{d}{dx}(x) + \frac{d}{dx}(1) = \frac{dt}{dx}$

or, $2x + 1 + 0 = \frac{dt}{dx}$

or $2x+1 = \frac{dt}{dx}$

$\therefore (2x+1)dx = dt$

So,

$\int (4x+2)\sqrt{x^2+x+1}\, dx = \int 2(2x+1)\sqrt{x^2+x+1}\, dx$

$= 2.\int (2x+1)dx\sqrt{x^2+x+1}$

$= 2.\int dt\sqrt{t}$

$= 2.\int t^{1/2}\, dt$

$= 2.\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$ $\left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right]$

$= \frac{2.t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{4t^{\frac{3}{2}}}{3} + c$

$\therefore \int (4x+2)\sqrt{x^2+x+1}\, dx = \frac{4(x^2+x+1)^{3/2}}{3} + c$

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c) $\int \frac{3x-1}{\sqrt{3x^2-2x+7}} dx$

Solution
Put $3x^2-2x+7=t$

Differentiating both sides with respect to 'x'.

$\frac{d}{dx}(3x^2-2x+7) = \frac{d}{dx}(t)$

or $\frac{d}{dx}(3x^2) - \frac{d}{dx}(2x) + \frac{d}{dx}(7) = \frac{dt}{dx}$

or, $6x - 2 + 0 = \frac{dt}{dx}$

$\therefore (6x-2)dx = dt$

So,

$\int \frac{3x-1}{\sqrt{3x^2-2x+7}} dx = \int \frac{6x-2}{2\sqrt{3x^2-2x+7}} dx$

$= \int \frac{dt}{2\sqrt{3x^2-2x+7}}$

$= \frac{1}{2}\int \frac{dt}{\sqrt{3x^2-2x+7}}$

$= \frac{1}{2}\int \frac{dt}{\sqrt{t}}$

$= \frac{1}{2}\int t^{-1/2} dt$

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$$= \frac{1}{2} \times \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= \frac{1}{2} \times \frac{t^{1/2}}{1/2} + c$$

$$= t^{1/2} + c$$

$$\therefore \int \frac{3x-1}{\sqrt{3x^2-2x+7}} = (3x^2-2x+7)^{1/2} + c$$

d) $\int \frac{2px+q}{\sqrt{px^2+qx+r}}\, dx$

Solution

Put $px^2+qx+r = t$

Differentiating both sides with respect to 'x'.

$$\frac{d}{dx}(px^2+qx+r) = \frac{d}{dx}(t)$$

or, $\frac{d}{dx}(px^2) + \frac{d}{dx}(qx) + \frac{d}{dx}(r) = \frac{dt}{dx}$

or, $2px + q + 0 = \frac{dt}{dx}$

or, $2px + q = \frac{dt}{dx}$

$\therefore (2px+q)\, dx = dt$

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So,

$$\int \frac{2px+q}{\sqrt{px^2+qx+r}}\,dx = \int \frac{1}{\sqrt{px^2+qx+r}} \cdot (2px+q)\,dx$$

$$= \int \frac{1}{\sqrt{t}} \cdot dt$$

$$= \int t^{-1/2}\,dt$$

$$= \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c \quad \left[\int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1\right]$$

$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2t^{\frac{1}{2}} + c$$

So,

$$\therefore \int \frac{2px+q}{\sqrt{px^2+qx+r}}\,dx = 2(px^2+qx+r)^{1/2} + c$$

---

e) $\int \frac{(1+x^2)}{x^2}\, e^{x-\frac{1}{x}}\,dx$

Solution

Put $x - \frac{1}{x} = t$

or, $x - x^{-1} = t$

Differentiating both sides with respect to 'x',

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$$\frac{d}{dx}(x-x^{-1}) = \frac{d}{dx}(t)$$

or, $\frac{d}{dx}(x) - \frac{d}{dx}(x^{-1}) = \frac{dt}{dx}$

or, $1 + x^{-2} = \frac{dt}{dx}$

$\therefore (1+x^{-2})\,dx = dt$

So,

$$\int \frac{(1+x^2)}{x^2} e^{x-\frac{1}{x}}\,dx = \int \frac{(x^{-2}.x^2 + x^2)}{x^2} e^{x-\frac{1}{x}}\,dx$$

$$= \int \frac{x^2(x^{-2}+1)}{x^2} e^{x-\frac{1}{x}}\,dx$$

$$= \int (1+x^{-2})\, e^{x-\frac{1}{x}}\,dx$$

$$= \int e^{x-\frac{1}{x}} \cancel{dx} (1+x^{-2})\,dx$$

$$= \int e^t . dt \quad \left[\because x - \tfrac{1}{x} = t,\ (1+x^{-2})\,dx = dt\right]$$

$$= e^t + c \quad \left[\because \int e^t\,dt = e^t + c\right]$$

Now,

$$\therefore \int \frac{(1+x^2)}{x^2} e^{x-\frac{1}{x}}\,dx = e^{x-\frac{1}{x}} + c$$

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b) $\int \frac{1}{x-\sqrt{x}}\,dx$

Solution

$$= \int \frac{1}{\sqrt{x}(\sqrt{x}-1)}\,dx$$

Put $\sqrt{x}-1 = t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(\sqrt{x}-1) = \frac{d}{dx}(t)$$

$$\text{or, } \frac{d}{dx}(x^{1/2}) - \frac{d}{dx}(1) = \frac{dt}{dx}$$

$$\text{or, } \frac{1}{2}x^{-1/2} - 0 = \frac{dt}{dx}$$

$$\text{or, } \frac{1}{2}x^{-1/2} = \frac{dt}{dx}$$

$$\therefore \frac{1}{2\sqrt{x}}\,dx = dt$$

So,

$$\int \frac{1}{x-\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x}(\sqrt{x}-1)}\,dx$$

$$= \int \frac{2}{2\sqrt{x}(\sqrt{x}-1)}\,dx$$

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$$= \int \frac{1}{2\sqrt{x}\cancel{(\sqrt{x}-1)}} dx \cdot \frac{2}{(\sqrt{x}-1)}$$

$$= \int dt \cdot \frac{2}{t}$$

$$= 2 \cdot \int \frac{dt}{t}$$

$$= 2 \cdot \ln t + c$$

Then,

$$\therefore \int \frac{1}{x-\sqrt{x}} dx = 2 \cdot \ln(\sqrt{x}-1) + c$$

g) $\int x^2 \sqrt{a^3+x^3}\, dx$

Solution

Put $a^3 + x^3 = t$

Differentiating both sides with respect to 'x',

$$\frac{d}{dx}(a^3+x^3) = \frac{d}{dx}(t)$$

$$\text{or, } \frac{d}{dx}(a^3) + \frac{d}{dx}(x^3) = \frac{dt}{dx}$$

$$\text{or, } 0 + 3x^2 = \frac{dt}{dx}$$

$$\text{or, } 3x^2 = \frac{dt}{dx}$$

$$\therefore 3x^2\, dx = dt$$

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Now,

$$\int x^2\sqrt{a^3+x^3}\,dx = \int \frac{3x^2\sqrt{a^3+x^3}}{3}\,dx$$

$$= \frac{1}{3}\int 3x^2 dx.\sqrt{a^3+x^3}$$

$$= \frac{1}{3}\int \sqrt{t}\,dt \quad \left[\because \begin{array}{l} 3x^2dx = dt, \\ a^3+x^3 = t \end{array}\right]$$

$$= \frac{1}{3}\int t^{1/2}\,dt$$

$$= \frac{1}{3}\times\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}+c \quad \left[\int x^n dx = \frac{x^{n+1}}{n+1}+c\right]$$

$$= \frac{1}{3}\times\frac{t^{3/2}}{3/2}+c$$

$$= \frac{2t^{3/2}}{9}+c$$

Then,

$$\therefore \int x^2\sqrt{a^3+x^3}\,dx = \frac{2(a^3+x^3)^{3/2}}{9}+c$$

---

ii) $\int \frac{(\sqrt{x}+5)^5}{\sqrt{x}}\,dx$

Solution

Put $\sqrt{x}+5 = t$

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Differentiating both sides with respect to 'x',

$$\frac{d}{dx}(x^{1/2}+5) = \frac{d}{dx}(t)$$

or $\frac{d}{dx}(x^{1/2}) + \frac{d}{dx}(5) = \frac{dt}{dx}$

or $\frac{1}{2}x^{-1/2} + 0 = \frac{dt}{dx}$

or; $\frac{1}{2}x^{-1/2}\,dx = dt$

So,

$$\int \frac{(\sqrt{x}+5)^{5}}{\sqrt{x}}\,dx = \int \frac{2(\sqrt{x}+5)^{5}}{2\sqrt{x}}\,dx$$

$$= \int \frac{1}{2\sqrt{x}}\,dx \cdot 2(\sqrt{x}+5)^{5}$$

$$= 2\int t^{5}\,dt \quad \left[\because \sqrt{x}+5=t,\ \frac{1}{2\sqrt{x}}dx = dt\right]$$

$$= 2\cdot\frac{t^{5+1}}{5+1} + c \quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1}+c\right]$$

$$= 2\cdot\frac{t^{6}}{6} + c$$

Then,

$$\int \frac{(\sqrt{x}+5)^{5}}{\sqrt{x}}\,dx = \frac{1}{3}(\sqrt{x}+5)^{6} + c$$

$$\therefore \int \frac{(\sqrt{x}+5)^{5}}{\sqrt{x}}\,dx = \frac{1}{3}(\sqrt{x}+5)^{6} + c$$

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2. Evaluate the following integrals by substitution method.

a) $\int \frac{x}{e^{x^2}} dx$

Solution

put $x^2 = t$

Differentiating both sides with respect to 'x'

$$\frac{d}{dx}(x^2) = \frac{d}{dx}(t)$$

or $2x = \frac{dt}{dx}$

$\therefore 2x\,dx = dt$

So,

$$\int \frac{x}{e^{x^2}} dx = \int \frac{2x}{2e^{x^2}} dx$$

$$= \int 2x\,dx \cdot \frac{1}{2e^{x^2}}$$

$$= \frac{1}{2} \int \frac{1}{e^{x^2}} \cdot 2x\,dx$$

$$= \frac{1}{2} \int e^{-x^2} \cdot 2x\,dx$$

$$= \frac{1}{2} \int e^{-t} \cdot dt \quad \left[\because x^2 = t,\ 2x\,dx = dt\right]$$

$$= \frac{1}{2}\left(-e^{-t} + c\right)$$

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$= -\frac{1}{2} e^{-t} + c$

$\therefore \int \frac{x}{e^{x^2}} = -\frac{1}{2} e^{-x^2} + c$

~~b) $\int \frac{dx}{1+\sqrt{x}}$~~

b) $\int \frac{(\ln x)^2}{x} dx$

Solution

Put $\ln x = t$

Differentiating both sides with respect to x,

$\frac{d}{dx}(\ln x) = \frac{d(t)}{dx}$

or, $\frac{1}{x} = \frac{dt}{dx}$

$\therefore dt = \frac{dx}{x}$

So, $\int \frac{(\ln x)^2}{x} dx = \int \frac{dx}{x} \cdot (\ln x)^2$

$= \int t^2 . dt$

$= \frac{t^{2+1}}{2+1} + c \quad [\because \int x^n dx = \frac{x^{n+1}}{n+1} + c]$

$= \frac{t^3}{3} + c$

$$\therefore \int \frac{(\ln x)^2}{x}\,dx = \frac{(\ln x)^3}{3} + c$$

c) $\displaystyle\int \frac{dx}{x(\ln x)^m}, \; x > 0$

Solution

Put $\ln x = t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(\ln x) = \frac{d}{dx}(t)$$

or, $\displaystyle\frac{1}{x} = \frac{dt}{dx}$

$\therefore \; dt = \dfrac{dx}{x}$

So,

$$\int \frac{dx}{x(\ln x)^m} = \int \frac{dx}{x} \cdot \frac{1}{(\ln x)^m}$$

$$= \int dt \cdot \frac{1}{(t)^m}$$

$$= \int \frac{dt}{t^m}$$

$$\cancel{= \ln(t^m) + c}$$

$$= \int t^{-m}\,dt$$

$$= \frac{t^{-m+1}}{m+1} + c \quad \left[\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + c\right]$$

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$$\therefore \int \frac{dx}{x(\ln x)^m} = \frac{(\ln x)^{-m+1}}{-m+1} + c$$

d) $\int \frac{dx}{x(1+\ln x)^2}$

Solution

Put $1+\ln x = t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(1+\ln x) = \frac{d}{dx}(t)$$

or $\frac{d}{dx}(1) + \frac{d}{dx}(\ln x) = \frac{dt}{dx}$

or, $0 + \frac{1}{x} = \frac{dt}{dx}$

$\therefore dt = \frac{dx}{x}$

So,

$$\int \frac{dx}{x(1+\ln x)^2} = \int \frac{dx}{x} \cdot \frac{1}{(1+\ln x)^2}$$

$$= \int dt \cdot \frac{1}{\cancel{(1+\ln x)^2}\, t^2}$$

$\left[\because \frac{dx}{x} = t,\ (1+\ln x) = t\right]$

$$= \int t^{-2}\, dt$$

$$= \frac{t^{-2+1}}{-2+1} + c \quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1\right]$$

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$$= -t^{-1} + c$$

$$\therefore \int \frac{dx}{x(1+\ln x)^2} = -(1+\ln x)^{-1} + c$$

e) $\int \frac{e^x}{e^x+2}\,dx$

Solution

Put $e^x + 2 = t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(e^x+2) = \frac{d}{dx}(t)$$

or, $\frac{d}{dx}(e^x) + \frac{d}{dx}(2) = \frac{dt}{dx}$

or, $e^x + 0 = \frac{dt}{dx}$

$\therefore e^x\,dx = dt$

Then,

$$\int \frac{e^x}{e^x+2}\,dx = \int \frac{dt}{t}$$

$$= \ln(t) + c$$

$$\therefore \int \frac{e^x}{e^x+2}\,dx = \ln(e^x+2) + c$$

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f) $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\,dx$

Solution

Put $e^{2x}+e^{-2x}=t$

Differentiating both sides with respect to 'x'.

$$\frac{d}{dx}(e^{2x}+e^{-2x})=\frac{d}{dx}(t)$$

or $\frac{d}{dx}(e^{2x})+\frac{d}{dx}(e^{-2x})=\frac{dt}{dx}$

or $\cancel{\frac{d}{dx}}\frac{d(e^{2x})}{d(2x)}\times\frac{d(2x)}{\cancel{d(x)}\,dx}+\frac{d(e^{-2x})}{d(-2x)}\times\frac{d(-2x)}{dx}=\frac{dt}{dx}$

or $2e^{2x}-2e^{-2x}=\frac{dt}{dx}$

~~or~~ $\therefore 2(e^{2x}-e^{-2x})dx=dt$

Then,

$$\int\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}=\int\frac{2(e^{2x}-e^{-2x})}{2(e^{2x}+e^{-2x})}\,dx$$

$$=\frac{1}{2}\int\frac{dt}{t}\quad\left[\begin{array}{l}\because 2(e^{2x}-e^{-2x})dx=dt\\ \because e^{2x}+e^{-2x}=t\end{array}\right]$$

$$=\frac{1}{2}\ln(t)+c\quad\left[\because\int\frac{dx}{x}=\ln x+c\right]$$

$$\therefore\int\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\,dx=\frac{1}{2}\ln(e^{2x}+e^{-2x})+c$$

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g) $\int \frac{e^x}{e^x + e^{-x}} dx$

Solution

$$\int \frac{e^x}{e^x + \frac{1}{e^x}} dx$$

$$= \int \frac{e^x}{\frac{e^{2x}+1}{e^x}} dx$$

$$= \int \frac{e^{2x}}{e^{2x}+1} dx$$

Put $e^{2x}+1 = t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(e^{2x}+1) = \frac{d}{dx}(t)$$

or, $\frac{d(e^{2x})}{d(2x)} \times \frac{d(2x)}{dx} = \frac{dt}{dx}$

or, $2e^{2x} = \frac{dt}{dx}$

$\therefore 2e^{2x} dx = dt$

Now,

$$\int \frac{e^x}{e^x + e^{-x}} dx = \int \frac{e^{2x}}{e^{2x}+1} dx$$

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$$= \int \frac{2e^{2x}}{2(e^{2x}+1)}\,dx$$

$$= \frac{1}{2}\int \frac{1}{(e^{2x}+1)} \times 2e^{2x}\,dx$$

$$= \frac{1}{2}\int \frac{1}{t} \times dt \quad [\because e^{2x}+1 = t,\ 2e^{2x}dx = dt]$$

$$= \frac{1}{2}\ln(t) + c$$

Then,

$$\therefore \int \frac{e^{x}}{e^{x}+e^{-x}} = \frac{1}{2}\ln(e^{2x}+1) + c$$

---

h) $\int \frac{(x+1)(x+\ln x)^2}{x}\,dx$

Solution

put $x + \ln x = t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(x+\ln x) = \frac{d}{dx}(t)$$

$$\text{or, } \frac{d}{dx}(x) + \frac{d}{dx}(\ln x) = \frac{dt}{dx}$$

$$\text{or, } 1 + \frac{1}{x} = \frac{dt}{dx}$$

$$\therefore \frac{x+1}{x}\,dx = dt$$

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Now,

$$\int \frac{(x+1)(x+\ln x)^2}{x}\,dx = \int (x+\ln x)^2 \cdot \frac{(x+1)}{x}\,dx$$

$$= \int t^2 \cdot dt \quad \left[\because x+\ln x = t,\ \because \frac{(x+1)}{x}dx = dt\right]$$

$$= \frac{t^{2+1}}{2+1} + c$$

$$= \frac{t^3}{3} + c$$

$$\therefore \int \frac{(x+1)(x+\ln x)^2}{x}\,dx = \frac{(x+\ln x)^3}{3} + c$$

i) $\int \frac{e^x - 1}{e^x + 1}\,dx$

Solution

put $e^x + 1 = t$

Differentiating both sides with respect to x,

$$\frac{d}{dx}(e^x+1) = \frac{d}{dx}(t)$$

or, $\frac{d}{dx}(e^x) + \frac{d}{dx}(1) = \frac{dt}{dx}$

or, $e^x + 0 = \frac{dt}{dx}$

$\therefore e^x\,dx = dt$

Now,

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$$\int \frac{e^x - 1}{e^x + 1} dx = \int \left(\frac{e^x}{e^x+1} - \frac{1}{e^x+1} \right) dx$$

$$= \int \frac{e^x}{e^x+1} dx - \int \frac{1}{e^x+1} dx$$

$$= \ln(1+e^x) \cancel{dx} - \int \frac{1+e^x-e^x}{1+e^x} dx$$

$$\left[\because \int \frac{f'(x)}{f(x)} = \ln[\cancel{a}f(x)] + C \right]$$

$$= \ln(1+e^x) \cancel{dx} - \int \left(1 - \frac{e^x}{1+e^x} \right) dx$$

$$= \ln(1+e^x) \cancel{dx} - \left[\int 1\, dx - \int \frac{e^x}{1+e^x} dx \right]$$

$$= \ln(1+e^x) \cancel{dx} - x + \int \frac{e^x}{1+e^x} dx$$

$$= \ln(1+e^x) \cancel{dx} - x + \int \frac{1}{t} \cdot dt \left[\because e^x+1 = t, \; e^x dx = dt \right]$$

$$= \ln(1+e^x) \cancel{dx} - x + \ln(t) + C$$

$$= \ln(1+e^x) \cancel{dx} - x + \ln(1+e^x) + C$$

$$\left[\because t = \cancel{e}\, 1+e^x \right]$$

$$= 2\ln(1+e^x) - x + C$$

$$\therefore \int \frac{e^x - 1}{e^x + 1} = 2\ln(1+e^x) - x + C$$

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j) $\int \frac{1}{1+e^x} dx$

Solution

$$\int \frac{1+e^x - e^x}{1+e^x} dx$$

$$= \int \left(\frac{1+e^x}{1+e^x} - \frac{e^x}{1+e^x}\right) dx$$

$$= \int \left(1 - \frac{e^x}{1+e^x}\right) dx$$

$$= 1\int dx - \int \frac{e^x}{1+e^x} dx$$

~~Let t~~

Put $1+e^x = t$

Differentiating both sides withrespect to 'x',

$$\frac{d}{dx}(1+e^x) = \frac{d}{dx}(t)$$

or, $e^x = \frac{dt}{dx}$

$\therefore e^x dx = dt$

Now,

$$\int \frac{1}{1+e^x} dx = \int dx - \int \frac{e^x}{1+e^x} dx$$

$$= x - \int \frac{dt}{t}$$

$$= x - \ln(t) + c$$

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$= x$

$\therefore \int \frac{1}{1+e^x} dx = x - \ln(1+e^x) + C$

K) $\int x^4 3^{2x^5+1} dx$

Solution

put $2x^5+1 = t$

Differentiating both sides with respect to x,

$\frac{d}{dx}(2x^5) + \frac{d}{dx}(1) = \frac{d}{dx}(t)$

or, $10x^4 + 0 = \frac{dt}{dx}$

$\therefore 10x^4 dx = dt$

Now,

$\int x^4 3^{2x^5+1} dx = \int x^4 dx \cdot 3^{2x^5+1}$

$= \int \frac{10x^4 dx}{10} \cdot 3^{2x^5+1}$

$= \frac{1}{10} \int 10x^4 dx \cdot 3^{2x^5+1}$

$= \frac{1}{10} \int dt \cdot 3^t \quad [\because 10x^4 dx = dt,\ 2x^5+1 = t]$

$= \frac{1}{10} \int 3^t \cdot dt$

$= \frac{1}{10} \times \frac{3^t}{\ln 3} + C \quad [\because \int a^x dx = \frac{a^x}{\ln a} + C,\ a > 0]$

$= \frac{3^t}{10 \ln 3} + C$

$\therefore \int x^4 3^{2x^5+1} dx = \frac{3^{2x^5+1}}{10 \ln 3} + C \quad [\because t = 2x^5+1]$

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(1) (h) $\int \frac{dx}{1+\sqrt{x}}$

Solution

Put $\sqrt{x} = t$

Differentiating both sides with respect to "x"

$\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(t)$

or, $\frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\therefore dx = 2\sqrt{x}\, dt$

Now,

$\int \frac{dx}{1+\sqrt{x}} = \int \frac{2\sqrt{x}\, dt}{1+\sqrt{x}}$ $[\because dx = 2\sqrt{x}\, dt]$

$= 2 \cdot \int \frac{\sqrt{x}}{1+\sqrt{x}} dt$

$= 2 \cdot \int \frac{t}{1+t} dt$ $[\because t = \sqrt{x}]$

$= 2 \cdot \int \frac{1+t-1}{1+t} dt$

$= 2 \cdot \int \left(1 - \frac{1}{1+t}\right) dt$

$= 2\left[\int dt - \int \frac{1}{1+t} dt\right]$

$= 2[t - \ln(1+t)] + c$ $\left[\because \int dx = x + c, \because \int \frac{dx}{x} = \ln x + c\right]$

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$$\therefore \int \frac{dx}{1+\sqrt{x}} = 2[\sqrt{x} - \ln(1+\sqrt{x})] + c \quad [\because t = \sqrt{x}]$$
$$= 2\sqrt{x} - 2\ln(1+\sqrt{x}) + c$$

3. Evaluate the following using integration by parts:

a) $\int x^2 e^x \, dx$

$$= x^2 \int e^x dx - \int \left(\frac{d}{dx}(x^2) \cdot \int e^x dx\right) dx$$

$$\left[\because \int u.v \, dx = u\int v\, dx - \int \left(\frac{d}{dx}(u) . \int v \, dx\right) dx\right]$$

$$= x^2 . e^x - \int (2x . e^x) dx$$
$$= x^2 . e^x - \left[(2x)\int e^x dx - \int \left(\frac{d}{dx}(2x) . \int e^x dx\right) dx\right]$$
$$= x^2 . e^x - \left[2x . e^x - \int (2 . e^x) dx\right]$$
$$= x^2 . e^x - \left[2x . e^x - 2\int e^x dx\right]$$
$$= x^2 . e^x - \left[2x . e^x - 2 . e^x\right] + c$$
$$\cancel{= x^2 . e^x - e^x(2x-2)}$$
$$\cancel{= x}$$
$$= x^2 e^x - 2xe^x + 2e^x + c$$

b) $\int x e^{4x} dx$

$$= x \int e^{4x} dx - \int \left(\frac{d}{dx}(x) . \int e^{4x} dx\right) dx$$

$$\left[\because \int u.v dx = u\int v \, dx - \int \left(\frac{d}{dx}(u) . \int v \, dx\right) dx\right]$$

$$= x . \frac{e^{4x}}{4} - \int \left(1 . \frac{e^{4x}}{4}\right) dx$$

$$= x . \frac{e^{4x}}{4} - \frac{1}{4}\int e^{4x} dx$$

$$= x . \frac{e^{4x}}{4} - \frac{1}{4} \times \frac{e^{4x}}{4} + c$$

$$= x . \frac{e^{4x}}{4} - \frac{e^{4x}}{16} + c$$

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$$= \frac{e^{4x}}{16}(4x-1) + C$$

c) $\int x^2 e^{3x}\, dx$

$$= x^2 \int e^{3x}\, dx - \int \left(\frac{d}{dx}(x^2) \cdot \int e^{3x}\, dx\right) dx$$

$$\left[\because \int u.v\, dx = u \int v\, dx - \int \left(\frac{d}{dx}(u) \cdot \int v\, dx\right) dx\right]$$

$$= x^2 \cdot \frac{e^{3x}}{3} - \int \left(2x \cdot \frac{e^{3x}}{3}\right) dx$$

$$= x^2 \cdot \frac{e^{3x}}{3} - \frac{2}{3} \int (x \cdot e^{3x})\, dx$$

$$= x^2 \cdot \frac{e^{3x}}{3} - \frac{2}{3}\left[x \cdot \int e^{3x}\, dx - \int \left(\frac{d}{dx}(x) \cdot \int e^{3x}\, dx\right) dx\right]$$

$$= x^2 \cdot \frac{e^{3x}}{3} - \frac{2}{3}\left[x \cdot \frac{e^{3x}}{3} - \int \left(1 \cdot \frac{e^{3x}}{3}\right) dx\right]$$

$$= x^2 \cdot \frac{e^{3x}}{3} - \frac{2}{3}\left[x \cdot \frac{e^{3x}}{3} - \frac{1}{3} \int e^{3x}\, dx\right]$$

$$= x^2 \cdot \frac{e^{3x}}{3} - \frac{2}{3}\left[x \cdot \frac{e^{3x}}{3} - \frac{1}{3} \cdot \frac{e^{3x}}{3}\right] + C$$

$$= x^2 \cdot \frac{e^{3x}}{3} - \frac{2}{3} \times \frac{e^{3x}}{3}\left(x - \frac{1}{3}\right) + C$$

$$= e^{3x}\left[\frac{x^2}{3} - \frac{2}{9}\left(x - \frac{1}{3}\right)\right] + C$$

$$= e^{3x}\left(\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27}\right) + C$$

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d) $\int (2x-1)\sqrt{3x+1}\,dx$

$= \int (2x-1)(3x+1)^{1/2}\,dx$

$= (2x-1)\int (3x+1)^{1/2}dx - \int\left(\frac{d}{dx}(2x-1).\int (3x+1)^{1/2}dx\right)dx$

$= (2x-1)\frac{(3x+1)^{\frac{1}{2}+1}}{3(\frac{1}{2}+1)} - \int\left(2.\frac{(3x+1)^{\frac{1}{2}+1}}{3(\frac{1}{2}+1)}\right)dx$

$= \frac{2}{9}(2x-1)(3x+1)^{\frac{3}{2}} - \int \frac{4(3x+1)^{3/2}}{9}dx$

$= \frac{2}{9}(2x-1)(3x+1)^{\frac{3}{2}} - \frac{4}{9}\int (3x+1)^{\frac{3}{2}}dx$

$= \frac{2}{9}(2x-1)(3x+1)^{\frac{3}{2}} - \frac{4}{9}\times\frac{(3x+1)^{\frac{3}{2}+1}}{3(\frac{3}{2}+1)} + C$

$= \frac{2}{9}(2x-1)(3x+1)^{\frac{3}{2}} - \frac{4}{9}\times\frac{2}{15}(3x+1)^{\frac{5}{2}} + C$

$= \frac{2}{9}(2x-1)(3x+1)^{\frac{3}{2}} - \frac{8}{135}(3x+1)^{\frac{5}{2}} + C$

e) $\int (5x+3)\sqrt{2x-1}\,dx$

$= \int (5x+3)(2x-1)^{1/2}dx$

$= (5x+3)\int (2x-1)^{1/2}dx - \int\left(\frac{d}{dx}(5x+3).\int (2x-1)^{1/2}dx\right)dx$

$= (5x+3)\frac{(2x-1)^{\frac{1}{2}+1}}{2(\frac{1}{2}+1)} - \int\left(5.\frac{(2x-1)^{\frac{1}{2}+1}}{2(\frac{1}{2}+1)}\right)dx$

$= \frac{1}{3}(5x+3)(2x-1)^{\frac{3}{2}} - \frac{5}{3}\int (2x-1)^{\frac{3}{2}}dx$

$= \frac{1}{3}(5x+3)(2x-1)^{\frac{3}{2}} - \frac{5}{3}\times\frac{(2x-1)^{\frac{3}{2}+1}}{2(\frac{3}{2}+1)} + C$

$= \frac{1}{3}(5x+3)(2x-1)^{\frac{3}{2}} - \frac{5}{3}\times\frac{1}{5}(2x-1)^{\frac{5}{2}} + C$

$= \frac{1}{3}(5x+3)(2x-1)^{\frac{3}{2}} - \frac{1}{3}(2x-1)^{\frac{5}{2}} + C$

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f) $\int \frac{x}{\sqrt{4x+1}} dx$

$= \int x.(4x+1)^{-\frac{1}{2}} dx$

$= x.\int (4x+1)^{-\frac{1}{2}} dx - \int \left(\frac{d}{dx}(x).\int (4x+1)^{-\frac{1}{2}} dx\right) dx$

$= x.\frac{(4x+1)^{-\frac{1}{2}+1}}{4(-\frac{1}{2}+1)} - \int \left(1.\frac{(4x+1)^{-1/2+1}}{4(-\frac{1}{2}+1)}\right) dx$

$= x.\frac{(4x+1)^{\frac{1}{2}}}{2} - \int \frac{(4x+1)^{\frac{1}{2}}}{2} dx$

$= x.\frac{(4x+1)^{\frac{1}{2}}}{2} - \frac{1}{2}\int (4x+1)^{\frac{1}{2}} dx$

$= \frac{1}{2}x(4x+1)^{\frac{1}{2}} - \frac{1}{2} \times \frac{(4x+1)^{\frac{1}{2}+1}}{4(\frac{1}{2}+1)} + c$

$= \frac{1}{2}x(4x+1)^{\frac{1}{2}} - \frac{1}{2} \times \frac{1}{6}(4x+1)^{\frac{3}{2}} + c$

$= \frac{1}{2}x(4x+1)^{\frac{1}{2}} - \frac{1}{12}(4x+1)^{\frac{3}{2}} + c$

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g) $\int \frac{x}{\sqrt{a-bx}} dx$

$= \int x.(a-bx)^{-\frac{1}{2}} dx$

$= x.\int (a-bx)^{-\frac{1}{2}} dx - \int \left(\frac{d}{dx}(x).\int (a-bx)^{-\frac{1}{2}} dx\right) dx$

$\left[\because \int u.v\, dx = u\int v dx - \int \left(\frac{d}{dx}(u).\int v\, dx\right) dx\right]$

$= x.\frac{(a-bx)^{-\frac{1}{2}+1}}{-b(-\frac{1}{2}+1)} - \int \left(1.\frac{(a-bx)^{-\frac{1}{2}+1}}{-b(-\frac{1}{2}+1)}\right) dx$

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$$= -2x \cdot \frac{(a-bx)^{1/2}}{b} - \int \frac{(a-bx)^{1/2}}{-\frac{b}{2}} dx$$

$$= -2x \cdot \frac{(a-bx)^{1/2}}{b} + \frac{2}{b} \int (a-bx)^{1/2} dx$$

$$= -2x \cdot \frac{(a-bx)^{1/2}}{b} + \frac{2}{b} \times \frac{(a-bx)^{\frac{1}{2}+1}}{-b(\frac{1}{2}+1)} + c$$

$$= -2x \cdot \frac{(a-bx)^{1/2}}{b} + \frac{2}{b} \times \frac{(a-bx)^{\frac{3}{2}}}{-b \times \frac{3}{2}} + c$$

$$= -\frac{2x}{b} (a-bx)^{1/2} - \frac{4}{3b^2} (a-bx)^{3/2} + c$$