Chapter-5 : Integration and Its Applications.

Exercise (5A)

1. Evaluate the following integrals :

a) $\int (2x+3)^4 \, dx$

Solution

$\int (2x+3)^4 \, dx$

$= \dfrac{(2x+3)^{4+1}}{2(4+1)} + c \quad \left[\because \int (ax+b)^n dx = \dfrac{(ax+b)^{n+1}}{a(n+1)} + c, n \neq -1\right]$

$= \dfrac{(2x+3)^5}{2 \times 5} + c$

$= \dfrac{(2x+3)^5}{10} + c$

b) $\int (2-3x)^6 \, dx$

Solution

$\int (2-3x)^6 \, dx$

$= \dfrac{(2-3x)^{6+1}}{-3(6+1)} + c \quad \left[\because \int (a-bx)^n dx = \dfrac{(a-bx)^{n+1}}{-b(n+1)} + c, n \neq -1\right]$

$= \dfrac{(2-3x)^7}{-21} + c$

c) $\int (3x^2 + \sqrt{x} - \frac{1}{x}) \, dx$

Solution

$\int (3x^2 + \sqrt{x} - \frac{1}{x}) \, dx$

$= \int 3x^2 \, dx + \int x^{1/2} \, dx - \int x^{-1} \, dx$

$= 3\int x^2 \, dx + \int x^{1/2} \, dx - \int x^{-1} \, dx$

$= 3 \cdot \dfrac{x^{2+1}}{2+1} + \dfrac{x^{1/2+1}}{\frac{1}{2}+1} - \dfrac{x^{-1+1}}{-1+1} + c$

$= 3 \cdot \dfrac{x^3}{3} + \dfrac{x^{3/2}}{\frac{3}{2}} - \dfrac{1}{0} + c$

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c) $\int (3x^2 + \sqrt{x} - \frac{1}{x})\, dx$

Solution

$\int (3x^2 + \sqrt{x} - \frac{1}{x})\, dx$

$= \int (3x^2 + x^{1/2} - \frac{1}{x})\, dx$

$= \int 3x^2 dx + \int x^{1/2} dx - \int \frac{1}{x} dx$

$= 3\int x^2 dx + \int x^{1/2} dx - \int \frac{1}{x} dx$

$= 3 \cdot \frac{x^{2+1}}{2+1} + \frac{x^{1/2+1}}{\frac{1}{2}+1} - \log x + C$ $\left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + c, \int \frac{1}{x} dx = \log x + c\right]$

$= x^3 + \frac{x^{3/2}}{\frac{3}{2}} - \log x + c$

$= x^3 + \frac{2x^{3/2}}{3} - \log x + c$

d) $\int \frac{(4x-3)^3}{x^2}\, dx$

Solution

$\int \frac{(4x-3)^3}{x^2}\, dx$

$= \int \left[\frac{(4x)^3 - 3\cdot(4x^2)\cdot(3) + 3\cdot 4x\cdot(3)^2 - (3)^3}{x^2}\right] dx$

$= \int \left(\frac{64x^3 - 144x^2 + 108x - 27}{x^2}\right) dx$

$= \int (64x - 144 + \frac{108}{x} - \frac{27}{x^2})\, dx$

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$$= 64\int x\,dx - 144\int dx + 108\int \frac{1}{x}\,dx - 27\int x^{-2}$$

$$= 64 \times \frac{x^{1+1}}{1+1} - 144x + \cancel{18}\,108\ln x - 27 \times \frac{x^{-2+1}}{-2+1} + C$$

$$= 64 \times \frac{x^2}{2} - 144x + 108\ln x - 27 \times \frac{x^{-1}}{-1} + C$$

$$= 32x^2 - 144x + 108\ln x \,\cancel{x}+ \frac{27}{x} + C$$

e) $\int \left(\sqrt{Q+1} + \frac{1}{\sqrt{Q}} \right) dQ$

Solution

$$= \int \left[(Q+1)^{1/2} + \frac{1}{(Q)^{1/2}} \right] dQ$$

$$= \int (Q+1)^{1/2}\,dQ + \int \frac{1}{(Q)^{1/2}}\,dQ$$

$$= \int (1.Q+1)^{1/2}\,dQ + \int \cancel{1}(Q)^{-1/2}\,dQ$$

$$= \frac{(Q+1)^{\frac{1}{2}+1}}{1.(\frac{1}{2}+1)} + \frac{(Q)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \cancel{dQ} + C \quad \left[\because \int x^n = \frac{x^{n+1}}{n+1}, n \neq -1\right]$$

$$= \frac{(Q+1)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{Q^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{2(Q+1)^{3/2}}{3} + 2Q^{1/2} + C$$

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f) $\int \frac{5\sqrt{4-y}}{3}\,dy$

Solution

$$\int \frac{5(4-y)^{1/2}}{3}\,dy$$

$$= \frac{5}{3}\int (4-y)^{1/2}\,dy$$

$$= \frac{5}{3}\int (4-1y)^{1/2}\,dy$$

$$= \frac{5}{3}\cdot\frac{(4-y)^{1/2+1}}{-1\left(\frac{1}{2}+1\right)} + c \quad \left[\because \int (a-bx)^n\,dx = \frac{(a-bx)^{n+1}}{-b(n+1)},\ n\neq -1\right]$$

$$= \frac{5}{3}\cdot\frac{(4-y)^{\frac{3}{2}}}{-\frac{3}{2}} + c$$

$$= \frac{5}{3}\times\frac{2(4-y)^{3/2}}{-3} + c$$

$$= -\frac{10(4-y)^{3/2}}{9} + c$$

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g) $\int 5(1-e^{-0.8y})\,dy$

Solution

$$\int (5-5e^{-0.8y})\,dy$$

$$= \int 5\,dy - \int 5e^{-0.8y}\,dy$$

$$= 5.\int dy - 5\int e^{-0.8y}\,dy$$

$$= 5.y - 5.\frac{e^{-0.8y}}{-0.8} + c \quad \left[\because \int e^{ax} = \frac{e^{ax}}{a} + c\right]$$

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$$= 5y + \frac{5 \cdot e^{-0.8y}}{0.8} + c$$

$$= 5\left(y + \frac{e^{-0.8y}}{0.8}\right) + c$$

h) $\int \frac{15\, dQ}{(Q-5)^7}$

Solution

$$\int [15 (Q-5)^{-7}] dQ$$

$$= 15 . \int (Q-5)^{-7} dQ$$

$$= 15 . \int (1Q-5)^{-7} dQ$$

$$= 15 . \frac{(Q-5)^{-7+1}}{-7+1} + c \qquad \left[\because \int (ax-b)^n = \frac{(ax-b)^{n+1}}{a(n+1)} + c \right]$$

$$= 15 . \frac{(Q-5)^{-6}}{-6} + c$$

$$= \frac{5}{-2(Q-5)^6} + c$$

i) $\int (1+0.8t)^{1.12} dt$

$$= \frac{(1+0.8t)^{1.12+1}}{0.8(1.12+1)} + c \left[\because \int (a+bx)^n dx = \frac{(a+bx)^{n+1}}{b(n+1)} + c, n \neq -1 \right]$$

$$= \frac{(1+0.8t)^{2.12}}{1.696} + c$$

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j) $\int \frac{dQ}{8Q+1}$

Solution

$= \frac{1}{8} \ln(8Q+1) + C \quad \left[\because \int \frac{dx}{ax+b} = \frac{1}{a} \ln(ax+b) + C\right]$

k) $\int \frac{dQ}{2-7Q}$

Solution

$= -\frac{1}{7} \ln(2-7Q) + C \quad \left[\because \int \frac{dx}{a-bx} = -\frac{1}{b} \ln(a-bx) + C\right]$

l) $\int \frac{dz}{4z+1}$

Solution

$= \frac{1}{4} \ln(4z+1) + C \quad \left[\because \int \frac{dx}{ax+b} = \frac{1}{a} \ln(ax+b) + C\right]$

m) $\int \left(\frac{3x^2}{x^3+4}\right) dx$

Solution

$= \ln(x^3+4) + C \quad \left[\because \int \frac{f'(x)}{f(x)} = \ln[f(x)] + C\right]$

n) $\int \frac{x^2+x+3}{x^2+3} dx$

Solution

$\int \left[\frac{(x^2+3)}{x^2+3} + \frac{x}{x^2+3}\right] dx$

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$$= \int \left(1 + \frac{x}{x^2+3}\right) dx$$

$$= \int 1\, dx + \int \frac{x}{x^2+3}\, dx$$

$$\cancel{= \int 1\, dx + \int \frac{dx}{x + \frac{3}{x}}}$$

$$= x + \int \frac{\frac{1}{2} \times 2x}{x^2+3}\, dx$$

$$= x + \frac{1}{2} \int \frac{2x}{x^2+3}\, dx$$

$$= x + \frac{1}{2} . \ln(x^2+3) + C \quad \left[\because \int \frac{f'(x)}{f(x)} = \ln[f(x)] + c\right]$$

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o) $\int \frac{dx}{3x-4}$

Solution

$$= \frac{1}{3} . \ln(3x-4) + C \quad \left[\because \int \frac{dx}{ax-b} = \frac{1}{a} . \ln(ax-b) + c\right]$$

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b) $\int 5^{2x+3}\,dx$

Solution

$= \int (5^{2x} \cdot 5^{3})\,dx$

$= 5^{3}\int 5^{2x}\,dx$

$= 5^{3} \cdot \int (5^{2})^{x}\,dx$

$= 5^{3} \cdot \dfrac{5^{2x}}{\ln 5^{2}} + c$

$= 5^{3} \cdot \dfrac{5^{2x}}{2\ln 5} + c$

2. Evaluate the following integrals:

a) $\int \dfrac{x+2}{x-2}\,dx$

Solution

$\int \dfrac{x+4-2}{x-2}\,dx$

$= \int \left(\dfrac{x-2}{x-2} + \dfrac{4}{x-2} \right) dx$

$= \int \left(1 + \dfrac{4}{x-2} \right) dx$

$= \int 1\,dx + \int \dfrac{4}{x-2}\,dx$

$= x + 4\int \dfrac{1}{x-2}\,dx \quad \left[\because \int dx = x + c\right]$

$= x + 4 \cdot \ln(x-2) + c \quad \left[\because \int \dfrac{dx}{ax-b} = \dfrac{1}{a}\ln(ax-b) + c\right]$

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b) $\int \frac{2x-1}{3x+7}\,dx$

Solution

$$\int \frac{\cancel{2x}\ 3x+7-x-8}{3x+7}\,dx$$

$$= \int \left(\frac{3x+7}{3x+7} - \frac{x+8}{3x+7}\right) dx$$

$$= \int \left(1 - \frac{x+8}{3x+7}\right) dx$$

$$= \int 1\,dx - \int \frac{x+8}{3x+7}\,dx$$

$$= 1.\int dx - \frac{1}{3}\int \frac{3x+24}{3x+7}\,dx$$

$$= x - \frac{1}{3}\int \frac{3x+7+17}{3x+7}\,dx$$

$$= x - \frac{1}{3}\int \left(\frac{3x+7}{3x+7} + \frac{17}{3x+7}\right) dx$$

$$= x - \frac{1}{3}\int \left(1 + \frac{17}{3x+7}\right) dx$$

$$= x - \frac{1}{3}\left[\int 1\,dx + \int \frac{17}{3x+7}\,dx\right]$$

$$= x - \frac{1}{3}\left[x + 17\int \frac{dx}{3x+7}\right] \quad \left[\because \int dx = 1\right]$$

$$= x - \frac{1}{3}\left[x + 17\cdot\frac{1}{3}\ln(3x+7)\right] + c \quad \left[\because \int \frac{dx}{ax+b} = \frac{1}{a}\ln(ax+b)+c\right]$$

$$= x - \frac{1}{3}x - \frac{17}{9}\ln(3x+7) + c$$

$$= \frac{2}{3}x - \frac{17}{9}\ln(3x+7) + c$$

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c) $\int \frac{1-x}{1+x}\,dx$

Solution

$$\int \frac{1+x-2x}{1+x}\,dx$$

$$= \int\left(\frac{1+x}{1+x} - \frac{2x}{1+x}\right)dx$$

$$= \int\left(1 - \frac{2x}{1+x}\right)dx$$

$$= 1\int dx - \int \frac{2x}{1+x}\,dx$$

$$\cancel{= x - \int \frac{2x-2x+1}{}}$$

$$= 1\int dx - 2\int \frac{x}{1+x}\,dx$$

$$\cancel{= x - 2\int \frac{dx}{\frac{1}{x}+1}}$$

$$= x - 2\int \frac{x+1-1}{x+1}\,dx \quad \left[\because \int dx = x\right]$$

$$= x - 2\int\left(\frac{x+1}{x+1} - \frac{1}{x+1}\right)dx$$

$$= x - 2\int\left(1 - \frac{1}{x+1}\right)dx$$

$$= x - 2\left[\int 1\,dx - \int \frac{dx}{x+1}\right]$$

$$= x - 2\left[x - \ln(x+1)\right] + C \quad \left[\because \int \frac{dx}{ax+1} = \frac{1}{a}\ln(ax+1) + C\right]$$

$$= x - 2x + 2\ln(x+1) + C$$

$$= -x + 2\ln(x+1) + C$$

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d) $\int \frac{x-2}{x+2}\, dx$

Solution

$$\int \frac{x+2-4}{x+2}\, dx$$

$$= \int \left(\frac{x+2}{x+2} - \frac{4}{x+2}\right) dx$$

$$= \int \left(1 - \frac{4}{x+2}\right) dx$$

$$= \int 1\, dx - \int \frac{4}{x+2}\, dx$$

$$= 1.\int dx - 4.\int \frac{dx}{x+2}$$

$$= x - 4.\ln(x+2) + C \quad \left[\because \int \frac{dx}{ax+b} = \frac{1}{a}\ln(ax+b) + C\right]$$

e) $\int \frac{x\,dx}{x-3}$

Solution

$$\int \frac{x+3-3}{x-3}\, dx$$

$$= \int \left(\frac{x-3}{x-3} + \frac{3}{x-3}\right) dx$$

$$= \int \left(1 + \frac{3}{x-3}\right) dx$$

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$= \int 1\,dx + \int \frac{3}{x-3}\,dx$

$= x + 3.\int \frac{dx}{x-3}$

$= x + 3.\ln(x-3) + C$

f) $\int \frac{x+3}{x+5}\,dx$

Solution

$\int \frac{x+5-2}{x+5}\,dx$

$= \int \left(\frac{x+5}{x+5} - \frac{2}{x+5}\right)dx$

$= \int \left(1 - \frac{2}{x+5}\right)dx$

$= \int 1\,dx - \int \frac{2}{x+5}\,dx$

$= x - 2.\int \frac{dx}{x+5}$

$= x - 2.\ln(x+5) + C$

g) $\int \frac{x+5}{x-5}\,dx$

Solution

$\int \frac{x-5+10}{x-5}\,dx$

$$= \int \left(\frac{x-5}{x-5} + \frac{10}{x-5} \right) dx$$

$$= \int \left(1 + \frac{10}{x-5} \right) dx$$

$$= \int 1\, dx + \int \frac{10}{x-5}\, dx$$

$$= 1. \int dx + 10 \int \frac{dx}{x-5}$$

$$= x + 10. \ln(x-5) + C \quad \left[\because \int dx = x + C, \int \frac{dx}{ax+b} = \frac{1}{a} \ln(x+b) \right]$$

h) $\int \frac{x+2}{x-3}\, dx$

Solution

$$\int \frac{x-3+5}{x-3}\, dx$$

$$= \int \left(\frac{x-3}{x-3} + \frac{5}{x-3} \right) dx$$

$$= \int \left(1 + \frac{5}{x-3} \right) dx$$

$$= \int 1\, dx + \int \frac{5}{x-3}\, dx$$

$$= 1. \int dx + 5 \int \frac{dx}{x-3}$$

$$= x + 5. \ln(x-3) + C \quad \left[\because \int dx = x + C, \int \frac{dx}{ax-b} = \frac{1}{a} \ln(ax-b) + C \right]$$

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3. Integrate the following!

a) $\int \frac{dx}{\sqrt{2x+1} - \sqrt{2x-3}}$

Solution

$$= \int\left(\frac{1}{\sqrt{2x+1} - \sqrt{2x-3}} \times \frac{\sqrt{2x+1} + \sqrt{2x-3}}{\sqrt{2x+1} + \sqrt{2x-3}}\right) dx$$

$$= \int\left(\frac{\sqrt{2x+1} + \sqrt{2x-3}}{2x+1-2x+3}\right) dx$$

$$= \int\left(\frac{\sqrt{2x+1} + \sqrt{2x-3}}{4}\right) dx$$

$$= \frac{1}{4} \int (\sqrt{2x+1} + \sqrt{2x-3})\, dx$$

$$= \frac{1}{4}\left[\int (2x+1)^{1/2} dx + \int (2x-3)^{1/2} dx\right]$$

$$= \frac{1}{4}\left[\frac{(2x+1)^{1/2+1}}{2(\frac{1}{2}+1)} + \frac{(2x-3)^{1/2+1}}{2(\frac{1}{2}+1)}\right]$$

$$\left[\because \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)},\ n \neq -1\right]$$

$$= \frac{1}{4}\left[\frac{(2x+1)^{3/2}}{3} + \frac{(2x-3)^{3/2}}{3}\right]$$

$$= \frac{(2x+1)^{3/2}}{12} + \frac{(2x-3)^{3/2}}{12}$$

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b) $\int \frac{dx}{\sqrt{x}+\sqrt{1+x}}$

Solution

$$= \int \left(\frac{1}{\sqrt{x}+\sqrt{1+x}} \times \frac{\sqrt{x}-\sqrt{1+x}}{\sqrt{x}-\sqrt{1+x}} \right) dx$$

$$= \int \left(\frac{\sqrt{x}-\sqrt{1+x}}{x-1-x} \right) dx$$

$$= \int \left(\frac{\sqrt{x}-\sqrt{1+x}}{-1} \right) dx$$

$$= -1 \int (\sqrt{x}-\sqrt{1+x})\, dx$$

$$= -1 \left[\int \sqrt{x}\, dx - \int \sqrt{1+x}\, dx \right]$$

$$= -1 \left[\int (x)^{1/2} dx - \int (1+x)^{1/2} dx \right]$$

$$= -1 \left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{(1+x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right] + C \quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1 \right]$$

$$= -1 \left[\frac{x^{3/2}}{3/2} - \frac{(1+x)^{3/2}}{3/2} \right] + C$$

$$= -\frac{2}{3} \left[x^{3/2} - (1+x)^{3/2} \right] + C$$

$$= \frac{2}{3} \left[(1+x)^{3/2} - x^{3/2} \right] + C$$

c) $\int \frac{dx}{\sqrt{x+1}-\sqrt{x}}$

Solution

$$\int \left(\frac{1}{\sqrt{x+1}-\sqrt{x}} \times \frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}\right) dx$$

$$= \int \left(\frac{\sqrt{x+1}+\sqrt{x}}{x+1-x}\right) dx$$

$$= \int (\sqrt{x+1}+\sqrt{x})\, dx$$

$$= \int \sqrt{x+1}\, dx + \int \sqrt{x}\, dx$$

$$= \int (x+1)^{1/2} dx + \int (x)^{1/2} dx$$

$$= \frac{(x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C \quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1\right]$$

$$\left[\because \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C,\ n \neq -1\right]$$

$$= \frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{(x)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{2(x+1)^{3/2}}{3} + \frac{2x^{3/2}}{3} + C$$

$$= \frac{2}{3}\left[(x+1)^{3/2} + x^{3/2}\right] + C$$

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d) $\int \frac{dx}{\sqrt{x+2}-\sqrt{x+3}}$

Solution

$$\int \left(\frac{1}{\sqrt{x+2}-\sqrt{x+3}} \times \frac{\sqrt{x+2}+\sqrt{x+3}}{\sqrt{x+2}+\sqrt{x+3}}\right) dx$$

$$= \int \left(\frac{\sqrt{x+2} + \sqrt{x+3}}{x+2-x-3} \right) dx$$

$$= \int \left(\frac{\sqrt{x+2} + \sqrt{x+3}}{-1} \right) dx$$

$$= -1 \int (\sqrt{x+2} + \sqrt{x+3}) \, dx$$

$$= -1 \left[\int (x+2)^{1/2} dx + \int (x+3)^{1/2} dx \right.$$

$$= -1 \left[\frac{(x+2)^{1/2+1}}{\frac{1}{2}+1} + \frac{(x+3)^{1/2+1}}{\frac{1}{2}+1} \right] + c$$

$$= -1 \left[\frac{2(x+2)^{3/2}}{3} + \frac{2(x+3)^{3/2}}{3} \right] + c$$

$$= -\frac{2}{3} \left[(x+2)^{3/2} + (x+3)^{3/2} \right] + C$$

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e) $\int \frac{dx}{\sqrt{x+1} + \sqrt{x+2}}$

Solution

$$\int \left(\frac{1}{\sqrt{x+1} + \sqrt{x+2}} \times \frac{\sqrt{x+1} - \sqrt{x+2}}{\sqrt{x+1} - \sqrt{x+2}} \right) dx$$

$$= \int \left(\frac{\sqrt{x+1} - \sqrt{x+2}}{x+1-x-2} \right) dx$$

$$= \int \left(\frac{\sqrt{x+1} - \sqrt{x+2}}{-1} \right) dx$$

$$= -1 \int (\sqrt{x+1} - \sqrt{x+2}) \, dx$$

$$= -1\left[\int (x+1)^{1/2}\,dx - \int (x+2)^{1/2}\,dx\right]$$

$$= -1\left[\frac{(x+1)^{1/2+1}}{\frac{1}{2}+1} - \frac{(x+2)^{1/2+1}}{\frac{1}{2}+1}\right] + c \quad \left[\because \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)},\ n \neq -1\right]$$

$$= -1\left[\frac{(x+1)^{3/2}}{\frac{3}{2}} - \frac{(x+2)^{3/2}}{\frac{3}{2}}\right] + c$$

$$= -\frac{2}{3}\left[(x+1)^{3/2} - (x+2)^{3/2}\right] + c$$

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f) $\displaystyle\int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}}$

Solution

$$\int \left(\frac{1}{\sqrt{x+a} + \sqrt{x+b}} \times \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}}\right) dx$$

$$= \int \left(\frac{\sqrt{x+a} - \sqrt{x+b}}{x+a-x-b}\right) dx$$

$$= \int \left(\frac{\sqrt{x+a} - \sqrt{x+b}}{a-b}\right) dx$$

$$= \frac{1}{a-b}\left[\int (x+a)^{1/2}\,dx - \int (x+b)^{1/2}\,dx\right]$$

$$= \frac{1}{a-b}\left[\frac{(x+a)^{1/2+1}}{\frac{1}{2}+1} - \frac{(x+b)^{1/2+1}}{\frac{1}{2}+1}\right] + c$$

$$\left[\because \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)},\ n \neq -1\right]$$

$$= \frac{1}{a-b}\left[\frac{2(x+a)^{3/2}}{3} - \frac{2(x+b)^{3/2}}{3}\right] + c$$

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$= \frac{2}{3(a-b)}\left[(x+a)^{3/2} - (x+b)^{3/2}\right] + c$

8) $\int \frac{dx}{\sqrt{x+1} - \sqrt{x+2}}$

Solution

$\int \left(\frac{1}{\sqrt{x+1} - \sqrt{x+2}} \times \frac{\sqrt{x+1} + \sqrt{x+2}}{\sqrt{x+1} + \sqrt{x+2}} \right) dx$

$= \int \left(\frac{\sqrt{x+1} + \sqrt{x+2}}{x+1-x-2} \right) dx$

$= \int \left(\frac{\sqrt{x+1} + \sqrt{x+2}}{-1} \right) dx$

$= -1\left[\int (x+1)^{1/2} dx + \int (x+2)^{1/2} dx\right]$

$= -1\left[\frac{(x+1)^{1/2+1}}{\frac{1}{2}+1} + \frac{(x+2)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right] + c$

$\left[\because \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c\right]$

$= -1\left[\frac{(x+1)^{3/2}}{\frac{3}{2}} + \frac{(x+2)^{3/2}}{\frac{3}{2}}\right] + c$

$= -\frac{2}{3}\left[(x+1)^{3/2} + (x+2)^{3/2}\right] + c$

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h) $\int \frac{dx}{\sqrt{ax+b} - \sqrt{ax+c}}$

Solution

$\int \left(\frac{1}{\sqrt{ax+b} - \sqrt{ax+c}} \times \frac{\sqrt{ax+b} + \sqrt{ax+c}}{\sqrt{ax+b} + \sqrt{ax+c}} \right) dx$

$= \int \left(\frac{\sqrt{ax+b} + \sqrt{ax+c}}{ax+b-ax-c} \right) dx$

$= \int \left(\frac{\sqrt{ax+b} + \sqrt{ax+c}}{b-c} \right) dx$

$= \frac{1}{b-c} \int (\sqrt{ax+b} + \sqrt{ax+c}) \, dx$

$= \frac{1}{b-c} \left[\int (ax+b)^{1/2} dx + \int (ax+c)^{1/2} dx \right]$

$= \frac{1}{b-c} \left[\frac{(ax+b)^{\frac{1}{2}+1}}{a(\frac{1}{2}+1)} + \frac{(ax+c)^{\frac{1}{2}+1}}{a(\frac{1}{2}+1)} \right] + K$

$\left[\because \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, \; n \neq -1 \right]$

$= \frac{1}{b-c} \left[\frac{(ax+b)^{\frac{3}{2}}}{a \cdot \frac{3}{2}} + \frac{(ax+c)^{\frac{3}{2}}}{a \cdot \frac{3}{2}} \right] + K$

$= \frac{2}{3a(b-c)} \left[(ax+b)^{3/2} + (ax+c)^{3/2} \right] + K$

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4. Integrate the following:

a) $\int (4e^{3x}+1)\,dx$

Solution

$\int 4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + 1.\int dx$

$= 4.\frac{e^{3x}}{3} + 1.x + c \quad \left[\because \int e^{ax}dx = \frac{e^{ax}}{a}+c,\ \int dx = x+c\right]$

$= \frac{4e^{3x}}{3} + x + c$

b) $\int e^{2x+3}\,dx$

Solution

$= \frac{e^{2x+3}}{2} + c \quad \left[\because \int e^{ax+b}\,dx = \frac{e^{ax+b}}{a}+c\right]$

c) $\int (e^{ax}-e^{-ax})^2\,dx$

Solution

$= \int (e^{2ax} - 2.e^{ax}.e^{-ax} + e^{-2ax})\,dx$

$= \int (e^{2ax} - 2 + e^{-2ax})\,dx$

$= \int e^{2ax}\,dx - 2\int dx + \int e^{-2ax}\,dx$

$= \frac{e^{2ax}}{2a} - 2x + \frac{e^{-2ax}}{-2a} + c \left[\because \int e^{ax}dx = \frac{e^{ax}}{a}+c\right]$

d) $\int \frac{x^2+e^{3x}}{x^2 e^{3x}}\,dx$

Solution

$= \int \left(\frac{x^2}{x^2 e^{3x}} + \frac{e^{3x}}{x^2 e^{3x}}\right) dx$

$$= \int \left(\frac{1}{e^{3x}} + \frac{1}{x^2} \right) dx$$

$$= \int e^{-3x}\, dx + \int x^{-2}\, dx$$

$$= \frac{e^{-3x}}{-3} + \frac{x^{-2+1}}{-2+1} + c \quad \left[\because \int e^{ax} dx = \frac{e^{ax}}{a} + c, \int x^n dx = \frac{x^{n+1}}{n+1} + c \right]$$

$$= -\frac{e^{-3x}}{3} - \frac{1}{x} + C$$

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e) $\int \frac{e^{5\ln x} - e^{4\ln x}}{e^{3\ln x} - e^{2\ln x}}\, dx$

Solution

$$\int \frac{e^{\ln x^5} - e^{\ln x^4}}{e^{\ln x^3} - e^{\ln x^2}}\, dx \quad \left[\because \ln m^n = n \ln m \right]$$

$$= \int \frac{x^5 - x^4}{x^3 - x^2} \quad \left[\because e^{\ln x} = x \right]$$

$$= \int \frac{x^4(x-1)}{x^2(x-1)}$$

$$= \int x^2$$

$$= \frac{x^{2+1}}{2+1} + c \quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1 \right]$$

$$= \frac{x^3}{3} + c$$

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f) $\int \frac{e^{2x} + e^{4x}}{e^{x} + e^{-x}} dx$

Solution

$$= \int \left(\frac{e^{2x} + e^{2x} \cdot e^{2x}}{e^{x} + \frac{1}{e^{x}}} \right) dx$$

$$= \int \left[\frac{e^{2x}(1 + e^{2x})}{\frac{e^{2x}+1}{e^{x}}} \right] dx$$

$$= \int \left[\frac{e^{3x}(e^{2x}+1)}{e^{2x}+1} \right] dx$$

$$= \int e^{3x} dx$$

$$= \frac{e^{3x}}{3} + c \quad \left[\because \int e^{ax} dx = \frac{e^{ax}}{a} + c \right]$$

g) $\int x^{3} e^{3x} \left(\frac{1}{x^{3}} + \frac{1}{e^{3x}} \right) dx$

Solution

$$\int \left(\frac{x^{3} e^{3x}}{x^{3}} + \frac{x^{3} e^{3x}}{e^{3x}} \right) dx$$

$$= \int (e^{3x} + x^{3}) dx$$

$$= \int e^{3x} dx + \int x^{3} dx$$

$$= \frac{e^{3x}}{3} + \frac{x^{3+1}}{3+1} + c \quad \left[\because \int e^{ax} dx = \frac{e^{ax}}{a} + c, \int x^{n} dx = \frac{x^{n+1}}{n+1} + c \right]$$

$$= \frac{e^{3x}}{3} + \frac{x^{4}}{4} + c$$

h) $\int e^{px+q}\, dx$

Solution

$= \frac{e^{px+q}}{p} + c \quad \left[\because \int e^{ax+b}\, dx = \frac{e^{ax+b}}{a} + c\right]$

i) $\int e^{2-7x}\, dx$

Solution

$= \frac{e^{2-7x}}{-7} + c \quad \left[\because \int e^{a-bx}\, dx = \frac{e^{a-bx}}{-b} + c\right]$

j) $\int \frac{e^{\log x^4} - e^{\log x^3}}{e^{\log x^2} - e^{\log x}}\, dx$

Solution

$= \int \left(\frac{x^4 - x^3}{x^2 - x}\right) dx \quad \left[\because e^{\log x} = x\right]$

$= \int \frac{x^3(x-1)}{x(x-1)}\, dx$

$= \int x^2\, dx$

$= \frac{x^{2+1}}{2+1} + c \quad \left[\int x^n\, dx = \frac{x^{n+1}}{n+1} + c\right]$

$= \frac{x^3}{3} + c$

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k) $\int \frac{e^{3x} + e^{x}}{e^{x} + e^{-x}} dx$

Solution

$$\int \left(\frac{e^{x} \cdot e^{2x} + e^{x}}{e^{x} + e^{-x}} \right) dx$$

$$= \int \left[\frac{e^{x}(e^{2x}+1)}{e^{x} + \frac{1}{e^{x}}} \right] dx$$

$$= \int \left[\frac{e^{x}(e^{2x}+1)}{\frac{e^{2x}+1}{e^{x}}} \right] dx$$

$$= \int \left[\frac{e^{x} \cdot e^{x}(e^{2x}+1)}{e^{2x}+1} \right] dx$$

$$= \int e^{2x} dx$$

$$= \frac{e^{2x}}{2} + c \quad \left[\because \int e^{2a} dx = \frac{e^{2a}}{a} + c \right]$$

l) $\int 3^{x+5} dx$

Solution

$$= \frac{3^{x+5}}{\ln 3} + c \quad \left[\because \int a^{x} dx = \frac{a^{x}}{\ln a} + c, a > 0 \right]$$