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Exercise 4(D)

1. The total cost of producing x units of a certain product is described by the function $C(x) = 10000 + 1500x + x^2$ where C is the total cost stated in dollars.

a) Determine the number of units of x that should be produced in order to minimizing the average cost.

b) Find the minimum average cost.

c) What is the total cost of production at this level of output?

Solution

Given,

$$C(x) = 10000 + 1500x + x^2$$

$$\text{Average Cost} = \frac{C(x)}{x}$$

$$= \frac{10000 + 1500x + x^2}{x}$$

$$\therefore AC(x) = \frac{10000}{x} + 1500 + x \quad \text{--- (1)}$$

a)

Differentiating both sides of (1) with respect to x,

$$\frac{d}{dx}[AC(x)] = \frac{d}{dx}\left(10000x^{-1} + 1500 + x\right)$$

$$= -10000x^{-2} + 0 + 1$$

$$\therefore AC'(x) = 1 - 10000x^{-2} \quad \text{--- (ii)}$$

At critical point, $AC'(x) = 0$,

or $1 - \frac{10000}{x^2} = 0$

or, $\frac{10000}{x^2} = 1$

$\therefore x = 100$

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Differentiating both sides of (II) with respect to x,

$$\frac{d}{dx}[AC'(x)] = \frac{d}{dx}(1 - 10000x^{-2})$$

$$= \frac{d}{dx}(1) - \frac{d}{dx}(10000x^{-2})$$

$$= 0 + 20000x^{-3}$$

$$\therefore AC''(x) = \frac{20000}{x^3}$$

At critical point $x = 100$,

$$AC''(100) = \frac{20000}{(100)^3}$$

$$\therefore AC''(100) = 0.02 > 0 \text{ (minimum)}$$

Hence, Average cost is minimum at $x = 100$.

b) When $x = 100$,

$$AC(100)_{min} = \frac{10000}{100} + 1500 + 100$$

$$= 100 + 1600$$

$$\therefore AC(100)_{min} = 1700$$

Therefore, the minimum average cost is 1700.

c) When $x = 100$,

$$C(100) = 10000 + 1500 \times (100) + (100)^2$$

$$= 10000 + 150000 + 10000$$

$$\therefore C(100) = 170000$$

Therefore, the total cost of production at this level of output is 170000 dollars.

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2. The total cost function TC of a firm for output Q is given by

$$TC = \frac{1}{3}Q^3 - 10Q^2 + 300Q$$

a) Find the minimum value of marginal cost function.
b) Find the minimum value of average cost function.
c) Show that marginal cost and average cost functions are equal at the point of minimum average cost.
d) Is it possible to find the maximum & minimum value of total cost function? Justify answer with reason.

Given,

$$TC = \frac{1}{3}Q^3 - 10Q^2 + 300Q$$

Differentiating both sides withrespect to Q,

$$\frac{d}{dQ}[TC(Q)] = \frac{d}{dQ}\left[\frac{1}{3}Q^3 - 10Q^2 + 300Q\right]$$

$$= \frac{1}{3} \times 3Q^2 - 20Q + 300$$

$$\therefore TC'(Q) = Q^2 - 20Q + 300$$

i.e. $MC(Q) = Q^2 - 20Q + 300$ —— (I)

a) Differentiating both sides of (I) with respect to Q,

$$\frac{d}{dQ}[MC(Q)] = \frac{d}{dQ}(Q^2 - 20Q + 300)$$

$$= 2Q - 20 + 0$$

$$\therefore MC'(Q) = 2Q - 20$$ —— (II)

At critical point, $MC'(Q) = 0$

or $2Q = 20$

$\therefore Q = 10$

Differentiating both sides of (ii) with respect to Q,

$$\frac{d}{dQ}[MC'(Q)] = \frac{d}{dQ}(2Q-20)$$

$$= 2-0$$

$$\therefore MC''(Q) = 2$$

When $Q = 10$,

$\therefore MC''(10) = 2 > 0$ (minimum)

Hence, Marginal cost is minimum at $Q = 10$.

<u>At critical point $Q = 10$</u>

$$MC(10)_{min} = (10)^2 - 20\times10 + 300$$

$$= 100 - 200 + 300$$

$\therefore MC(10)_{min} = 200$

Therefore, minimum value of marginal cost function is 200.

b)

$$\text{Average Cost} = \frac{TC(Q)}{Q}$$

$$= \frac{\frac{Q^3}{3} - 10Q^2 + 300Q}{Q}$$

$$= \frac{Q^3}{3Q} - \frac{10Q^2}{Q} + \frac{300Q}{Q}$$

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$= \frac{Q^2}{3} - 10Q + 300$

$\therefore AC(Q) = \frac{Q^2}{3} - 10Q + 300$ —— (i)

Differentiating both sides of (i) with respect to Q,

$\frac{d}{dQ}[AC(Q)] = \frac{d}{dQ}\left[\frac{Q^2}{3} - 10Q + 300\right]$

$= \frac{1}{3} \times 2Q - 10 + 0$

$\therefore AC'(Q) = \frac{2Q}{3} - 10$ —— (ii)

At critical point, $AC'(Q) = 0$

or, $\frac{2Q}{3} - 10 = 0$

or, $\frac{2Q - 30}{3} = 0$

or, $2Q = 30$

$\therefore Q = 15$

~~Hence~~

Differentiating both sides of (ii) with respect to Q,

$\frac{d}{dQ}[AC'(Q)] = \frac{d}{dQ}\left[\frac{2Q}{3} - 10\right]$

$= \frac{2}{3} - 0$

$\therefore AC''(Q) = \frac{2}{3} > 0$ (minimum)

Hence, Average cost is minimum at $Q = 15$.

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At critical point $Q = 15$

$$AC(15)_{min} = \frac{15^2}{3} - 10 \times 15 + 300$$

$$= 75 - 150 + 300$$

$$= \cancel{-375}\ 75$$

$\therefore AC(15)_{min} = 225$

Therefore, minimum value of average cost function is Rs 225.

c)

~~when~~

when $Q = 15$,

$$MC(15) = (15)^2 - 20 \times 15 + 300$$

$$= 225 - 300 + 300$$

$$= 225$$

$$AC(15) = \frac{15^2}{3} - 10 \times 15 + 300$$

$$= 75 - 150 + 300$$

$$= 225$$

$AC(15)_{min} = 225$

Hence, from the above result we can see that $MC(15) = AC(15) = AC(15)_{min}$.

Hence, proved

d)

$$TC = \frac{1}{3}Q^3 - 10Q^2 + 300Q$$

$$TC'(Q) = Q^2 - 20Q + 300$$

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At critical point, $TC'(Q) = 0$

or $Q^2 - 20Q + 300 = 0$

~~or~~ $\therefore Q = 10 + 10\sqrt{2}i, \ 10 - 10\sqrt{2}i$

~~Therefore~~ Hence, we can see that critical point is not real.

3. A given product can be manufactured at a total cost

$C(x) = \frac{x^3}{3} - \frac{7x^2}{2} + 10x - 100$ and demand function is

$P = 235 - \frac{7}{2}x$, where x is output.

a) Find the profit function
b) Find the level of output at which profit is maximum.
c) Find the maximum profit.
d) Find the level of output at which average cost is equal to marginal cost.
e) Find the break even point.

Given,

$$C(x) = \frac{x^3}{3} - \frac{7x^2}{2} + 10x - 100$$

$$P = 235 - \frac{7}{2}x$$

We know,

$$R(x) = P \times Q$$

$$= \left(235 - \frac{7}{2}x\right) \times x$$

$$\therefore R(x) = 235x - \frac{7}{2}x^2$$

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a) Profit Function = Revenue Function − Cost Function

i.e. $\pi(x) = R(x) - C(x)$

$$= 235x - \frac{7}{2}x^2 - \frac{x^3}{3} + \frac{7x^2}{2} - 10x + 100$$

$$\therefore \pi(x) = -\frac{x^3}{3} + 225x + 100 \quad \text{— (i)}$$

b)

Differentiating both sides with respect to x,

$$\frac{d}{dx}[\pi(x)] = \frac{d}{dx}\left[-\frac{x^3}{3} + 225x + 100\right]$$

$$= \frac{d}{dx}\left(-\frac{x^3}{3}\right) + \frac{d}{dx}(225x) + \frac{d}{dx}(100)$$

$$= -\frac{1}{3} \times 3x^2 + 225 + 0$$

$$\therefore \pi'(x) = -x^2 + 225 \quad \text{— (ii)}$$

At critical point, $\pi'(x) = 0$

or $-x^2 + 225 = 0$

or, $x^2 = 225$

$\therefore x = 15$

c) Differentiating both sides of (ii) with respect to x,

$$\frac{d}{dx}[\pi'(x)] = \frac{d}{dx}(-x^2 + 225)$$

$$= -2x + 0$$

$$\therefore \pi''(x) = -2x$$

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<u>At critical point $x=15$,</u>

$\pi''(15) = -2\times15$

$\therefore \pi''(15) = -30 < 0$ (maximum)

Hence, profit is maximum at $x=15$.

c) when $x=15$,

$\pi(15)_{max} = -\frac{(15)^3}{3} + 225\times15 + 100$

$= -1125 + 3375 + 100$

$\therefore \pi(15)_{max} = 2350$

Therefore, the maximum profit is Rs 2350.

d) We know,

Average Cost $= \frac{C(x)}{x}$

$= \frac{\frac{x^3}{3} - \frac{7x^2}{2} + 10x - 100}{x}$

$= \frac{x^3}{3x} - \frac{7x^2}{2x} + \frac{10x}{x} - \frac{100}{x}$

$\therefore AC(x) = \frac{x^2}{3} - \frac{7x}{2} + 10 - \frac{100}{x}$

Also,

Marginal Cost $= \frac{d}{dx}[C(x)]$

$= \frac{d}{dx}\left[\frac{x^3}{3} - \frac{7x^2}{2} + 10x - 100\right]$

$= x^2 - 7x + 10 - 0$

$\therefore MC(x) = x^2 - 7x + 10$

When $AC = MC$,

$$\frac{x^2}{3} - \frac{7x}{2} + 10 - \frac{100}{x} = x^2 - 7x + 10$$

or, $\frac{x^2}{3} - x^2 - \frac{7x}{2} + 7x - \frac{100}{x} = 0$

or, $-\frac{2}{3}x^2 + \frac{\cancel{21+14}x}{2} - \frac{100}{x} = 0$

or, $-\frac{2}{3}x^2 + \frac{7x}{2} - \frac{100}{x} = 0$

~~or,~~ $\cancel{x\left[-\frac{2}{3}x + 7 - \frac{100}{x^2}\right]}$

or, $-\frac{2}{3}x^2 + \frac{7x}{2} = \frac{100}{x}$

or, $x\left[-\frac{2}{3}x + \frac{7}{2}\right] = \frac{100}{x}$

or, $-\frac{2}{3}x + \frac{7}{2} = \frac{100}{x^2}$

or, $-\frac{2}{3}x^3 + \frac{7}{2}x^2 = 100$

or, $-\frac{2}{3}x^3 + \frac{7}{2}x^2 + 0x - 100 = 0$

$\therefore x = \cancel{-3.29}, \cancel{8.34}\ -4.0221, 4.636$

Here, $\overset{-4.0221}{\cancel{-3.29}}$ is rejected.

$\therefore x = \cancel{8.34}\ 4.636$

Therefore, the level of output at which average cost is equal to marginal cost is 4.636.

e) We know,
For Breakeven point,
$\pi(x) = 0$
or, $-\frac{x^3}{3} + 225x + 100 = 0$

or, $-x^3 + 675x + 300 = 0$
$\therefore x = -25.75, 26.20$
Here, $x = -25.75$ is rejected.
Hence, the breakeven point is at 26.20

4. A given product can be manufactured at a total cost $C(x) = \frac{x^2}{100} + 100x + 40$ where x is the number produced. The price of which each unit can be sold is given by $P = Rs. \left(200 - \frac{x}{400}\right)$

a) Determine the total revenue
b) Find the maximum revenue.
c) Determine the production level x at which the profit is maximum. Hence find the maximum profit.
d) What is the price per unit and the total profit at this level of production?
e) Find the break even point.

Given,

$$C(x) = \frac{x^2}{100} + 100x + 40$$

$$P = Rs. \left(200 - \frac{x}{400}\right)$$

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a) Total Revenue = P x Q

$$= \left(200 - \frac{x}{400}\right) \times x$$

$$\therefore TR(x) = 200x - \frac{x^2}{400}$$

b)

Differentiating both sides with respect to x,

$$\frac{d}{dx}[TR(x)] = \frac{d}{dx}\left[200x - \frac{x^2}{400}\right]$$

$$= \frac{d}{dx}(200x) - \frac{d}{dx}\left(\frac{x^2}{400}\right)$$

$$= 200 - \frac{1}{400} \times 2x$$

$$\therefore TR'(x) = 200 - \frac{x}{200} \text{ --- } (1)$$

At critical point $TR'(x) = 0$

$TR'(x) = 0$

or $200 - \frac{x}{200} = 0$

or, $\frac{40000 - x}{200} = 0$

$\therefore x = 40000$

Differentiating both sides of (1) with respect to x, we get,

$$\frac{d}{dx}[TR'(x)] = \frac{d}{dx}\left(200 - \frac{x}{200}\right)$$

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$= 0 - \frac{1}{200}$

$= -\frac{1}{200} < 0$ ~~fm~~

$\therefore TR''(x) = -\frac{1}{200}$

At critical point $x = 40000$

$TR(40000) = -\frac{1}{200} < 0$ (Maximum)

Hence, total revenue is maximum at $x = 40000$.

When $x = 40,000$,

$TR(40000)_{max} = 200 \times 40000 - \frac{(40000)^2}{400}$

$= 8000000 - 4000000$

$= 4000000$

$\therefore TR(40000)_{max} = 4000000$

Therefore ~~total~~ maximum revenue is Rs 4000000.

c) We know,

Total Profit = Total Revenue - Total cost

i.e. $\pi(x) = TR(x) - TC(x)$

$= 200x - \frac{x^2}{400} - \frac{x^2}{100} - 100x - 40$

$\therefore \pi(x) = 100x - \frac{x^2}{80} - 40$

Differentiating both sides with respect to x,

$\frac{d}{dx}[\pi(x)] = \frac{d}{dx}\left[100x - \frac{x^2}{80} - 40\right]$

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$$= 100 - \frac{1}{40}x - 0$$

$$\therefore \pi'(x) = 100 - \frac{1}{40}x \text{ —— } (A)$$

At critical point $\pi'(x) = 0$

or, $100 - \frac{1}{40}x = 0$

or, $\frac{1}{40}x = 100$

or, $x = 100 \times 40$

$\therefore x = 4000$

Differentiating both sides of (A) with respect to x,

$$\frac{d}{dx}[\pi'(x)] = \frac{d}{dx}\left(100 - \frac{1}{40}x\right)$$

$$= 0 - \frac{1}{40}$$

$$\therefore \pi''(x) = -\frac{1}{40}$$

At critical point $x = 40000$

$$\pi''(4000) = -\frac{1}{40} < 0 \text{ (maximum)}$$

Hence, profit is maximum at $x = 4000$

When $x = 4000$,

$$\pi(4000)_{max} = 100 \times 4000 - \frac{(4000)^2}{80} - 40$$

$$= 400000 - 200000 - 40$$

$$= 199960$$

Hence the maximum profit is Rs 199960.

d) When $x = 4000$,

$$P = Rs.\left(200 - \frac{4000}{400}\right)$$

$$= Rs.\left(200 - 10\right)$$

$$= Rs.\,190$$

$$\pi(4000) = 100 \times 4000 - \frac{(4000)^2}{80} - 40$$

$$= 199960$$

Therefore, the price per unit is Rs 190 and the total profit is Rs 199960.

e) For break even point,

$\pi(x) = 0$

or $100x - \frac{x^2}{80} - 40 = 0$

or, $\frac{8000x - x^2 - 3200}{80} = 0$

or, $-x^2 + 8000x - 3200 = 0$

or, $x^2 - 8000x + 3200 = 0$

$\therefore x = 7999.59,\ 0.40$

Therefore the break even point is either 7999.59 or 0.40

5. Denim Jeans manufacturing company presents the price demand equation $P = 108 - 5x$ for a Jeans, where P is the unit price (in Rs.) and x is the quantity demanded (in units). The financial department provides the cost function. $C(x) = -12x + x^2$, where $C(x)$ is the cost in rupees for manufacturing and selling the jeans (x) in units.

a) Find the marginal revenue function at $x = 8$.

b) Find the profit function as a function of x.

c) Find the break even point.

d) Find the best production level of jeans to produce maximum profit. What is the company's maximum profit?

e) What is the price per jeans that produces maximum profit?

f) Find the maximum revenue.

Solution

Given,

$P = 108 - 5x$

$C(x) = -12x + x^2$

$R(x) = P \times Q = (108 - 5x) \times x = 108x - 5x^2$

a) We know,

Marginal Revenue (MR) $= \dfrac{d}{dx}[C(x)]$

$= \dfrac{d}{dx}(-12x + x^2)$

$\therefore MR(x) = -12 + 2x$

When $x = 8$,

$MR(8) = -12 + 2 \times 8$

$= -12 + 16$

$\therefore MR(8) = 4$

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a) We know,
~~Rt~~ Marginal Revenue (MR) $= \frac{d}{dx}[R(x)]$

$$= \frac{d}{dx}(108x - 5x^2)$$

$$\therefore MR(x) = 108 - 10x$$

When $x = 8$,
$MR(8) = 108 - 10 \times 8$
$= 108 - 80$
$\therefore MR(8) = 28$
Therefore the marginal revenue is 28 at $x = 8$.

b) Also we know,
Profit Function = Revenue Function - Cost Function
i.e. $\pi(x) = R(x) - C(x)$
$= 108x - 5x^2 + 12x - x^2$
$= 108x - 6x^2 + 12x$
$\therefore \pi(x) = -6x^2 + 120x$

c) For break-even point, we know,
$\pi(x) = 0$
or, $-6x^2 + 120x = 0$
or, $6x^2 - 120x = 0$
or, $6x(x - 20) = 0$

Either $6x = 0$ $\quad \therefore x = 0$
Or, $x - 20 = 0$
$\therefore x = 20$

$\therefore x = 0, 20$
~~Here, 20 is rejected.~~
Therefore, the break even point is either 0 or 20.

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d) $\pi(x) = -6x^2 + 120x$

Differentiating both sides with respect to x,

$$\frac{d}{dx}[\pi(x)] = \frac{d}{dx}(-6x^2 + 120x)$$

$$\therefore \pi'(x) = -12x + 120 \quad \text{——— (1)}$$

At critical point, $\pi'(x) = 0$

or, $-12x + 120 = 0$

or, $12x = 120$

$\therefore x = 10$

Differentiating both sides of (1) with respect to x,

$$\frac{d}{dx}[\pi'(x)] = \frac{d}{dx}(-12x + 120)$$

$$= -12 + 0$$

$$\therefore \pi''(x) = -12$$

At critical point $x = 10$

$\pi''(10) = -12 < 0$ (Maximum)

Therefore profit is maximum at $x = 10$.

When $x = 10$,

$\pi(10)\,max = -6 \times (10)^2 + 120 \times 10$

$= -600 + 1200$

$\therefore \pi(10)\,max = 600$

Therefore the maximum profit is Rs 600.

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e) ~~Price per jeans that produces maximum profit~~ $= \frac{x(x)}{x}$

$= \frac{x(10)}{10}$

When $x = 10$,

$P = 108 - 5 \times 10$

$= 108 - 50$

$\therefore P = 58$

Therefore price per jeans that produces maximum profit is Rs. 58

f) we have,

$R(x) = 108x - 5x^2$

Differentiating both sides with respect to x,

$$\frac{d}{dx}[R(x)] = \frac{d}{dx}(108x - 5x^2)$$

$$= \frac{d}{dx}(108x) - \frac{d}{dx}(5x^2)$$

$$\therefore R'(x) = 108 - 10x \quad \text{——— (A)}$$

At critical point, $R'(x) = 0$,

i.e. $108 - 10x = 0$

or, $10x = 108$

$\therefore x = 10.8$

Differentiating both sides of (A) with respect to x,

$$\frac{d}{dx}[R'(x)] = \frac{d}{dx}(108 - 10x)$$

$$= 0 - 10$$

$$\therefore R''(x) = -10$$

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At the critical point $x = 10.8$

$\therefore R''(10.8) = -10 < 0$ (Maximum)

Hence, there is maximum revenue at $x = 10.8$

when $x = 10.8$,

$R(10.8)max = 108 \times 10.8 - 5 \times (10.8)^2$

$= 1166.4 - 583.2$

$\therefore R(108)\ max = 583.2$

Therefore the maximum revenue is Rs 583.2

6. A firm has the following total cost and demand function for a company are

$TC = \frac{1}{3}Q^3 - 15Q^2 + 480Q + 750$

$P = 536 - 2Q$

a) Find the revenue function and profit function.
b) Determine the level of output Q for which profit is maximized and minimized.
c) Also, find the marginal profit and find the maximum value of marginal profit.
~~d) Determine the intervals at which the curve for the profit is concave upward and concave downward.~~
e) Find the break even points.

Given,

$TC = \frac{1}{3}Q^3 - 15Q^2 + 480Q + 750$

$P = 536 - 2Q$

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a) We know,

Revenue Function $= P \times Q$

$= (536 - 2Q) \times Q$

$\therefore R(Q) = 536Q - 2Q^2$

Also,

Profit Function = Revenue Function − Cost Function

i.e. $\pi(Q) = R(Q) - C(Q)$

$= 536Q - 2Q^2 - \frac{1}{3}Q^3 + 15Q^2 - 480Q - 750$

$\therefore \pi(Q) = 56Q + 13Q^2 - \frac{1}{3}Q^3 - 750$

b) We have,

$\pi(Q) = -\frac{1}{3}Q^3 + 13Q^2 + 56Q - 750$

Differentiating both sides with respect to Q,

$\frac{d}{dx}[\pi(Q)] = \frac{d}{dQ}\left(-\frac{1}{3}Q^3 + 13Q^2 + 56Q - 750\right)$

$\therefore \pi'(Q) = -Q^2 + 26Q + 56$ —— (1)

At the critical point, $\pi'(Q) = 0$

or, $-Q^2 + 26Q + 56 = 0$

or, $Q^2 - 26Q - 56 = 0$

or, $Q^2 - 28Q + 2Q - 56 = 0$

or, $Q(Q - 28) + 2(Q - 28) = 0$

or, $(Q - 28)(Q + 2) = 0$

Either,

$Q - 28 = 0$

$\therefore Q = 28$

or,

$Q + 2 = 0$

$\therefore Q = -2$ (Rejected)

$\therefore Q = 28$

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Differentiating both sides of ① with respect to Q,

$$\frac{d}{dQ}[\pi'(Q)] = \frac{d}{dQ}(-Q^2+26Q+56)$$

$\therefore \pi''(Q) = -2Q+26$

At critical point Q = 28

$\pi''(28) = -2\times28+26$
$= -56+26$
$\therefore \pi''(28) = -30 < 0$ (Maximum)

Hence profit is maximum at ~~x~~ = Q = 28

c) We know,

Marginal Profit = $\pi'(Q)$
$\therefore M[P(Q)] = -Q^2+26Q+56$

Also,
$M'[P(Q)] = \pi''(Q)$
$\therefore M'[P(Q)] = -2Q+26$ —— Ⓐ

At the critical point $M'[P(Q)] = 0$

$-2Q+26 = 0$
or, $2Q = 26$
$\therefore Q = 13$

Differentiating both sides of Ⓐ with respect to Q,

$$\frac{d}{dQ}[M'(P(Q))] = \frac{d}{dQ}(-2Q+26)$$

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$\therefore M''[P(Q)] = -2$

At the critical point $Q=13$

$M''[P(13)] = -2 < 0$ (Maximum)

Hence profit is maximum at $Q=13$.

When $Q=13$,

$M[P(13)] = \cancel{(-13)^2} - (13)^2 + 26 \times 13 + 56$

$= -169 + 338 + 56$

$\therefore M[P(13)] = 225$

Therefore, the maximum profit is Rs. 225

e) For break even points,

$\pi(Q) = 0$

~~or, $56Q + 13Q^2 =$~~

or, $-\frac{1}{3}Q^3 + 13Q^2 + 56Q - 750 = 0$

$\therefore Q = -8.83, 41.73$

Here, $Q=-8.83$ is rejected.

Therefore, there is a break even point at $Q = 41.73$

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7. A Television manufacturing company produces x sets per week at a total cost of Rs. $(x^2+78x+2500)$ and the demand function for its product is $\frac{600-P}{8}$ where the price is P per set.

a) Find the profit function.
b) Find the output x for which profit is maximum
c) Find the maximum profit
d) Find the corresponding price of each TV set.
e) Find the break even points.

Given,

$C(x) = x^2 + 78x + 2500$

$x\ \cancel{Q} = \frac{600-P}{8}$

Or, $8\cancel{Q} = 600 - P$

$\therefore$ ~~or~~ $P = 600 - \cancel{8Q}\ 8x$

We know,

$R(x) = P \times Q$

$= (600 - 8\cancel{Q}) \times \cancel{Q}\ x$

~~$\therefore R(Q) = 600Q - 8Q^2$~~

$\therefore R(x) = 600x - 8x^2$

a) We have,

Profit Function = Revenue Function − Cost Function

i.e. $\pi(x) = R(x) - C(x)$

$= 600x - 8x^2 - x^2 - 78x - 2500$

$\therefore \pi(x) = -9x^2 + 522x - 2500$

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b)

$\pi = \pi(x) = -9x^2 + 522x - 2500$

Differentiating both sides with respect to x,

$\frac{d}{dx}[\pi(x)] = \frac{d}{dx}(-9x^2 + 522x - 2500)$

$= -18x + 522 - 0$

$\therefore \pi'(x) = -18x + 522$ —— ①

At critical point $\pi'(x) = 0$,

or $-18x + 522 = 0$

or $18x = 522$

$\therefore x = 29$

Differentiating both sides of ① with respect to x,

$\frac{d}{dx}[\pi'(x)] = \frac{d}{dx}(-18x + 522)$

$= -18 + 0$

$\therefore \pi''(x) = -18$

At critical point $x = 29$

$\pi''(29) = -18 < 0$ (maximum)

Hence, profit is maximum at $x = 29$.

c) when $x = 29$

$\pi(29)\,max = -9 \times (29)^2 + 522 \times 29 - 2500$

$= -7569 + 15138 - 2500$

$\therefore \pi(29)\,max = 5069$

Therefore the maximum profit is Rs 5069.

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d) When $x = 29$,

$P = 600 - 8 \times 29$

$= 600 - 232$

$\therefore P = 368$

Therefore the corresponding price of each TV set is Rs 368.

e) For break even points,

$\pi(x) = 0$

or $-9x^2 + 522x - 2500 = 0$

or, $9x^2 - 522x + 2500 = 0$

~~or~~ $\therefore x = 52.73, 5.26$

Therefore the break even point is either 52.73 or 5.26

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Exercise 4(D)

[Book Qsn No: 11]

1. If the demand function of monopolistic is given by $P = 100 - x$ and its cost function is given by $C = 100 + 5x + 4x^2$.

a) Find the value of x at which the revenue is maximum.
b) Find the maximum revenue.
c) Find the value of x at which the profit is maximum.
d) Find the price (P) at which profit is maximum.
e) Find the maximum profit.
f) Find the price where the revenue is maximum.
g) Find the price where the profit is maximum.
h) Find the break even point.

Solution

Given, Demand Function is : $P = 100 - x$

Cost Function is : $C = 100 + 5x + 4x^2$.

We know,

a) ~~Total~~ Revenue $= P \times Q$

$= (100 - x) \times x$

$\therefore R(x) = 100x - x^2$ ——— (1)

Differentiating both sides of (1) with respect to x.

$$\frac{d}{dx}[R(x)] = \frac{d}{dx}(100x - x^2)$$

$$\therefore R'(x) = 100 - 2x \quad \text{——— (II)}$$

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At the critical point $R'(x) = 0$

or, $100 - 2x = 0$

or, $2x = 100$

$\therefore x = 50$

Differentiating both sides of (11) with respect to x,

$$\frac{d}{dx}[R'(x)] = \frac{d}{dx}(100 - 2x)$$

$$= \frac{d}{dx}(100) - \frac{d}{dx}(2x)$$

$$= 0 - 2$$

$$\therefore R''(x) = -2$$

At the critical point, $x = 50$

when $x = 50$, $R''(50) = -2 < 0$ (maximum)

Therefore, the revenue (R) is maximum at $x = 50$.

b) We know,

~~The revenue is maximum at $x = 50$,~~

when $x = 50$,

$\cancel{P = 100 - x}$ $\quad R(50)_{max} = 100 \times (50) - (50)^2$

$\cancel{= 100 - 50}$ $\quad = 5000 - 2500$

$\cancel{= 50}$ $\quad = 2500$

Therefore, the maximum revenue is Rs2500.

c)

Profit Function = Revenue Function - Cost Function

$= 100x - x^2 - 100 - 5x - 4x^2$

$\therefore \pi(x) = 95x - 5x^2 - 100$ —— (1)

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Differentiating both sides of ① with respect to x,

$$\frac{d}{dx}[\pi(x)] = \frac{d}{dx}(95x - 5x^2 - 100)$$

$$= 95 - 10x - 0$$

$$\therefore \pi'(x) = 95 - 10x \quad \text{—— (ii)}$$

At critical point $\pi'(x) = 0$,

$95 - 10x = 0$

or, $10x = 95$

$\therefore x = 9.5$

Differentiating both sides of (ii) with respect to x,

$$\frac{d}{dx}[\pi'(x)] = \frac{d}{dx}(95 - 10x)$$

$$= 0 - 10$$

$$\therefore \pi''(x) = -10$$

e) At the critical point, $x = 9.5$

When $x = 9.5$, $\pi''(9.5) = -10 < 0$ (maximum)

Therefore, profit is maximum at $x = 9.5$

When $x = 9.5$, $\pi(9.5)_{max} = 95 \times (9.5) - 5 \times (9.5)^2 - 100$

$= 902.5 - 451.25 - 100$

$= 351.25$

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d) when $x = 9.5$, Price $= \cancel{95}\ 100 - 9.5$
$= 90.5$

Therefore, profit is maximum when the price is Rs 90.5

f) ~~When~~ We know that the revenue is maximum at $x = 50$. So,

$P = 100 - x$
$= 100 - 50$
$= 50$

Therefore, the revenue is maximum when price is Rs. 50

g) we know that the profit is maximum at $x = 9.5$. So,

$P = 100 - 9.5$
$=$ Rs 90.5 [Repeated same qsn: 'd']

h) we know,

At Break Even Point,

Revenue Function = Cost Function

i.e. $R(x) = C(x)$

or, $100x - x^2 = 100 + 5x + 4x^2$

or, $100x - 5x - x^2 - 4x^2 - 100 = 0$

or, $-5x^2 + 95x - 100 = 0$

or, $5x^2 - 95x + 100 = 0$

or, $x = \dfrac{-95 \pm \sqrt{(95)^2 - 4 \times 5 \times 100}}{2 \times 5}$

$\therefore x = 17.88, 1.11$

Therefore, the breakeven points are either 17.88 or 1.11

[Book Question Number: 8]

2. The demand function for a commodity is $p = 15 - q$ and the cost function is $TC = q^2 - 3q - 20$, where q is output, find:

a) marginal profit when 6 units are produced.
b) break even point
c) The output at which profit is maximum.
d) The output at which revenue is maximum.
e) The maximum revenue & profit.

Solution
Given,

$p = 15 - q$

$TC = q^2 - 3q - 20$

we know,

$TR = p \times q$

$= (15 - q) \times q$

$\therefore TR(q) = 15q - q^2$ —— (i)

Differentiating (i) with respect to q, we get

$$\frac{d}{dq}[TR(q)] = \frac{d}{dq}(15q - q^2)$$

$$= \frac{d}{dq}(15q) - \frac{d}{dq}(q^2)$$

$$\therefore TR'(q) = 15 - 2q \quad \text{—— (ii)}$$

At critical point, $TR'(q) = 0$

$15 - 2q = 0$

or, $2q = 15$

$\therefore q = 7.5$

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Differentiating both sides of (II) with respect to q, we get

$$\frac{d}{dq}[TR'(q)] = \frac{d}{dq}(15-2q)$$

$$= \frac{d}{dq}(15) - \frac{d}{dq}(2q)$$

$$= 0 - 2$$

$$\therefore TR''(q) = -2 < 0 \text{ (maximum)}$$

Therefore revenue is maximum at $q = 7.5$, since $TR''(7.5) = -2 < 0$.

Now,

Profit Function = Revenue Function − Cost Function

i.e. $\pi(q) = TR(q) - TC(q)$

$$= 15q - q^2 - q^2 + 3q + 20$$

$$\therefore \pi(q) = -2q^2 + 18q + 20 \quad \text{—— (III)}$$

Differentiating both sides of (III) with respect to q,

$$\frac{d}{dq}[\pi(q)] = \frac{d}{dq}(-2q^2 + 18q + 20)$$

$$= \frac{d}{dq}(-2q^2) + \frac{d}{dq}(18q) + \frac{d}{dq}(20)$$

$$= -4q + 18 + 0$$

$$\therefore \pi'(q) = -4q + 18 \quad \text{—— (IV)}$$

At critical point $\pi'(q) = 0$

$-4q + 18 = 0$

or, $4q = 18$

$\therefore q = 4.5$

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Differentiating both sides of (iv) with respect to q,

$$\frac{d}{dq}[\pi'(q)] = \frac{d}{dq}(-4q+18)$$

$$= \frac{d}{dq}(-4q) + \frac{d}{dq}(18)$$

$$= -4+0$$

$\therefore \pi''(q) = -4 < 0$ (maximum)

Therefore, profit is maximum at $q = 4.5$ since $\pi''(4.5) = -4 < 0$.

Now,

a) When $q = 6$,

Marginal Profit $= -4q+18$

$= -4\times6+18$

$= -24+18$

$= -6$

Hence, the marginal profit when 6 units are produced is -6.

b) For break-even points,

$TR = TC$

or, $15q - q^2 = q^2 - 3q - 20$

or, $-q^2 - q^2 + 15q + 3q + 20 = 0$

or, $-2q^2 + 18q + 20 = 0$

or, $2q^2 - 18q - 20 = 0$

$\therefore q = 10, -1$

Here, -1 is rejected.

$\therefore q = 10$ units

c) Output at which profit is maximum = 4.5

d) Output at which revenue is maximum = 7.5

e)

when $q = 7.5$,

$TR(7.5)_{max} = (15 \times 7.5) - (7.5)^2$

$= 112.5 - 56.25$

$= 56.25$

when $q = 4.5$

$\pi(4.5)_{max} = -2 \times (4.5)^2 + 18 \times 4.5 + 20$

$= -40.5 + 81 + 20$

$= 60.5$

Hence the maximum revenue is 56.25 and maximum profit is 60.5

[Book question Number: 9]

3. A factory produces Q tons of status metal per month at a total cost $C(Q) = \frac{2}{3}Q^3 - 20Q^2 + 11Q + 10$ and the demand function equation
$P = \frac{1}{3}Q^2 - 10Q + 75$

a) Find the revenue function
b) Find the profit function
c) Find the level of output Q. for which revenue is maximum.
~~d) Determine the interval at which revenue is increasing or decreasing.~~

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Solution

Given,

$$C(Q) = \frac{2}{3}Q^3 - 20Q^2 + 11Q + 10$$

$$P = \frac{1}{3}Q^2 - 10Q + 75$$

a) we know,

$$\text{Revenue Function} = P \times Q$$

$$= \left[\frac{1}{3}Q^2 - 10Q + 75\right] \times Q$$

$$\therefore R(Q) = \frac{1}{3}Q^3 - 10Q^2 + 75Q$$

b) Also,

Profit Function = Revenue Function - Cost Function

$$\cancel{\text{i.e. } \pi(Q) = R(Q) - C(Q)}$$

$$\cancel{= \frac{Q^3}{3} - 10Q^2 + 75Q - \frac{1}{3}Q^2 + 10Q - 75}$$

$$\cancel{\therefore \pi(Q) = \frac{Q^3}{3} - \frac{31}{3}Q^2 + 85Q - 75}$$

$$\text{i.e. } \pi(Q) = R(Q) - C(Q)$$

$$= \frac{1}{3}Q^3 - 10Q^2 + 75Q - \frac{2}{3}Q^3 + 20Q^2 - 11Q - 10$$

$$\therefore \pi(Q) = -\frac{1}{3}Q^3 + 10Q^2 + 64Q - 10$$

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c)

$$R(Q) = \frac{1}{3}Q^3 - 10Q^2 + 75Q$$

Differentiating both sides with respect to Q,

$$\frac{d}{dQ}(R(Q)) = \frac{d}{dQ}\left[\frac{1}{3}Q^3 - 10Q^2 + 75Q\right]$$

$$= \frac{d}{dQ}\left(\frac{1}{3}Q^3\right) - \frac{d}{dQ}(10Q^2) + \frac{d}{dQ}(75Q)$$

$$= \frac{1}{3} \times 3Q^2 - 20Q + 75$$

$$\therefore R'(Q) = Q^2 - 20Q + 75 \quad \text{--- (1)}$$

At critical point, $R'(Q) = 0$,

$Q^2 - 20Q + 75 = 0$

or $Q^2 - 15Q - 5Q + 75 = 0$

or, $Q(Q-15) - 5(Q-15) = 0$

or $(Q-15)(Q-5) = 0$

$\therefore Q = 15, 5$

Differentiating (1) with respect to Q,

$$\frac{d}{dQ}[R'(Q)] = \frac{d}{dQ}[Q^2 - 20Q + 75]$$

$$= \frac{d}{dQ}(Q^2) - \frac{d}{dQ}(20Q) + \frac{d}{dQ}(75)$$

$$= 2Q - 20 + 0$$

$$\therefore R''(Q) = 2Q - 20$$

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When $Q = 15$,

$R''(15) = 2 \times 15 - 20$
$= 30 - 20$
$= 10 > 0$ (Minimum)

Revenue is minimum at $Q = 15$

When $Q = 5$,
$R''(5) = 2 \times 5 - 20$
$= 10 - 20$
$= -10 < 0$ (Maximum)

Hence, Revenue is maximum at $Q = 5$. ~~since~~

[Book Question Number: 10]

4. The demand respectively and cost functions of a commodity is $P = 20 - Q$ and $C(Q) = Q^2 + 8Q + 2$ respectively.
a) Find the profit function
b) Find the break-even point.
c) Find the maximum revenue.
d) Find the maximum ~~fat~~ profit.

Solution
Given,
$P = 20 - Q$
$C(Q) = Q^2 + 8Q + 2$
Then,
Total Revenue $= P \times Q$
$= (20 - Q) \times Q$
$\therefore TR(Q) = 20Q - Q^2$

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We know,

a) Profit Function = Total Revenue − Total cost

i.e. $\pi(Q) = TR(Q) - TC(Q)$

$= 20Q - Q^2 - Q^2 - 8Q - 2$

$\therefore \pi(Q) = -2Q^2 + 12Q - 2$

b) For break-even point,

$TR = TC$

or, $20Q - Q^2 = Q^2 + 8Q + 2$

or, $20Q - Q^2 - Q^2 - 8Q - 2 = 0$

or, $-2Q^2 + 12Q - 2 = 0$

~~or, $-2Q^2 + 12Q = 2$~~

~~or, $2(Q^2 + 6Q) =$~~

or, $-Q^2 + 6Q - 1 = 0$

$\therefore Q = 5.82, 0.17$

Therefore break even point is either 5.82 or 0.17.

c) $TR(Q) = 20Q - Q^2$

Differentiating both sides with respect to Q,

$\frac{d}{dQ}[TR(Q)] = \frac{d}{dQ}(20Q - Q^2)$

$= \frac{d}{dQ}(20Q) - \frac{d}{dQ}(Q^2)$

$\therefore TR'(Q) = 20 - 2Q$ —— (1)

At critical point, $TR'(Q) = 0$

$20 - 2Q = 0$

or, $2Q = 20$

$\therefore Q = 10$

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Differentiating ① with respect to Q,

$$\frac{d}{dQ}[TR'(Q)] = \frac{d}{dQ}(20-2Q)$$

$$= \frac{d}{dQ}(20) - \frac{d}{dQ}(2Q)$$

$$= 0-2$$

$$\therefore TR''(Q) = -2 < 0 \text{ (maximum)}$$

When $Q=10$, At the critical point

$$\frac{d}{dQ} \quad TR''(10) = -2 < 0 \text{ (maximum)}$$

Therefore, revenue is maximum at $Q=10$.

When $Q=10$,

$$TR(10)_{max} = 20 \times 10 - (10)^2$$

$$= 200 - 100$$

$$\therefore TR(10)_{max} = 100$$

d)

$$\pi(Q) = -2Q^2 + 12Q - 2$$

Differentiating both sides with respect to Q,

$$\frac{d}{dQ}[\pi(Q)] = \frac{d}{dQ}(-2Q^2+12Q-2)$$

$$= \frac{d}{dQ}(-2Q^2) + \frac{d}{dQ}(12Q) - \frac{d}{dQ}(2)$$

$$= -4Q + 12 - 0$$

$$\therefore \pi'(Q) = -4Q + 12 \quad \text{——— (11)}$$

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At critical point, $\pi'(Q) = 0$

$-4Q + 12 = 0$
or $4Q = 12$
$\therefore Q = 3$

Differentiating both sides of (11) with respect to Q, we get,

$$\frac{d}{dQ}[\pi'(Q)] = \frac{d}{dQ}(-4Q + 12)$$
$$= \frac{d}{dQ}(-4Q) + \frac{d}{dQ}(12)$$
$$= -4 + 0$$
$$\therefore \pi''(Q) = -4$$

At the critical point, when $Q = 3$,

$\pi''(3) = -4 < 0$ (maximum)

Therefore, profit is maximum at $Q = 3$.

When $Q = 3$.

~~$\pi(3) =$~~

$\pi(3)_{max} = -2 \times (3)^2 + (12 \times 3) - 2$
$= -18 + 36 - 2$
$\therefore \pi(3)_{max} = 16$