~~Exercise 4(B)~~ Exercise 4(B)

~~Example-1~~

1. The cost function $C = 5Q + \frac{1}{6}Q^2$. Find the average and marginal cost when the quantity production $Q = 4$.

Solution

Given,

$$C = 5Q + \frac{1}{6}Q^2$$

i.e. $$C(Q) = 5Q + \frac{1}{6}Q^2$$

We know,

$$AC(Q) = \frac{C(Q)}{Q}$$

$$= \frac{5Q + \frac{1}{6}Q^2}{Q}$$

$$= \frac{5Q}{Q} + \frac{Q^2}{6Q}$$

$$\therefore AC(Q) = 5 + \frac{Q}{6}$$

When $Q = 4$,

$$AC(4) = 5 + \frac{4}{6}$$

$$= \frac{\cancel{8}34}{6}$$

$$= \frac{17}{3}$$

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Also,

$$MC(x) = \frac{d}{dQ}[C(x)]$$

$$= \frac{d}{dQ}\left[5Q + \frac{1}{6}Q^2\right]$$

$$= \frac{d}{dQ}(5Q) + \frac{d}{dQ}\left(\frac{1}{6}Q^2\right)$$

$$= 5 + \frac{1}{6} \times 2Q$$

$$= 5 + \frac{Q}{3}$$

when $Q = 4$,

$$MC(4) = 5 + \frac{4}{3}$$

$$= \frac{15+4}{3}$$

$$= \frac{19}{3}$$

Therefore, the average cost and marginal cost when the quantity production $Q = 4$ is $\frac{17}{3}$ and $\frac{19}{3}$ respectively.

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2. Find an expression for the MC when ~~AC =~~ $AC = 2Q + 5 + \frac{30}{Q}$, Calculate the value of marginal cost when $Q = 50$.

Solution

Given,

$$AC = 2Q + 5 + \frac{30}{Q}$$

i.e. $AC(Q) = 2Q + 5 + \frac{30}{Q}$

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We Know,

$$AC(Q) = \frac{TC(Q)}{Q}$$

or, $TC(Q) = AC(Q) \times Q$

or, $TC(Q) = (2Q + 5 + \frac{30}{Q}) \times Q$

$\therefore$ ~~or,~~ $TC(Q) = 2Q^2 + 5Q + 30$

Now,

$$MC(Q) = \frac{d}{dQ}[TC(Q)]$$

$$= \frac{d}{dQ}(2Q^2 + 5Q + 30)$$

$\therefore MC(Q) = 4Q + 5$

When $Q = 50$,

$MC(50) = 4 \times 50 + 5$

$= 205$

$\therefore$ The value of marginal cost is 205 when $Q = 50$.

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3. The demand function for a good is $P = 125 - 3Q$

a) Find expressions for TR, MR and AR.

b) Evaluate TR, MR and AR at $Q = 10$ and $Q = 25$

c) Calculate the value of Q for which $MR = 0$, $AR = 0$, $TR = 0$

Solution

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Given demand function: $P = 125 - 3Q$

a) $TR = P \times Q$

$= (125 - 3Q) \times Q$

$\therefore TR(Q) = 125Q - 3Q^2$

Also,

$$MR = \frac{d}{dQ} \cancel{[MR(Q)]} [TR(Q)]$$

$$= \frac{d}{dQ}(125Q - 3Q^2)$$

$\therefore MR(Q) = 125 - 6Q$

Also,

$$AR = \frac{TR(Q)}{Q}$$

$$= \frac{125Q - 3Q^2}{Q}$$

$$= 125 - 3Q$$

b)

When $Q = 10$,

$TR(10) = (125 \times 10) - (3 \times 10^2) = 1250 - 300 = 950$

$MR(10) = 125 - 6 \times 10 = 125 - 60 = 65$

$AR(10) = 125 - 3 \times 10 = 125 - 30 = 95$

When $Q = 25$,

$TR(25) = (125 \times 25) - (3 \times 25^2) = 3125 - \cancel{5625}\ 1875$

$= \cancel{-2500}\ 1250$

$MR(25) = 125 - 6 \times 25 = 125 - 150 = -25$

$AR(25) = 125 - 3 \times 25 = 125 - 75 = 50$

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c)

When,

$MR = 0$

or, $\frac{d}{dQ}[TR(Q)] = 0$

or, $\frac{d}{dQ}(125Q - 3Q^2) = 0$

or, $125 - 6Q = 0$

or, $6Q = 125$

$\therefore Q = \frac{125}{6}$

When $AR = 0$

or, $\frac{TR(Q)}{Q} = 0$

or, $\frac{125Q - 3Q^2}{Q} = 0$

or, $125 - 3Q = 0$

or, $3Q = 125$

$\therefore Q = \frac{125}{3}$

When $TR = 0$

$125Q - 3Q^2 = 0$

or, $3Q^2 - 125Q = 0$

or, $Q(3Q - 125) = 0$

Either,

$Q = 0$

Or,

$3Q - 125 = 0$

or, $3Q = 125$

$\therefore Q = \frac{125}{3}$

4. The revenue function of a commodity is $R(x) = x^3 - 3x^2 - 9$; find the average and marginal revenue when the level of output is 5.

Solution

Given,

$$R(x) = x^3 - 3x^2 - 9$$

$$AR(x) = \frac{R(x)}{x}$$

$$= \frac{x^3 - 3x^2 - 9}{x}$$

when the output is 5,

$$AR(5) = \frac{5^3 - 3\times(5)^2 - 9}{5}$$

$$= \frac{41}{5}$$

Again,

$$MR(x) = \frac{d}{dx}[R(x)]$$

$$= \frac{d}{dx}(x^3 - 3x^2 - 9)$$

$$= 3x^2 - 6x$$

when the output is 5,

$$MR(5) = 3\times(5)^2 - 6\times5$$

$$= 75 - 30$$

$$= 45$$

Therefore, the average and marginal revenue when the level of output is 5 are $\frac{41}{5}$ and 45 respectively.

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5. A firm forms the demand function $Q = 150 - P$. Its total cost function is $C = \frac{Q^2}{2}$. Find the profit function and the marginal ~~function~~ profit function and the average profit and their values when the level of production is 30.

Solution

Given demand function: $Q = 150 - P$ or, $P = 150 - Q$

Total cost function: $C = \frac{Q^2}{2}$

$$\text{Total Revenue (TR)} = P \times Q$$
$$= (150 - Q) \times Q$$
$$= 150Q - Q^2$$

$$\text{Total Profit} = TR - TC$$
$$= 150Q - Q^2 - \frac{Q^2}{2}$$
$$= \frac{300Q - 2Q^2 - Q^2}{2}$$
$$\therefore \pi(Q) = \frac{300Q - 3Q^2}{2}$$

$$\text{Marginal Profit} = \frac{d}{dQ}[\pi(Q)]$$

i.e. $$M[\pi(Q)] = \frac{d}{dQ}\left[\frac{300Q - 3Q^2}{2}\right]$$
$$= \frac{1}{2} \times \frac{d}{dQ}(300Q - 3Q^2)$$
$$= \frac{1}{2} \times (300 - 6Q)$$
$$\therefore M[\pi(Q)] = 150 - 3Q$$

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Average Profit $= \dfrac{\pi(Q)}{Q}$

$= \dfrac{300Q - 3Q^2}{2Q}$

When the level of production is 30,

$\pi(30) = \dfrac{(300 \times 30) - 3 \times (30)^2}{2}$

$= \dfrac{9000 - 2700}{2}$

$= 3150$

$M[\pi(30)] = 150 - 3 \times 30$

$= 150 - 90$

$= 60$

$A[\pi(30)] = \dfrac{(300 \times 30) - 3 \times (30)^2}{2 \times 30}$

$= \dfrac{9000 - 2700}{60}$

$= 105$

Therefore, the value of profit, marginal profit and average profit when the level of profit is 30 is 3150, 60 and 105 respectively.

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6. The demand function for a good is $P = \frac{1}{3}Q^2 - 10Q + 75$. Find the marginal revenue function and its value at $Q = 3$.

Solution

Given demand function: $P = \frac{1}{3}Q^2 - 10Q + 75$.

$$TR(Q) = P \times Q$$
$$= \left(\frac{1}{3}Q^2 - 10Q + 75\right) \times Q$$
$$= \frac{1}{3}Q^3 - 10Q^2 + 75Q$$

Now,

$$MR(Q) = \frac{d}{dQ}[TR(Q)]$$
$$= \frac{d}{dQ}\left[\frac{1}{3}Q^3 - 10Q^2 + 75Q\right]$$
$$= \frac{1}{3} \times 3Q^2 - 20Q + 75$$
$$= Q^2 - 20Q + 75$$

when $Q = 3$,

$$MR(3) = (3)^2 - 20 \times 3 + 75$$
$$= 9 - 60 + 75$$
$$= 24$$

$\therefore$ The marginal revenue when $Q = 3$ is 24.

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7. If the total cost is $C = Q^2 + 8Q + 2$, find the average cost, marginal cost when $Q = 5$ and $Q = 10$.

Solution

Given,

~~C(Q) =~~

$C(Q) = Q^2 + 8Q + 2$

Average cost i.e. $AC(Q) = \frac{C(Q)}{Q}$

$= \frac{Q^2 + 8Q + 2}{Q}$

Marginal cost i.e. $MC(Q) = \frac{d}{dQ}[C(Q)]$

$= \frac{d}{dQ}(Q^2 + 8Q + 2)$

$= 2Q + 8$

When $Q = 5$,

$AC(5) = \frac{5^2 + 8 \times 5 + 2}{5} = \frac{67}{5}$

$MC(5) = 2 \times 5 + 8 = 18$

When $Q = 10$,

$AC(10) = \frac{10^2 + 8 \times 10 + 2}{10} = \frac{91}{5}$

$MC(10) = 2 \times 10 + 8 = 28$

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8. The total cost is $C = 200x - 30x^2 + \frac{2}{3}x^{-3}$. Find the average cost and marginal cost.

Solution

Given,

$$C(x) = 200x - 30x^2 + \frac{2}{3}x^{-3}$$

$$AC(x) = \frac{C(x)}{x}$$

$$= \frac{200x - 30x^2 + \frac{2}{3}x^{-3}}{x}$$

$$\therefore AC(x) = 200 - 30x + \frac{2}{3}x^{-4}$$

$$MC(x) = \frac{d}{dx}[C(x)]$$

$$= \frac{d}{dx}\left(200x - 30x^2 + \frac{2}{3}x^{-3}\right)$$

$$= 200 - 60x + \frac{2}{3} \times (-3)\, x^{-4}$$

$$= 200 - 60x - 2x^{-4}$$

$$\therefore MC(x) = 200 - 60x - 2x^{-4}$$