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$= \frac{2\times1+\sqrt{2+2\times(1)^2}}{1+\sqrt{2-(1)^2}}$

$= \frac{2+\sqrt{4}}{1+\sqrt{1}}$

$= \frac{2+2}{1+1}$

$= \frac{4}{2}$

$= 2$ proved.

Continuity

Exercise 3(D)

3. A function $y=f(x)$ is defined by $f(x)=\begin{cases} 2x^2+1 & \text{for } x\le 2 \\ 4x+1 & \text{for } x>2 \end{cases}$ at $x=2$

Is it continuous at $x=2$?

Solution

Given,

$$y=f(x)=\begin{cases} 2x^2+1 & \text{for } x\le 2 \\ 4x+1 & \text{for } x>2 \end{cases}$$

$$\therefore y=f(x)=\begin{cases} 2x^2+1 & \text{if } x<2 \\ 2x^2+1 & \text{if } x=2 \\ 4x+1 & \text{if } x>2 \end{cases}$$

Left Hand Limit (LHL)

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (2x^2+1)$$

$= 2\times(2)^2+1$

$= 8+1$

$= 9$

Right Hand Limit (RHL)

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (4x+1)$$
$$= 4 \times 2 + 1$$
$$= 9$$

Functional value (FV)

when $x = 2$,

$$f(x) = 2 \times 2^2 + 1$$
$$= 8 + 1$$
$$= 9$$

From the above result, we can see that LHL = RHL = FV.
So, the above function is continuous at $x = 2$.

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4. A function $y = f(x)$ is defined by ~~$f(x) =$~~

$$f(x) = \begin{cases} 3 - 2x & \text{for } 0 \le x \le 3/2 \\ -3x - 2 & \text{for } x \ge 3/2 \end{cases} \quad \text{at } x = 3/2$$

Is it continuous at $x = \frac{3}{2}$?

Given,

$$y = f(x) = \begin{cases} 3 - 2x & \text{for } 0 \le x < 3/2 \\ -3x - 2 & \text{for } x \ge 3/2 \end{cases}$$

i.e.

$$y = f(x) = \begin{cases} 3 - 2x & \text{for } x = 0,\ x > 0,\ x < 3/2 \\ -3x - 2 & \text{for } x \ge 3/2 \end{cases}$$

$$y = f(x) = \begin{cases} 3-2x & \text{if } x < 3/2 \\ -3x-2 & \text{if } x \geq 3/2 \end{cases}$$

Left Hand Limit (LHL)

$$\lim_{x \to 3/2^-} f(x) = \lim_{x \to 3/2^-} (3-2x)$$

$$= 3 - 2 \times \frac{3}{2}$$

$$= 0$$

Right Hand Limit (RHL)

$$\lim_{x \to 3/2^+} f(x) = \lim_{x \to 3/2^+} (-3x-2)$$

$$= -3 \times \frac{3}{2} - 2$$

$$= \frac{-9}{2} - 2$$

$$= \frac{13}{2}$$

From the above result, we can see that LHL $\neq$ RHL.
So, the above function is discontinuous at $x = \frac{3}{2}$.

5. ~~A function $y =$~~

5. A function $y = f(x)$ is defined by

$$f(x) = \begin{cases} x-1 & \text{for } 1 \le x < 2 \\ 2x-3 & \text{for } 2 \le x \le 3 \end{cases} \quad \text{at } x = 2$$

Is it continuous at $x = 2$?

Given,

$$y = f(x) = \begin{cases} x-1 & \text{for } 1 \le x < 2 \\ 2x-3 & \text{for } 2 \le x \le 3 \end{cases}$$

i.e. $$y = f(x) = \begin{cases} x-1 & \text{if } x = 1,\ x > 1,\ x < 2 \\ 2x-3 & \text{if } x = 2,\ x > 2,\ x = 3,\ x < 3 \end{cases}$$

$$y = f(x) = \begin{cases} x-1 & \text{if } x < 2 \\ 2x-3 & \text{if } x = 2,\ x > 2 \end{cases}$$

Left Hand Limit (LHL)

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x-1)$$
$$= 2 - 1$$
$$= 1$$

Right Hand Limit (RHL)

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (2x-3)$$
$$= 2 \times 2 - 3$$
$$= 4 - 3$$
$$= 1$$

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Functional value (FV)

When $x=2$,

$f(2) = 2\times2-3$
$= 4-3$
$= 1$

From the above result we can see that LHL = RHL = FV.
So, the above function is continuous at $x=2$.

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7. A function $f(x)$ is defined by $f(x) = \begin{cases} 3x+2 & \text{for } -2\le x<0 \\ 2-5x & \text{for } 0\le x<2 \\ 5-2x & \text{for } x\ge 2 \end{cases}$
Is it continuous at $x=0$?

Given,

$$f(x) = \begin{cases} 3x+2 & \text{for } -2\le x<0 \\ 2-5x & \text{for } 0\le x<2 \\ 5-2x & \text{for } x\ge 2 \end{cases}$$

$$\text{i.e. } f(x) = \begin{cases} 3x+2 & \text{for } x=-2,\ x>-2,\ x<0 \\ 2-5x & \text{for } x=0,\ x>0,\ x<2 \\ 5-2x & \text{for } x=2,\ x>2 \end{cases}$$

$$f(x) = \begin{cases} 3x+2 & \cancel{\text{for}} \text{ if } x<0 \\ 2-5x & \text{if } x=0,\ x>0 \end{cases}$$

Left Hand Limit (LHL)

$$\lim_{x\to0^-} f(x) = \lim_{x\to0^-} (3x+2)$$
$$= 3\times0+2$$
$$= 2$$

Right Hand Limit (RHL)

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (2-5x)$$
$$= 2 - 5 \times 0$$
$$= 2$$

Functional value (FV)

When $x = 0$,
$$f(0) = 2 - 5 \times 0$$
$$= 2$$

From the above result, we can see that LHL = RHL = FV.
So, the above function is continuous at $x = 0$.

8. A function $y = f(x)$ is defined by

$$f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x-3}, & x \neq 3 \\ 5, & x = 3 \end{cases}$$

Is it continuous at $x = 3$?

Given,

$$y = f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x-3}, & x \neq 3 \\ 5, & x = 3 \end{cases}$$

$$y = f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x-3}, & x > 3 \\ 5, & x = 3 \\ \dfrac{x^2 - x - 6}{x-3}, & x < 3 \end{cases}$$

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Left Hand Limit (LHL)

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \left[\frac{x^2 - x - 6}{x - 3} \right]$$

$$\neq \frac{3^2 - 3 - 6}{3 - 3}$$

$$= \lim_{x \to 3^-} \left[\frac{x^2 - 3x + 2x - 6}{x - 3} \right]$$

$$= \lim_{x \to 3^-} \left[\frac{x(x-3) + 2(x-3)}{(x-3)} \right]$$

$$= \lim_{x \to 3^-} (x+2)$$

$$= 3 + 2$$

$$= 5$$

Right Hand Limit (RHL)

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \left[\frac{x^2 - x - 6}{x - 3} \right]$$

$$= \lim_{x \to 3^+} \left[\frac{x^2 - 3x + 2x - 6}{x - 3} \right]$$

$$= \lim_{x \to 3^+} \left[\frac{x(x-3) + 2(x-3)}{(x-3)} \right]$$

$$= \lim_{x \to 3^+} (x+2)$$

$$= 3 + 2$$

$$= 5$$

Functional value (FV)

When $x = 3$,

$\therefore f(3) = 5$

From the above result, we can see that LHL = RHL = FV.

So, the above function is continuous at $x = 5$.

9. A function $y = f(x)$ is defined by,

$$y = f(x) = \begin{cases} x-1, & x<1 \\ 1, & x=1 \\ 2x-2, & x>1 \end{cases}$$

Is it continuous at $x=1$?

Given,

$$y = f(x) = \begin{cases} x-1, & x<1 \\ 1, & x=1 \\ 2x-2, & x>1 \end{cases}$$

Left Hand Limit (LHL)

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \cancel{(2x-2)}\,(x-1)$$
$$= \cancel{2\times1-2}\ 1-1$$
$$= 0$$

Right Hand Limit (RHL)

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2x-2)$$
$$= 2\times1-2$$
$$= 2-2$$
$$= 0$$

Functional value (FV)

When $x=1$,

$f(1) = 1$

From the above result we can see that LHL = RHL $\neq$ FV.

So, the above function is discontinuous at $x=1$.

10. A function $y = f(x)$ is defined by

$$f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \frac{1}{2}, & x = \frac{1}{2} \\ \frac{3}{2} - x, & x > \frac{1}{2} \end{cases}$$

Test the continuity of a function at a point $x = \frac{1}{2}$.

Given,

$$y = f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \frac{1}{2}, & x = \frac{1}{2} \\ \frac{3}{2} - x, & x > \frac{1}{2} \end{cases}$$

Left Hand Limit (LHL)

$$\lim_{x \to 1/2^-} f(x) = \lim_{x \to 1/2^-} \left[\frac{1}{2} - x\right]$$

$$= \frac{1}{2} - \frac{1}{2}$$

$$= 0$$

Right Hand Limit (RHL)

$$\lim_{x \to 1/2^+} f(x) = \lim_{x \to 1/2^+} \left[\frac{3}{2} - x\right]$$

$$= \frac{3}{2} - \frac{1}{2}$$

$$= 1$$

Functional value (FV)

When $x = \frac{1}{2}$

$\therefore f(\frac{1}{2}) = \frac{1}{2}$

From the above result, we can see that LHL $\neq$ RHL $\neq$ FV

So, the above function is discontinuous at $x = \frac{1}{2}$

12. Find the value of k for which

$$f(x) = \begin{cases} \frac{x^2 - 2x - 3}{x+1} & \text{when } x \neq -1 \\ k & \text{when } x = -1 \end{cases}$$

is continuous at $x = -1$.

Given,

$$f(x) = \begin{cases} \frac{x^2 - 2x - 3}{x+1} & \text{when } x \neq -1 \\ k & \text{when } x = -1 \end{cases}$$

Since the above function is continuous, then

Left Hand Limit = Right Hand Limit = Functional value

LHL

or, $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \left[\frac{x^2 - 2x - 3}{x+1}\right]$

$= \left[\frac{(-1)^2 - 2\times(-1) - 3}{(-1)+1}\right]$

$= \lim_{x \to -1^-} \left[\frac{x^2 - 2x - x - 3}{x+1}\right]$

$= \lim_{x \to -1^-} \left[x(x-2) - \right.$

$= \lim_{x \to -1^-} \left[\frac{x^2 - 3x + x - 3}{x+1}\right]$

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$$= \lim_{x \to -1^-} \left[\frac{x(x-3)+1(x-3)}{(x+1)} \right]$$

$$= \lim_{x \to -1^-} \left[\frac{(x-3)(x+1)}{(x+1)} \right]$$

$$= \lim_{x \to -1^-} (x-3)$$

$$= -1-3$$

$$= -4$$

Since, LHL = RHL = FV.

$f(x) = -4$

$\therefore K = -4$

---

13. A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 2x-3 & \text{for } x<2 \\ 2 & \text{for } x=2 \\ 3x-5 & \text{for } x>2 \end{cases}$$

Is the function $f(x)$ continuous at $x=2$? If not, how can the function $f(x)$ be made continuous at $x=2$.

Given,

$$f(x) = \begin{cases} 2x-3 & \text{for } x<2 \\ 2 & \text{for } x=2 \\ 3x-5 & \text{for } x>2 \end{cases}$$

Left Hand Limit (LHL)

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x-3)$$

$$= 2 \times 2 - 3$$

$$= 1$$

Right Hand Limit (RHL)

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (3x-5)$$
$$= 3\times 2 - 5$$
$$= 6-5$$
$$= 1$$

Functional value (FV)

When $x=2$,
$f(2) = 2$
From the above result we can see that LHL = RHL $\neq$ FV.
So, the above function is discontinuous at $x=2$.

To make the above function continuous, the above function should be redefined as:

$$f(x) = \begin{cases} 2x-3 & \text{for } x<2 \\ 1 & \text{for } x=2 \\ 3x-5 & \text{for } x>2 \end{cases}$$

---

15. A function $y=f(x)$ is defined by

$$f(x) = \begin{cases} kx-3 & , x<2 \\ 3-x+2x^2 & , x=2 \\ 3-x+2x^2 & , x>2 \end{cases}$$

if it is continuous at $x=2$, find the value of k.

Given,

$$y = f(x) = \begin{cases} kx-3, & x<2 \\ 3-x+2x^2, & x=2 \\ 3-x+2x^2, & x>2 \end{cases}$$

Since, the above function is continuous, then
Left Hand Limit = Right Hand Limit = Functional value

i.e. $\lim_{x \to 2^-} (kx-3) = \lim_{x \to 2^+} (3-x+2x^2) = f(2)$

~~When $x=2$,~~

~~$f(2) = k \times 2 - 3$~~

~~$= 2k - 3$~~

or, $\lim_{x \to 2^-} (kx-3) = \lim_{x \to 2^+} (3-x+2x^2) = 3-x+2x^2$

Or, $2k-3 = \cancel{1+2}\ 3-2+(2\times 4) = 3-x+2x^2$

Or, $2k-3 = 9 = 3-x+2x^2$

<u>Taking 1st & 2nd</u>

$2k-3=9$

or, $2k=12$

$\therefore k=6$

---

16. Function $y = f(x)$ is defined by

$$f(x) = \begin{cases} kx+3, & x>3 \\ 3x, & x<3 \\ 9, & x=3 \end{cases}$$

If it is continuous at $x=3$, find k.

Given,

$$y = f(x) = \begin{cases} kx+3, & x>3 \\ 3x, & x<3 \\ 9, & x=3 \end{cases}$$

Since, the above function is continuous, then
Left Hand Limit = Right Hand Limit = Functional value
i.e. $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$

or, $\lim_{x \to 3^-} (3x) = \lim_{x \to 3^+} (kx+3) = 9$

or, $9 = \lim_{x \to 3} (3k+3) = 9$

Taking 1st & 2nd

$3k+3 = 9$
or $3k = 6$
$\therefore k = 2$

14. A function $y = f(x)$ is defined by

$$f(x) = \begin{cases} kx^2+5x-9, & x<1 \\ J, & x=1 \\ (3-x)(k-2x), & x>1 \end{cases}$$

If it is continuous for all x, find the value of k and J.

Given,

$$y = f(x) = \begin{cases} kx^2+5x-9, & x<1 \\ J, & x=1 \\ (3-x)(k-2x), & x>1 \end{cases}$$

Since, the above function is continuous, then
Left Hand Limit = Right Hand Limit = Functional value
i.e. $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

or, $\lim_{x \to 1^-} (kx^2+5x-9) = \lim_{x \to 1^+} [(3-x)(k-2x)] = J$

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or, $k+5-9 = 2(k-2) = J$

or, $k-4 = 2k-4 = J$

Taking 1st & 2nd

$k-4 = 2k-4$

or, $-k = 0$

$\therefore k = 0$

Taking 2nd & 3rd

$2k-4 = J$

or, $J = 2\times0-4$ [$\because k=0$]

$\therefore J = -4$

---

6. A function $y = f(x)$ is defined by

$$f(x) = \begin{cases} \frac{|x-4|}{x-4} & \text{if } x \neq 4 \\ 1 & \text{if } x = 4 \end{cases}$$

Is it continuous at $x = 4$?

Given,

$$y = f(x) = \begin{cases} \frac{|x-4|}{x-4} & \text{if } x \neq 4 \\ 1 & \text{if } x = 4 \end{cases}$$

$$y = f(x) = \begin{cases} \frac{|x-4|}{x-4} & \text{if } x < 4 \\ \frac{|x-4|}{x-4} & \text{if } x > 4 \\ 1 & \text{if } x = 4 \end{cases}$$

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$$y = f(x) = \begin{cases} \dfrac{-(x-4)}{x-4} & \text{if } x < 4 \\ \dfrac{(x-4)}{x-4} & \text{if } x > 4 \\ 1 & \text{if } x = 4 \end{cases}$$

Left Hand Limit (LHL)

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \left[\frac{-(x-4)}{(x-4)} \right]$$

$$= -1$$

Right Hand Limit (RHL)

$$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} \left[\frac{(x-4)}{x-4} \right]$$

$$= 1$$

From the above result, we can see that LHL $\neq$ RHL.
So, the above function is discontinuous at $x = 4$.

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11. A function $y = f(x)$ is defined by

$$f(x) = \begin{cases} 2x-1, & x \leq 1 \\ x, & x > 1 \end{cases}$$

Is it continuous at $x = 1$?

Given,

$$y = f(x) = \begin{cases} 2x-1, & x \leq 1 \\ x, & x > 1 \end{cases}$$

$$y = f(x) = \begin{cases} 2x-1, & x = 1, x < 1 \\ x, & x > 1 \end{cases}$$

$$y = f(x) = \begin{cases} 2x-1, & x<1 \\ x, & x>1 \\ 2x-1, & x=1 \end{cases}$$

Left Hand Limit (LHL)

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2x-1)$$
$$= 2\times 1 - 1$$
$$= 1$$

Right Hand Limit (RHL)

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x)$$
$$= 1$$

Functional value (FV)

when $x=1$,

$$f(1) = 2\times 1 - 1$$
$$= 1$$

From the above result, we can see that LHL = RHL = FV.
So, the above function is continuous at $x=1$.

---

1. Test the continuity of the following functions at the specified points:

a) $f(x) = 2x^2 - 3x + 1$ at $x = 1$.

Given,

$f(x) = 2x^2 - 3x + 1$

Left Hand Limit (LHL)

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2x^2 - 3x + 1)$$
$$= 2\times 1 - 3\times 1 + 1$$
$$= 3 - 3$$
$$= 0$$

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<u>Right Hand Limit (RHL)</u>

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (2x^2 - 3x + 1)$$
$$= 2 \times 1 - 3 \times 1 + 1$$
$$= 3 - 3$$
$$= 0$$

<u>Functional value (FV)</u>

When $x = 1$,
$$f(1) = 2 \times (1)^2 - 3 \times (1) + 1$$
$$= 2 - 3 + 1$$
$$= 0$$

From the above result we can see that LHL = RHL = FV.
So, the above function is continuous at $x = 1$.

---

b) $f(x) = \frac{x^2 - 4}{x - 2}$ at $x = 2$.

Given,
$$f(x) = \frac{x^2 - 4}{x - 2}$$

<u>Left Hand Limit (LHL)</u>

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \left[\frac{x^2 - 4}{x - 2} \right]$$
$$= \lim_{x \to 2^-} \left[\frac{(x+2)(x-2)}{(x-2)} \right]$$
$$= \lim_{x \to 2^-} (x + 2)$$
$$= 2 + 2$$
$$= 4$$

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Right Hand Limit (RHL)

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}\left[\frac{x^2-4}{x-2}\right]$$

$$= \lim_{x\to 2^+}\left[\frac{(x+2)(x-2)}{(x-2)}\right]$$

$$= \lim_{x\to 2^+}(x+2)$$

$$= 2+2$$

$$= 4$$

Functional value (FV)

when $x=2$,

$$f(2) = \frac{2^2-4}{2-2}$$

$$= \frac{4-4}{2-2}$$

$$= \frac{0}{0}$$

From the above result, we can see that $LHL = RHL \neq FV$.

So, the above function is ~~conti~~ discontinuous at $x=2$.

---

c) $f(x) = \frac{1}{\sqrt{x}}$ at $x=0$

Given,

$$f(x) = \frac{1}{\sqrt{x}}$$

Left Hand Limit (LHL)

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\left[\frac{1}{\sqrt{x}}\right]$$

$$= \frac{1}{\sqrt{0}}$$

$$= \infty$$

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Right Hand Limit (RHL)

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left[\frac{1}{\sqrt{x}} \right]$$

$$= \frac{1}{\sqrt{0}}$$

$$= \infty$$

From the above result, we can see that LHL $\neq$ RHL.
So, the above function is discontinuous at $x = 0$

d) $|x-2|$ at $x = 2$.
Given,
$f(x) = |x-2|$

Left Hand Limit (LHL)

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} [|x-2|]$$

$$= \lim_{x \to 2^-} -(x-2)$$

$$= -(2-2)$$

$$= 0$$

Right Hand Limit (RHL)

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} [|x-2|]$$

$$= \lim_{x \to 2^+} (x-2)$$

$$= 2-2$$

$$= 0$$

Functional value (FV)

When $x = 2$,

$$f(2) = |2-2|$$

$$= |0|$$

$$= 0$$

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From the result, we can see that LHL = RHL = FV.
So, the function is continuous at $x = 2$.

2. A function $y = f(x)$ is defined by

$$f(x) = \begin{cases} \dfrac{x^2-1}{x-1} & \text{for } x \neq 1 \\ 2 & \text{for } x = 1 \end{cases}$$

Is it continuous at $x = 1$?

Given,

$$y = f(x) = \begin{cases} \dfrac{x^2-1}{x-1} & \text{for } x \neq 1 \\ 2 & \text{for } x = 1 \end{cases}$$

$$y = f(x) = \begin{cases} \dfrac{x^2-1}{x-1} & \text{if } x < 1 \\ \dfrac{x^2-1}{x-1} & \text{if } x > 1 \\ 2 & \text{if } x = 1 \end{cases}$$

Left Hand Limit (LHL)

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \left[\frac{x^2-1}{x-1} \right]$$

$$= \lim_{x \to 1^-} \left[\frac{(x+1)(x-1)}{(x-1)} \right]$$

$$= \lim_{x \to 1^-} (x+1)$$

$$= 1+1$$

$$= 2$$

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Right Hand Limit (RHL)

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \left[\frac{x^2 - 1}{x - 1} \right]$$

$$= \lim_{x \to 1^+} \left[\frac{(x+1)(x-1)}{(x-1)} \right]$$

$$= \lim_{x \to 1^+} (x+1)$$

$$= 1 + 1$$

$$= 2$$

Functional value (FV)

When $x = 1$,

$f(1) = 2$

From the above result, we can see that LHL = RHL = FV.
So, the above function is continuous at $x = 1$.

Signature of Subject Teacher:

Signature of Director: