Exercise 3(B)

1. If the total cost function $C(x) = 2000 + 30x$ and the revenue function $R(x) = 10x$, find the breakeven point.

Given,

$C(x) = 2000 + 30x$

$R(x) = 10x$

At breakeven point

$R(x) = C(x)$

or, $10x = 2000 + 30x$

or, $20x = -2000$

or, $x = -\frac{2000}{20}$

$\therefore x = -100$

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2. If fixed cost of 30 articles is Rs. 40,000 and variable cost for each additional article is Rs 35, find the total cost of 100 articles.

Given,

Fixed cost of 30 articles = Rs. 40,000

Variable cost of 1 article = Rs 35

Then, Variable cost of 70 articles = Rs 35×70

= Rs 2450

Now,

Total cost = Fixed cost + Variable cost

= Rs 40,000 + Rs 2450

= Rs 42,450

$\therefore$ The cost of 100 articles is Rs 42,450.

3. A certain car rental agency charges $25 per day plus 30 cents per mile. Express the cost of renting a car from this agency for 1 day as a function of the number of miles driven.

Solution

We know,

100 cents = $1

∴ 30 cents = $0.3

Given,

Fixed cost = $25

Let the number of miles driven be x.

Variable Cost for 1 mile = 30 cents = $0.3

Variable Cost for x miles = $0.3 × x

Now,

Total Cost = Fixed Cost + variable Cost

= $25 + $0.3x

∴ T(x) = $(25 + 0.3x)

Therefore, the cost of renting a car is $(25 + 0.3x)

4. The fixed cost of 20 units of electricity for a month is Rs. 80 and the variable cost for each extra unit of electricity is Rs. 7.30. Find the total cost of expenditure. Also, obtain the total cost when 50 units of electricity is consumed.

Given,

Fixed Cost of 20 units of electricity = Rs. 80

Variable Cost of 1 unit of electricity = Rs. 7.30

Let the number of units of electricity more than 20 units be x,

Variable cost of x unit of electricity = Rs 7.30 × x

Now,

Total Cost = Fixed Cost + variable cost

= Rs (80 + 7.30x)

When 50 units of electricity is consumed i.e. $x = 30$,

Total cost = Rs (80 + 7.30 × 30)
= Rs (80 + 219)
= Rs 299

5. A firm has a fixed cost of Rs. 4000 for plant and equipment and an extra cost of Rs. 300 for each additional unit produced.
a) What is the total cost function?
b) What are the cost of producing 25 units, and 45 units respectively?
c) Find the break-even point for the firm if each unit sold at Rs. 500.

Given,
Fixed Cost = Rs. 4000
Variable Cost for 1 unit = Rs 300
Variable Cost for x unit = Rs $300 \times x$

We know;

a) Total Cost = Fixed Cost + variable Cost
$\therefore TC(x) = Rs(4000 + 300x)$

b) Cost of producing 25 units = Rs(4000 + 300 × 25)
i.e. $TC(25) = Rs(4000 + 7500) = Rs\ 11,500$

Cost of producing 45 units = Rs(4000 + 300 × 45)
i.e. $TC(45) = Rs(4000 + 13500) = Rs.\ 17,500$

c) For Break-Even point,
$R(x) = C(x)$
or, $500x = 4000 + 300x$
or, $200x = 4000$
$\therefore x = 20$

6. A firm operating in pure competition receives Rs. 25 for each unit of output sold. It has an extra cost of Rs. 15 per item and a fixed cost of Rs. 1200.

a) What is the profit function?

b) Find the profit when the sells 200 items, 300 items and 100 items respectively.

Given,

Fixed cost = Rs 1200

Variable cost for 1 item = Rs 15

Let the number of items be x,

Variable cost for x items = Rs $15 \times x$

Then,

Total cost = Fixed cost + variable cost

= Rs $(1200 + 15x)$

Total Revenue = Rs $25 \times x$

Now,

a) Profit Function = Total Revenue - Total cost

i.e. $\pi(x) = TR(x) - TC(x)$

$= 25x - 1200 - 15x$

$= 10x - 1200$

b) When sells 200 items,

$\pi(200) =$ Rs $(10 \times 200 - 1200)$

$=$ Rs $(2000 - 1200)$

$=$ Rs 800

When sells 300 items,

$\pi(300) =$ Rs $(10 \times 300 - 1200)$

$=$ Rs $(3000 - 1200)$

$=$ Rs 1800

When sells 100 items,

$$\pi(100) = Rs(10 \times 100 - 1200)$$
$$= Rs\ 1000 - Rs\ 1200$$
$$= -Rs\ 200$$

7. A book publisher finds that the production cost of a book is Rs. 30 and the fixed cost per year amounts to Rs. 25,000. If each book is sold at the rate of Rs. 50, find the total cost function and the revenue function. Also find the minimum number of books to be sold per year in order that there is no loss.

Given,

Fixed cost = Rs. 25,000

let the number of books be x,

Variable cost for production of 1 book = Rs. 30

variable cost for production of x books = Rs. $30 \times x$

we know,

Total Cost = Fixed Cost + variable cost

i.e. $TC(x) = Rs\ (25000 + 30x)$

Total Revenue = Price per book x Quantity sold

i.e. $TR(x) = Rs\ 50 \times x$

Also,

To ensure no loss,

$$TR(x) \geq TC(x)$$
$$\Rightarrow 50x \geq 25000 + 30x$$
$$\Rightarrow 50x - 30x \geq 25000 + 30x - 30x$$
$$\Rightarrow 20x \geq 25000$$
$$\Rightarrow x \geq 1250.$$

8. A company finds that the production cost associated with each article is Rs 40 and fixed cost is Rs. 30000. If each article is sold for Rs. 60, find

a) total cost function

b) profit function.

c) break-even point and

d) how many articles should be produced to make a profit of Rs. 50000?

Given,

Fixed Cost = Rs. 30,000

Let the number of articles be x.

Variable cost for production of 1 articles = Rs 40

Variable cost for production of x articles = Rs $40 \times x$

We know,

a) Total Cost = Fixed cost + Variable cost

i.e. $TC(x) = Rs\ (30000 + 40x)$

Also,

Total Revenue = Price per article x Quantity sold

i.e. $TR(x) = Rs\ 60 \times x$

b) Profit = Total Revenue − Total cost

i.e. $\pi(x) = 60x - 30000 - 40x$

$= 20x - 30000$

c) For Break-even point,

$TR = TC$

or, $60x = 30000 + 40x$

or, $20x = 30000$

$\therefore x = 1500$

d) When profit = Rs 50,000,

$\pi = TR - TC$

or, $50000 = 20x - 30000$

or, $20x = 80000$

$\therefore x = 4000$

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9. The demand for a certain item is given by $q = 150 - 3p$, where q is the amount demanded in unit and P, the price per unit. It costs Rs 4 to produce each unit. What is the profit function of the firm for this item? At what minimum price the firm should sell the item to receive a profit of Rs. 720?

Given,

Demand Function i.e. $q = 150 - 3p$

Cost per unit = Rs 4.

Total Cost = $150 - 3 \times 4$

$= 150 - 12$

$= 138$

Demand Function $q = 150 - 3p$

Cost Per unit = Rs 4

Total Cost = $4 \times (150 - 3P)$

$= 600 - 12P$

Total Revenue (TR) = $P \times (150 - 3P)$

$= 150P - 3P^2$

We know,

$\pi = TR - TC$

$= 150P - 3P^2 - 600 + 12P$

$= -3P^2 + 162P - 600$

From the question, when Profit = Rs. 720

$-3P^2 + 162P - 600 = 720$

or, $-3P^2 + 162P = 1320$ — (1)

or, $3(-P^2 + 54P) = 1320$

or, $-P^2 + 54P = 440$

or, $P^2 - 54P = -440$

or, $P^2 - 54P + 440 = 0$ — (1)

Comparing (1) with $ax^2 + bx + c = 0$,

$a = 1, b = -54, c = 440$

$$P = \frac{-(-54) \pm \sqrt{(-54)^2 - 4 \times 1 \times 440}}{2 \times 1}$$

$$\text{or, } P = \frac{54 \pm \sqrt{2916 - 1760}}{2}$$

$$\text{or, } P = \frac{54 \pm 34}{2}$$

$\therefore P = 44, 10$

Here, $P = 44$ is rejected.

Minimum price = Rs 10

Therefore, the firm should sell the item at Rs 10 to receive a profit of Rs 720.

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10. A profit-making company want to launch a new product. It observes that the fixed cost of the new product is Rs. 100000 and variable cost per unit is Rs 1000. The revenue function for the sale of x units is given by $200x + 500x^2$, Find

a) cost function

b) profit function

c) cost at the production level 50

d) revenue at the production level 50

e) profit at the production level 50 and

f) break-even point

Given,

Fixed cost = Rs 100000

Let the number of new product be x.

Variable cost for the production of 1 unit = Rs. 1000

Variable cost for the production of x units = Rs. $1000 \times x$

Also we have,

Revenue function for the sales of x units = $200x + 500x^2$

i.e. $TR(x) = 500x^2 + 200x$

We know,

a) Total Cost = Fixed cost + variable cost

i.e. $TC(x) = Rs\ (100000 + 1000x)$

b) Profit = Total Revenue − Total cost

i.e. $\pi(x) = TR(x) - TC(x)$

$= 500x^2 + 200x - 100000 - 1000x$

$= 500x^2 - 800x - 100000$

c) When $x = 50$,

$TC(50) = Rs\ (100000 + 1000 \times 50)$

$= Rs\ (100000 + 50000)$

$\therefore TC(50) = Rs\ 150000$

d) When $x = 50$,

$TR(50) = [500 \times (50)^2 + 200 \times (50)]$

$= [(500 \times 2500) + 10000]$

$= Rs\ (1250000 + 10000)$

$\therefore TR(50) = Rs\ 1260000$

e) When $x = 50$,

$\pi(50) = TR(50) - TC(50)$

$= Rs\ 1260000 - Rs\ 150000$

$= Rs\ 1110000$

b) For break-even point

$TR = TC$

or, $500x^2 + 200x = 100000 + 1000x$

or, $500x^2 - 800x = 100000$

or, $100(5x^2 - 8x) = 100000$

or, $5x^2 - 8x = 1000$

$\therefore x = 14.96$

11. A carpenter finds that each chair cost Rs. 35 and that the fixed cost is Rs. 5000 per month. If each chair sells for Rs 60, find the following:

a) Total cost function.

b) The break-even point.

c) The numbers of chairs that must be produced and sold each month to get profit of Rs. 10000.

Given,

Fixed cost = Rs 5000

Let the number of chairs produced be x.

variable cost for the production of 1 chair = Rs. 35

variable cost for the production of x chairs = Rs. $35 \times x$

We know,

a) Total Cost = Fixed cost + variable cost

i.e. $TC(x) =$ Rs $(5000 + 35x)$

Also,

Total Revenue = Price Per Unit X Quantity Sold

i.e. $TR(x) =$ Rs $60 \times x$

b) For break-even point,

$TR = TC$

or, $60x = 5000 + 35x$

or, $60x - 35x = 5000$

or, $25x = 5000$

$\therefore x = 200$

c) When profit = Rs. 10000,

$\pi = TR - TC$

Or, $10000 = 60x - 5000 - 35x$

or, $25x = 15000$

$\therefore x = 600$

$\therefore$ 600 chairs must be produced and sold to get profit of Rs. 10000.

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12. A calculator manufacturing company finds that the daily cost of producing x calculators is given by $C(x) = 1200x + 15680$

a) If each calculator is sold for Rs 1340, determine the minimum number of calculators that must be produced and sold daily to ensure no loss.

b) If the selling price is increased by Rs. 20 per piece, what would be the break-even point?

c) If it is known that a least 80 calculators can be sold daily, what price the company should charge per piece of calculator to guarantee no loss?

Given,

$C(x) = 1200x + 15680$

a) If each calculator is sold for Rs 1340,

$R(x) = 1340x$

To ensure no loss,

$R(x) \geq C(x)$

$\Rightarrow 1340x \geq 1200x + 15680$

$\Rightarrow 1340x - 1200x \geq 1200x + 15680 - 1200x$

$\Rightarrow 140x \geq 15680$

$\therefore x \geq 112$

Therefore, 112 calculators must be produced and sold daily to ensure no loss if each is sold for Rs 1340.

b) If the selling price is increased by Rs 20 i.e. sold for Rs 1360,

$R(x) = 1360x$

For break-even point,

$R(x) = C(x)$

or, $1360x = 1200x + 15680$

or, $160x = 15680$

$\therefore x = 98$

Therefore, 98 would be the break-even point if the selling price is increased by Rs. 20 per piece.

c) If 80 calculators can be sold daily,

$C(80) = 1200 \times 80 + 15680$

$= 96000 + 15680$

$= \text{Rs } 111680$

$C(1) = \text{Rs } \frac{111680}{80}$

$\therefore C(1) = \text{Rs } 1396$

$\therefore$ The company should charge Rs 1396 per piece of calculator to guarantee no loss if it is known that at least 80 calculators can be sold daily.