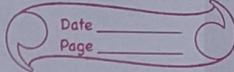


Chapter-3

Functions, Limits and Continuity

Homework



Date: 2020/09/12

Exercise 3(A)

1. Decide whether the following relations are functions or not.

a) $y = 4x - 3$

Given,

$$y = 4x - 3$$

For every value of x , there is a unique value of y .
So, it is a function.

b) $y = \frac{1}{9}x^2 + \frac{1}{5}x + 6$

Given,

$$y = \frac{1}{9}x^2 + \frac{1}{5}x + 6$$

For every value of x , there is a unique value of y .
So, it is a function.

c) $y = \frac{1}{\sqrt{x+5}}$, $x > -5$

Given,

$$y = \frac{1}{\sqrt{x+5}}, x > -5$$

For every value of x , there is a unique value of y .
So, it is a function.

d) $y^2 = 25 - x$

Given,

$$\begin{aligned} y^2 &= 25 - x \\ \therefore y &= \pm \sqrt{25 - x} \end{aligned}$$

For every value of x , there is no unique value of y .
So, it is not a function.

2. If $f(x) = x^2 - 2x + 2$, find $f(0)$ and $f(-\sqrt{2})$.

Given,

$$f(x) = x^2 - 2x + 2$$

$$\begin{aligned} \text{when } x = 0, f(0) &= (0)^2 - 2 \times 0 + 2 \\ &= 0 - 0 + 2 \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{when } x = -\sqrt{2}, f(-\sqrt{2}) &= (-\sqrt{2})^2 - 2 \times (-\sqrt{2}) + 2 \\ &= 2 + 2\sqrt{2} + 2 \\ &= 4 + 2\sqrt{2} \\ &= 2(2 + \sqrt{2}) \end{aligned}$$

$$\therefore f(0) = 2 \text{ & } f(-\sqrt{2}) = 2 + 2(2 + \sqrt{2})$$

3. If $f(x) = \begin{cases} 3x+2 & \text{for } x \geq 0, \\ 5x-1 & \text{for } x < 0 \end{cases}$,

compute $f(2)$, $f\left(-\frac{1}{5}\right)$

Given,

$$f(x) = \begin{cases} 3x+2 & \text{for } x \geq 0 \text{ i.e. } x=0, x>0 \\ 5x-1 & \text{for } x < 0 \end{cases}$$

$$\text{When } x=2, f(2) = 3x2 + 2 \\ = 6+2 \\ = 8$$

$$\text{When } x=-\frac{1}{5}, f(-\frac{1}{5}) = 3x(-\frac{1}{5}) + 2 \\ = -\frac{3}{5} + 2 \\ = \frac{7}{5} \\ \therefore f(2) = 8 \text{ & } f(-\frac{1}{5}) = \frac{7}{5}$$

4. If $f(x) = 3x^2 + x + 1$, find $\frac{f(x+h) - f(x)}{h}$

Given,

$$f(x) = 3x^2 + x + 1$$

$$\begin{aligned} f(x+h) &= 3[(x+h)^2] + (x+h) + 1 \\ &= 3(x^2 + 2xh + h^2) + x + h + 1 \\ &= 3x^2 + 6xh + 3h^2 + x + h + 1 \end{aligned}$$

Now,

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{3x^2 + 6xh + 3h^2 + x + h + 1 - 3x^2 - x - 1}{h} \\ &= \frac{6xh + 3h^2 + h}{h} \\ &= \frac{h(6x + 3h + 1)}{h} \\ &= 6x + 3h + 1 \end{aligned}$$

5. If $f(x) = x^2 - \frac{1}{x^2}$, show that $f(x) + f\left(\frac{1}{x}\right) = 0$

Given,

$$f(x) = x^2 - \frac{1}{x^2}$$

$$f\left(\frac{1}{x}\right) = \frac{1}{x^2} - \frac{1}{\frac{1}{x^2}}$$

$$= \frac{1}{x^2} - x^2$$

Now,

$$\begin{aligned} f(x) + f\left(\frac{1}{x}\right) &= x^2 - \frac{1}{x^2} + \frac{1}{x^2} - x^2 \\ &= 0 \quad \text{proved} \end{aligned}$$

6. If $f(x) = \frac{x-1}{x+1}$, show that, if

$$\frac{f(x) - f(y)}{1 + f(x) \cdot f(y)} = \frac{x-y}{1+xy}$$

Given,

$$f(x) = \frac{x-1}{x+1}$$

Then,

$$f(y) = \frac{y-1}{y+1}$$

Now,

$$\frac{f(x) - f(y)}{1 + f(x) \cdot f(y)}$$

$$\text{LHS} = \frac{f(x) - f(y)}{1 + f(x) \cdot f(y)}$$

$$= \frac{\frac{x-1}{x+1} - \frac{y-1}{y+1}}{1 + \frac{x-1}{x+1} \cdot \frac{y-1}{y+1}}$$

$$= \frac{(x-1)(y+1) - (y-1)(x+1)}{(x+1)(y+1) + (x-1)(y-1)}$$

$$= \frac{(x-1)(y+1) - (y-1)(x+1)}{(x+1)(y+1) + (x-1)(y-1)}$$

$$= \frac{xy + x - y - 1 - (xy + y - x - 1)}{xy + x + y + 1 + xy - x - y + 1}$$

$$= \frac{xy + x - y - 1 - xy - y + x + 1}{2xy + 2}$$

$$= \frac{2x - 2y}{2xy + 2}$$

$$= \frac{2(x-y)}{2(xy+1)}$$

$$= \frac{x-y}{xy+1}$$

$$= \frac{x-y}{1+xy}$$

proved