Chapter-3

Functions, Limits and Continuity

Homework

Date: 2080/09/12

Exercise 3(A)

1. Decide whether the following relations are functions or not,

a) $y = 4x - 3$

Given,

$y = 4x - 3$

For every value of x, there is a unique value of y.
So, it is a function.

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b) $y = \frac{1}{9}x^2 + \frac{1}{5}x + 6$

Given,

$y = \frac{1}{9}x^2 + \frac{1}{5}x + 6$

For every value of x, there is a unique value of y.
So, it is a function.

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c) $y = \frac{1}{\sqrt{x+5}}, \; x > -5$

Given,

$y = \frac{1}{\sqrt{x+5}}, \; x > -5$

For every value of x, there is a unique value of y.
So, it is a function.

d) $y^2 = 25 - x$

Given,

$y^2 = 25 - x$

$\therefore y = \pm\sqrt{25 - x}$

For every value of x, there is no unique value of y.
So, it is not a function.

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2. If $f(x) = x^2 - 2x + 2$, find $f(0)$ and $f(-\sqrt{2})$.

Given,

$f(x) = x^2 - 2x + 2$

When $x = 0$, $f(0) = (0)^2 - 2\times 0 + 2$

$= 0 - 0 + 2$

$= 2$

When $x = -\sqrt{2}$, $f(-\sqrt{2}) = (-\sqrt{2})^2 - 2\times(-\sqrt{2}) + 2$

$= 2 + 2\sqrt{2} + 2$

$= 4 + 2\sqrt{2}$

$= 2(2+\sqrt{2})$

$\therefore f(0) = 2$ & $f(-\sqrt{2}) = 2(2+\sqrt{2})$

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3. If $f(x) = \begin{cases} 3x+2 & \text{for } x \geq 0, \\ 5x-1 & \text{for } x < 0 \end{cases}$

compute $f(2)$, $f\left(-\frac{1}{5}\right)$

Given,

$f(x) = \begin{cases} 3x+2 & \text{for } x \geq 0 \text{ i.e. } x = 0, x > 0 \\ 5x-1 & \text{for } x < 0 \end{cases}$

When $x = 2$, $f(2) = 3\times 2 + 2$

$= 6 + 2$

$= 8$

When $x = -\frac{1}{5}$, $f(-\frac{1}{5}) = \cancel{3\times(-\frac{1}{5})}\ 5\times(-\frac{1}{5}) - 1$

$= -1 - 1$

$= -2$

$\therefore f(2) = 8$ & $f(-\frac{1}{5}) = -2$

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4. If $f(x) = 3x^2 + x + 1$, find $\frac{f(x+h) - f(x)}{h}$

Given,

$f(x) = 3x^2 + x + 1$

$f(x+h) = 3[(x+h)^2] + (x+h) + 1$

$= 3(x^2 + 2xh + h^2) + x + h + 1$

$= 3x^2 + 6xh + 3h^2 + x + h + 1$

Now,

$$\frac{f(x+h) - f(x)}{h} = \frac{3x^2 + 6xh + 3h^2 + x + h + 1 - 3x^2 - x - 1}{h}$$

$$= \frac{6xh + 3h^2 + h}{h}$$

$$= \frac{h(6x + 3h + 1)}{h}$$

$$= 6x + 3h + 1$$

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5. If $f(x) = x^2 - \frac{1}{x^2}$, show that $f(x) + f\left(\frac{1}{x}\right) = 0$

Given,

$$f(x) = x^2 - \frac{1}{x^2}$$

$$f\left(\frac{1}{x}\right) = \frac{1}{x^2} - \frac{1}{\frac{1}{x^2}}$$

$$= \frac{1}{x^2} - x^2$$

Now,

$$f(x) + f\left(\frac{1}{x}\right) = x^2 - \frac{1}{x^2} + \frac{1}{x^2} - x^2$$

$$= 0 \quad \underline{\text{proved}}$$

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6. If $f(x) = \frac{x-1}{x+1}$, show that, if

$$\frac{f(x) - f(y)}{1 + f(x) \cdot f(y)} = \frac{x-y}{1+xy}$$

Given,

$$f(x) = \frac{x-1}{x+1}$$

Then,

$$f(y) = \frac{y-1}{y+1}$$

Now,

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$$\cancel{\frac{f(x) - f(y)}{1 + f(x)}}$$

$$LHS = \frac{f(x) - f(y)}{1 + f(x) \cdot f(y)}$$

$$= \frac{\frac{x-1}{x+1} - \frac{y-1}{y+1}}{1 + \frac{x-1}{x+1} \cdot \frac{y-1}{y+1}}$$

$$= \frac{\frac{(x-1)(y+1) - (y-1)(x+1)}{(x+1)(y+1)}}{\frac{(x+1)(y+1) + (x-1)(y-1)}{(x+1)(y+1)}}$$

$$= \frac{(x-1)(y+1) - (y-1)(x+1)}{(x+1)(y+1) + (x-1)(y-1)}$$

$$= \frac{xy + x - y - 1 - (xy + y - x - 1)}{xy + x + y + 1 + xy - x - y + 1}$$

$$= \frac{xy + x - y - 1 - xy - y + x + 1}{2xy + 2}$$

$$= \frac{2x - 2y}{2xy + 2}$$

$$= \frac{2(x-y)}{2(xy+1)}$$

$$= \frac{x-y}{xy+1}$$

$$= \frac{x-y}{1+xy}$$

proved