Date
Page

Exercise (2B)

1. Apply De-Moivre's Theorem to compute.

a) $(\cos 8^\circ + i\sin 8^\circ)^{30}$

Solution

For any positive integer n, De-moivre's theorem states that:

$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

i.e. $[1(\cos 8^\circ + i\sin 8^\circ)]^{30} = 1^{30}(\cos 30\times 8 + i\sin 30\times 8)$

$= \cos 240^\circ + i\sin 240^\circ$

$= -\frac{1}{2} - i.\frac{\sqrt{3}}{2}$ where,

$x = -\frac{1}{2}$ & $y = -\frac{\sqrt{3}}{2}$

b) $[3(\cos 15^\circ + i\sin 15^\circ)]^8$

Solution

For any positive integer n, De-moivre's theorem states that:

$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

i.e. $[3(\cos 15^\circ + i\sin 15^\circ)]^8 = 3^8(\cos 8\times 15 + i\sin 8\times 15)$

$= 6561(\cos 120 + i\sin 120)$

$= 6561\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= \frac{-6561}{2} + \frac{6561\sqrt{3}}{2} i$

$= 6561\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$

$= 3^8\left(\frac{-1 + i\sqrt{3}}{2}\right)$

Date
Page

c) $\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^{20}$

Solution

Let $z=-\frac{\sqrt{3}}{2}-\frac{1}{2}i = r(\cos\theta + i\sin\theta)$ —— ①

where, $x=-\frac{\sqrt{3}}{2}$ & $y=-\frac{1}{2}$

we know that,

$r=\sqrt{x^2+y^2}$

$=\sqrt{\left(-\frac{\sqrt{3}}{2}\right)^2+\left(-\frac{1}{2}\right)^2}$

$=\sqrt{\frac{3}{4}+\frac{1}{4}}$

$\therefore r=1$

Also,

$\tan\theta=\frac{y}{x}$

or, $\tan\theta = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

or, $\tan\theta = \frac{1}{\sqrt{3}}$

or, $\tan\theta = \tan 210^\circ$

$\therefore \theta = 210^\circ$

Now,

$z=-\frac{\sqrt{3}}{2}-\frac{1}{2}i = 1(\cos 210^\circ + i\sin 210^\circ)$

$\therefore \left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^{20} = [1(\cos 210^\circ + i\sin 210^\circ)]^{20}$

$= 1^{20}(\cos 20\times 210 + i\sin 20\times 210^\circ)$

$[\because [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)]$

$= \cos 4200^\circ + i\sin 4200^\circ$

$= -\frac{1}{2}-\frac{\sqrt{3}}{2}i$

Where $x=-\frac{1}{2}$ & $y=-\frac{\sqrt{3}}{2}$

Date
Page

d) $(1+i)^6$

Solution

Let $z = (1+i)^{6} = r(\cos\theta + i\sin\theta)$ — ①

where, $x = 1$ & $y = 1$

We know that,

$r = \sqrt{x^2+y^2}$

$= \sqrt{(1)^2+(1)^2}$

$\therefore r = \sqrt{2}$

Also,

$\tan\theta = \frac{y}{x}$

or, $\tan\theta = \frac{1}{1}$

or, $\tan\theta = 1$

or, $\tan\theta = \tan 45^\circ$

$\therefore \theta = 45^\circ$

Now,

$z = 1+i = \sqrt{2}(\cos 45^\circ + i\sin 45^\circ)$

$\therefore (1+i)^6 = [\sqrt{2}(\cos 45^\circ + i\sin 45^\circ)]^6$

$= (\sqrt{2})^6 (\cos 6\times45^\circ + i\sin 6\times45^\circ)$

$[\because [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)]$

$= 8(\cos 270^\circ + i\sin 270^\circ)$

$= 8(0 + (-1)i)$

$= -8i$

$= 0 - 8i$ is in the form of $A+iB$

where, $A = 0$, & $B = -8$

Date
Page

e) $(-1+i)^{14}$

Solution

let $z = (-1+i)^{\cancel{14}} = r(\cos\theta + i\sin\theta)$ —— (1)

where, $x = -1$ & $y = 1$

we know that,

$r = \sqrt{x^2+y^2}$

$= \sqrt{(-1)^2 + (1)^2}$

$= \sqrt{1+1}$

$= \sqrt{2}$

Also,

$\tan\theta = \frac{y}{x}$

or $\tan\theta = \frac{1}{-1}$

or $\tan\theta = -1$

or, $\tan\theta = \tan 135^\circ$

$\therefore \theta = 135^\circ$

Now,

$z = -1+i = \sqrt{2}(\cos 135^\circ + i\sin 135^\circ)$

$\therefore (-1+i)^{14} = [\sqrt{2}(\cos 135^\circ + i\sin 135^\circ)]^{14}$

$= (\sqrt{2})^{14}(\cos 14\times 135^\circ + i\sin 14\times 135^\circ)$

$[\because [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)]$

$= 128(\cos 1890^\circ + i\sin 1890^\circ)$

$= 128(0 + i\times 1)$

$= 128i$

$= 0 + 128i$ is in the form of $A + iB$

Where, $A = 0$ & $B = 128$

Date
Page

2. Find the square roots of the following using De - Moivre's theorem:

a) $4+4\sqrt{3}\,i$

Solution

Let $z = 4+4\sqrt{3}\,i$ — ① where $x=4$ & $y=4\sqrt{3}$

We know that,

$$r = \sqrt{x^2+y^2}$$
$$= \sqrt{(4)^2+(4\sqrt{3})^2}$$
$$= \sqrt{16+48}$$
$$= \sqrt{64}$$
$$\therefore r = 8$$

Also,

$$\tan\theta = \frac{y}{x}$$
$$\text{or, } \tan\theta = \frac{4\sqrt{3}}{4}$$
$$\text{or, } \tan\theta = \sqrt{3}$$
$$\text{or, } \tan\theta = \tan 60^\circ$$
$$\therefore \theta = 60^\circ$$

We know,

$$z_k = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta + k\times 360^\circ}{n}\right) + i\sin\left(\frac{\theta + k\times 360^\circ}{n}\right)\right],$$

where $k = 0, 1, 2, \ldots (n-1)$

For square root put $n=2$, $k=0, 1$

Date
Page

when $n=2, k=0, r=8$ and $\theta=60^\circ$

$$z_0 = 8^{\frac{1}{2}}\left[\cos\left(\frac{60^\circ+0\times360^\circ}{2}\right)+i\sin\left(\frac{60^\circ+0\times360^\circ}{2}\right)\right]$$

$$= 2\sqrt{2}\,(\cos 30^\circ + i\sin 30^\circ)$$

$$= 2\sqrt{2}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$$

$$\therefore z_0 = \sqrt{2}\,(\sqrt{3}+i)$$

Again,

when $n=2, K=1, r=8$ and $\theta=60^\circ$

$$z_1 = 8^{\frac{1}{2}}\left[\cos\left(\frac{60^\circ+1\times360^\circ}{2}\right)+i\sin\left(\frac{60^\circ+1\times360^\circ}{2}\right)\right]$$

$$= 2\sqrt{2}\,(\cos 210^\circ + i\sin 210^\circ)$$

$$= 2\sqrt{2}\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)$$

$$\therefore z_1 = -\sqrt{2}\,(\sqrt{3}+i)$$

Therefore,

$$\sqrt{4+4\sqrt{3}\,i} = \pm\sqrt{2}\,(\sqrt{3}+i)$$

---

b) $-1+\sqrt{3}\,i$

Solution

Let $z = -1+\sqrt{3}\,i$ — (1) where, $x=-1$ & $y=\sqrt{3}$

$$r = \sqrt{x^2+y^2}$$

$$= \sqrt{(-1)^2+(\sqrt{3})^2}$$

$$= \sqrt{1+3}$$

$$\therefore r = 2$$

Also,

$$\tan\theta = \frac{y}{x}$$

or, $\tan\theta = \frac{\sqrt{3}}{-1}$

or, $\tan\theta = -\sqrt{3}$

or, $\tan\theta = \tan 120^\circ$

$$\therefore \theta = 120^\circ$$

We know,

$$z_k = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta + k\times 360^\circ}{n}\right) + i\sin\left(\frac{\theta + k\times 360^\circ}{n}\right)\right]$$

where, $k = 0, 1, 2, \ldots (n-1)$

For square root put $k = 0, 1$

When $n = 2, k = 0, r = 2$ & $\theta = 120^\circ$

$$z_0 = 2^{\frac{1}{2}}\left[\cos\left(\frac{120 + 0\times 360^\circ}{2}\right) + i\sin\left(\frac{120 + 0\times 360^\circ}{2}\right)\right]$$

$$= \cancel{\sqrt{2}}\ \sqrt{2}\,(\cos 60^\circ + i \sin 60^\circ)$$

$$= \sqrt{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$$\therefore z_0 = \frac{1}{\sqrt{2}}(1 + \sqrt{3}i)$$

When $n = 2, k = 1, r = 2$ & $\theta = 120^\circ$

$$z_1 = 2^{\frac{1}{2}}\left[\cos\left(\frac{120^\circ + 1\times 360^\circ}{2}\right) + i\sin\left(\frac{120^\circ + 1\times 360^\circ}{2}\right)\right]$$

$$= \sqrt{2}\,(\cos 240^\circ + i\sin 240^\circ)$$

$$= \sqrt{2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$= \frac{1}{\sqrt{2}}(-1 - \sqrt{3}i)$$

$$\therefore z_1 = -\frac{1}{\sqrt{2}}(1 + \sqrt{3}i)$$

Therefore,

$$\sqrt{-1 + \sqrt{3}i} = \pm\frac{1}{\sqrt{2}}(1 + \sqrt{3}i)$$

Date
Page

c) $1+\sqrt{3}i$

Solution

Let $z = 1+\sqrt{3}i$ — ①, where $x=1, y=\sqrt{3}$

$$r = \sqrt{x^2+y^2}$$
$$= \sqrt{(1)^2+(\sqrt{3})^2}$$
$$= \sqrt{1+3}$$
$$\therefore r = \cancel{4}\ 2$$

Also,

$$\tan\theta = \frac{y}{x}$$
or, $\tan\theta = \frac{\sqrt{3}}{1}$

or, $\tan\theta = \sqrt{3}$

or, $\tan\theta = \tan 60^\circ$

$\therefore \theta = 60^\circ$

We know,

$$z_k = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta + k\times 360^\circ}{n}\right) + i\sin\left(\frac{\theta + k\times 360^\circ}{n}\right)\right]$$

where $k = 0, 1, 2, --- (n-1)$

For square root put $k = 0, 1$

When $n=2$, $k=0$, $r=2$ & $\theta = 60^\circ$

$$z_0 = 2^{\frac{1}{2}}\left[\cos\left(\frac{60^\circ + 0\times 360^\circ}{2}\right) + i\sin\left(\frac{60^\circ + 0\times 360^\circ}{2}\right)\right]$$
$$= \sqrt{2}(\cos 30^\circ + i\sin 30^\circ)$$
$$= \sqrt{2}\left(\frac{\sqrt{3}}{2} + i\times\frac{1}{2}\right)$$

$$\therefore z_0 = \frac{1}{\sqrt{2}}(\sqrt{3}+i)$$

when $n=2$, $k=1$, $r=2$ & $\theta=60^\circ$

$$z_1 = 2^{\frac{1}{2}}\left[\cos\left(\frac{60^\circ+1\times360^\circ}{2}\right)+i\sin\left(\frac{60^\circ+1\times360^\circ}{2}\right)\right]$$

$$= \sqrt{2}\,(\cos 210^\circ + i\sin 210^\circ)$$

$$= \sqrt{2}\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)$$

$$= \frac{1}{\sqrt{2}}\,(-\sqrt{3}-i)$$

$$\therefore z_1 = -\frac{1}{\sqrt{2}}\,(\sqrt{3}+i)$$

Therefore,

$$\sqrt{1+\sqrt{3}i} = \pm\frac{1}{\sqrt{2}}\,(\sqrt{3}+i)$$

d) $-1-\sqrt{3}i$

Solution

Let $z=-1-\sqrt{3}i$ —— ①, where $x=-1$, $y=-\sqrt{3}$

$$r = \sqrt{x^2+y^2}$$

$$= \sqrt{(-1)^2+(-\sqrt{3})^2}$$

$$= \sqrt{1+3}$$

$$\therefore r = 2$$

Also,

$$\tan\theta = \frac{y}{x}$$

or, $\tan\theta = \frac{-\sqrt{3}}{-1}$

or, $\tan\theta = \sqrt{3}$

or, $\tan\theta = \tan 240^\circ$

$$\therefore \theta = 240^\circ$$

Date
Page

We know,

$$z_k = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta + k \times 360^\circ}{n}\right) + i\sin\left(\frac{\theta + k\times 360^\circ}{n}\right)\right]$$

where $k = 0, 1, 2, \ldots (n-1)$

For square roots put $k = 0, 1$

When $n = 2$, $k = 0$, $r = 2$ & $\theta = 240^\circ$

$$z_0 = 2^{\frac{1}{2}}\left[\cos\left(\frac{240^\circ + 0\times 360^\circ}{2}\right) + i\sin\left(\frac{240^\circ + 0 \times 360^\circ}{2}\right)\right]$$

$$= \sqrt{2}\,(\cos 120^\circ + i \sin 120^\circ)$$

$$= \sqrt{2}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$$

$$\therefore z_0 = \frac{1}{\sqrt{2}}(-1 + \sqrt{3}\, i)$$

When $n = 2$, $k = 1$, $r = 2$ & $\theta = 240^\circ$

$$z_1 = 2^{\frac{1}{2}}\left[\cos\left(\frac{240^\circ + 1\times 360^\circ}{2}\right) + i\sin\left(\frac{240^\circ + 1\times 360^\circ}{2}\right)\right]$$

$$= \sqrt{2}\,(\cos 300^\circ + i\sin 300^\circ)$$

$$= \sqrt{2}\left(\frac{1}{2} + i\cdot\left(-\frac{\sqrt{3}}{2}\right)\right)$$

$$= \sqrt{2}\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$$

$$= \frac{1}{\sqrt{2}}(1 - \sqrt{3}\, i)$$

$$\therefore z_1 = -\frac{1}{\sqrt{2}}(-1 + \sqrt{3}\, i)$$

Therefore,

$$\sqrt{-1 - \sqrt{3}\, i} = \pm \frac{1}{\sqrt{2}}(-1 + \sqrt{3}\, i)$$

3. Find the square roots of the following (without using De-Moivre's Theorem):

a) $5+12i$

Solution

Let $\sqrt{5+12i} = x+iy$ — ①, where $a=5$ & $b=12$

We know;

$$x^2 = \frac{\sqrt{a^2+b^2}+a}{2}$$

or $$x^2 = \frac{\sqrt{(5)^2+(12)^2}+5}{2}$$

or, $$x^2 = \frac{13+5}{2}$$

or, $x^2 = 9$

$\therefore x = \pm 3$

And,

$$y^2 = \frac{\sqrt{a^2+b^2}-a}{2}$$

or, $$y^2 = \frac{\sqrt{(5)^2+(12)^2}-5}{2}$$

or, $$y^2 = \frac{13-5}{2}$$

or, $y^2 = 4$

$\therefore y = \pm 2$

Since, "b" is positive, x & y must have the same sign,

When $x=3$, $y=2$

$\sqrt{5+12i} = 3+i\times 2$

$= 3+2i$

When $x=-3, y=-2$

$$\begin{aligned}\sqrt{5+12i} &= -3+i\times(-2)\\ &= -3-2i\\ &= -(3+2i)\end{aligned}$$

Therefore, $\sqrt{5+12i} = \pm(3+2i)$

---

b) $3-4i$

Solution

Let $\sqrt{3-4i} = x+iy$ —— ①, where $a=3$ & $b=-4$

We know;

$$x^2 = \frac{\sqrt{a^2+b^2}+a}{2}$$

or, $x^2 = \frac{\sqrt{(3)^2+(-4)^2}+3}{2}$

or, $x^2 = \frac{5+3}{2}$

or, $x^2 = 4$

$\therefore x = \pm 2$

And,

$$y^2 = \frac{\sqrt{a^2+b^2}-a}{2}$$

or, $y^2 = \frac{\sqrt{(3)^2+(4)^2}-3}{2}$

or, $y^2 = \frac{5-3}{2}$

or, $y^2 = 1$

$\therefore y = \pm 1$

Date
Page

Since, "b" is negative, x & y must have the opposite sign.

When $x = 2, y = -1$

$\sqrt{3-4i} = 2 + i \times (-1)$
$= 2 - i$

When $x = -2, y = 1$

$\sqrt{3-4i} = -2 + i \times 1$
$= -2 + i$
$= -(2 - i)$

Therefore, $\sqrt{3-4i} = \pm (2 - i)$

---

c) $-i$

Solution

Let $\sqrt{-i} = x + iy$ — ①, where $a = 0$ & $b = -1$

we know;

$$x^2 = \frac{\sqrt{a^2+b^2} + a}{2}$$

or, $x^2 = \frac{\sqrt{0^2+(-1)^2} + 0}{2}$

or, $x^2 = \frac{1}{2}$

$\therefore x = \pm \frac{1}{\sqrt{2}}$

And,

$$y^2 = \frac{\sqrt{a^2+b^2} - a}{2}$$

or, $y^2 = \frac{\sqrt{(0)^2+(-1)^2} - 0}{2}$

or, $y^2 = \frac{1}{2}$

$\therefore y = \pm \frac{1}{\sqrt{2}}$

Since, "b" is negative. x & y must have the opposite sign.

when $x = \frac{1}{\sqrt{2}}$, $y = -\frac{1}{\sqrt{2}}$,

$$\sqrt{-i} = \frac{1}{\sqrt{2}} + i \times \left(-\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i$$

when $x = -\frac{1}{\sqrt{2}}$, $y = \frac{1}{\sqrt{2}}$

$$\sqrt{-i} = -\frac{1}{\sqrt{2}} + i \times \frac{1}{\sqrt{2}}$$

$$= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$$

$$= -\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i\right)$$

Therefore, $\sqrt{-i} = \pm\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i\right)$

---

d) $\frac{5+12i}{3-4i}$

Let $z = \frac{5+12i}{3-4i}$

$$= \frac{5+12i}{3-4i} \times \frac{3+4i}{3+4i}$$

$$= \frac{15+20i+36i+48i^2}{(3)^2-(4i)^2}$$

$$= \frac{15+56i-48}{9-16i^2} \quad [\because i^2 = -1]$$

$$= \frac{56i-33}{9+16}$$

Date
Page

$= \frac{56i - 33}{25}$

$= -\frac{33}{25} + \frac{56}{25}i$

let $\sqrt{-\frac{33}{25} + \frac{56}{25}i} = x + iy$ — ①, where $a = -\frac{33}{25}$ & $b = \frac{56}{25}$

We know,

$x^2 = \frac{\sqrt{a^2 + b^2} + a}{2}$

or, $x^2 = \frac{\sqrt{\left(-\frac{33}{25}\right)^2 + \left(\frac{56}{25}\right)^2} + \left(-\frac{33}{25}\right)}{2}$

or, $x^2 = \frac{\frac{13}{5} - \frac{33}{25}}{2}$

or, $x^2 = \frac{\frac{32}{25}}{2}$

or, $x^2 = \frac{16}{25}$

$\therefore x = \pm \frac{4}{5}$

And,

$y^2 = \frac{\sqrt{a^2 + b^2} - a}{2}$

or, $y^2 = \frac{\sqrt{\left(-\frac{33}{25}\right)^2 + \left(\frac{56}{25}\right)^2} - \left(-\frac{33}{25}\right)}{2}$

or, $y^2 = \frac{\frac{13}{5} + \frac{33}{25}}{2}$

or, $y^2 = \frac{\frac{98}{25}}{2}$

or, $y^2 = \frac{49}{25}$

$\therefore y = \pm \frac{7}{5}$

Since "b" is positive, x & y must have the same sign.

when $x = \frac{4}{5}$, $y = \frac{7}{5}$,

$$\sqrt{-\frac{33}{25} + \frac{56}{25}i} = \frac{4}{5} + i \times \frac{7}{5}$$

$$= \frac{4}{5} + \frac{7}{5}i$$

when $x = -\frac{4}{5}$, $y = -\frac{7}{5}$,

$$\sqrt{-\frac{33}{25} + \frac{56}{25}i} = -\frac{4}{5} + i \times \left(-\frac{7}{5}\right)$$

$$= -\left(\frac{4}{5} + \frac{7}{5}i\right)$$

Therefore, $\sqrt{-\frac{33}{25} + \frac{56}{25}i} = \pm\left(\frac{4}{5} + \frac{7}{5}i\right)$

---

e) $-5 + 12i$

Solution

Let $\sqrt{-5+12i} = x + iy$ — (1), where $a = -5$ & $b = 12$

we know;

$$x^2 = \frac{\sqrt{a^2+b^2} + a}{2}$$

or, $x^2 = \frac{\sqrt{(-5)^2 + (12)^2} + (-5)}{2}$

or, $x^2 = \frac{13 - 5}{2}$

or, $x^2 = \frac{8}{2}$

or, $x^2 = \cancel{2\sqrt{2}}\ 4$

$\therefore x = \pm \cancel{\sqrt{2}}\ 2$

And,

$$y^2 = \frac{\sqrt{a^2+b^2} - a}{2}$$

or, $$y^2 = \frac{\sqrt{(-5)^2 + (12)^2} - (-5)}{2}$$

or, $$y^2 = \frac{13+5}{2}$$

or, $y^2 = 9$

$\therefore y = \pm 3$

Since, "b" is positive, x & y must have the same sign.

When $x = \cancel{\sqrt{2}}\,2$, $y = 3$,

$$\sqrt{-5+12i} = \cancel{\sqrt{2}}\,2 + i \times 3$$
$$= \cancel{\sqrt{2}}\,2 + 3i$$

When $x = \cancel{\sqrt{2}}\,-2$, $y = -3$,

$$\sqrt{-5+12i} = \cancel{\sqrt{2}}\,-2 + i \times (-3)$$
$$= \cancel{\sqrt{2} - 3i}\ -2-3i$$
$$= -(\cancel{\sqrt{2}}\,2 + 3i)$$

Therefore, $\sqrt{-5+12i} = \cancel{\pm(\sqrt{2}+3i)}\ \pm(2+3i)$

b) $-8+6i$

Solution

Let $\sqrt{-8+6i} = x + iy$ — ①, where, $a = -8$ & $b = 6$

We know;

$$x^2 = \frac{\sqrt{a^2+b^2} + a}{2}$$

or, $$x^2 = \frac{\sqrt{(-8)^2 + (6)^2} + (-8)}{2}$$

or, $$x^2 = \frac{10-8}{2}$$

$\therefore x = \pm 1$

Date
Page

And,

$$y^2 = \frac{\sqrt{a^2+b^2} - a}{2}$$

or, $$y^2 = \frac{\sqrt{(-8)^2+(6)^2} - (-8)}{2}$$

or $$y^2 = \frac{10+8}{2}$$

or $y^2 = 9$

$\therefore$ $y = \pm 3$

Since, "b" is positive, x & y must have the same sign,

When $x = 1$, $y = 3$,

$$\sqrt{-8+6i} = 1 + i \times 3$$
$$= 1 + 3i$$

When $x = -1$, $y = -3$

$$\sqrt{-8+6i} = -1 + i \times (-3)$$
$$= -1 - 3i$$
$$= -(1+3i)$$

Therefore, $\sqrt{-8+6i} = \pm(1+3i)$

Date
Page

4. If ω be a complex cube root of unity. Show that:

a) $\omega^{302} = \omega^2$

Solution

$$\begin{aligned} LHS &= \omega^{302} \\ &= (\omega^3)^{100} \cdot \omega^2 \\ &= (1)^{100} \cdot \omega^2 \quad [\because \omega^3 = 1] \\ &= 1 \cdot \omega^2 \\ &= \omega^2 \ RHS. \end{aligned}$$

$\therefore$ LHS = RHS proved

b) $\omega^{-200} = \omega$

Solution

$$\begin{aligned} LHS &= \omega^{-200} \\ &= \frac{1}{\omega^{200}} \\ &= \frac{1}{\omega^{198} \cdot \omega^2} \times \frac{\omega}{\omega} \\ &= \frac{\omega}{(\omega^3)^{66} \cdot \omega^3} \\ &= \frac{\omega}{(1)^{66} \cdot \cancel{\omega^3}\,1} \quad [\because \omega^3 = 1] \\ &\quad \omega \end{aligned}$$

$\therefore$ LHS = RHS proved.

c) $(1+\omega^2)^3 - (1+\omega)^3 = 0$

Solution

We have,

$$1 + \omega + \omega^2 = 0$$

$$\therefore 1 + \omega = -\omega^2$$

$$\therefore 1 + \omega^2 = -\omega$$

Now,

$LHS = (1+\omega^2)^3 - (1+\omega)^3$
$= (-\omega)^3 - (-\omega^2)^3$
$= -\omega^3 - (-\omega^6)$
$= -\omega^3 + \omega^6$
$= -\omega^3 + \omega^3 \cdot \omega^3$
$= -1 + 1 \quad [\because \omega^3 = 1]$
$= 0$ RHS

$\therefore$ LHS = RHS proved.

d) $(1-\omega+\omega^2)^4 + (1+\omega-\omega^2)^4 = -16$

Solution

we have,

$1+\omega+\omega^2 = 0$
$\therefore 1+\omega = -\omega^2$
$\therefore 1+\omega^2 = -\omega$

Now,

$LHS = (1-\omega+\omega^2)^4 + (1+\omega-\omega^2)^4$
$= (1+\omega^2-\omega)^4 + (1+\omega-\omega^2)^4$
$= (-\omega-\omega)^4 + (-\omega^2-\omega^2)^4$
$= (-2\omega)^4 + (-2\omega^2)^4$
$= 16\omega^4 + 16\omega^8$
$= 16\omega^3 \cdot \omega + 16\omega^3 \cdot \omega^3 \cdot \omega^2$
$= 16\omega + 16\omega^2$
$= 16(\omega+\omega^2)$
$= 16 \times (-1)$
$= -16$ RHS

$\therefore$ LHS = RHS proved.

e) $(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11}) = 49$

Solution

$$LHS = (2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11})$$
$$= (2-\omega)(2-\omega^2)(2-\omega^3.\omega^3.\omega^3.\omega)(2-\omega^3.\omega^3.\omega^3.\omega^2)$$
$$= (2-\omega)(2-\omega^2)(2-\omega)(2-\omega^2)$$
$$= (2-\omega)(2-\omega)(2-\omega^2)(2-\omega^2)$$
$$= \{(2-\omega)\}^2 \{(2-\omega^2)\}^2$$
$$= (4-4\omega+\omega^2)(4-4\omega^2+\omega^4)$$
$$= (4-4\omega+\omega^2)(4-4\omega^2+\omega)$$
$$\cancel{= (1+3-4\omega}$$
$$= (3+1-4\omega+\omega^2)(3+1-4\omega^2+\omega)$$
$$= (1+\omega^2+3-4\omega)(1+\omega+3-4\omega^2)$$
$$= (-\omega+3-4\omega)(-\omega^2+3-4\omega^2)$$
$$= (3-5\omega)(3-5\omega^2)$$
$$= 9-15\omega^2-15\omega+25\omega^3$$
$$= 9-15\omega^2-15\omega+25$$
$$= 34-15\omega^2-15\omega$$
$$= 34-15(\omega^2+\omega)$$
$$= 34-15(1+\omega+\omega^2-1)$$
$$= 34-15(-1)$$
$$= 34+15$$
$$= 49 \text{ RHS}$$

$\therefore$ LHS = RHS proved.

---

f) $(1+\omega-\omega^2)^3 - (1-\omega+\omega^2)^3 = 0$

Solution

$$LHS = (1+\omega-\omega^2)^3 - (1-\omega+\omega^2)^3$$
$$= (1+\omega+\omega^2-2\omega^2)^3 - (1+\omega+\omega^2-2\omega)^3$$
$$= (-2\omega^2)^3 - (-2\omega)^3$$
$$= -8\omega^6 - (-8\omega^3)$$
$$= -8\omega^6 + 8\omega^3$$
$$= -8\omega^3.\omega^3 + 8\omega^3$$

Date
Page

$= -8+8$

$= 0$ RHS

$\therefore$ LHS = RHS proved.

g) $(2+\omega+\omega^2)^3 + (1+\omega-\omega^2)^8 - (1-3\omega+\omega^2)^4 = 1$

Solution

LHS $= (2+\omega+\omega^2)^3 + (1+\omega-\omega^2)^8 - (1-3\omega+\omega^2)^4$

$= (2-1)^3 + (-\omega^2-\omega^2)^8 - (1-3\omega-1-\omega)^4$ $[\because 1+\omega+\omega^2=0]$

$= (1)^3 + (-2\omega^2)^8 - (-4\omega)^4$

$= 1 + 256\omega^{16} - 256\omega^4$

$= 1 + 256(\omega^3)^5 \cdot \omega - 256\omega^3 \cdot \omega$

$= 1 + 256\omega - 256\omega$ $[\because \omega^3 = 1]$

$= 1$ RHS

$\therefore$ LHS = RHS proved.

h) $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8) = 9$

Solution

LHS $= (1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8)$

$= (1-\omega)(1-\omega^2)(1-\omega^3 \cdot \omega)(1-\omega^3 \cdot \omega^3 \cdot \omega^2)$

$= (1-\omega)(1-\omega^2)(1-\omega)(1-\omega^2)$ $[\because \omega^3 = 1]$

$= (1-\omega)(1-\omega)(1-\omega^2)(1-\omega^2)$

$= \{(1-\omega)\}^2 \{(1-\omega^2)\}^2$

$= \{1-2\omega+\omega^2\}\{1-2\omega^2+\omega^4\}$

$= \{1-2\omega+\omega^2\}\{1-2\omega^2+\omega^3 \cdot \omega\}$

$= (1-2\omega+\omega^2)(1-2\omega^2+\omega)$

$= (1+\omega^2-2\omega)(1+\omega-2\omega^2)$

$= (-\omega-2\omega)(-\omega^2-2\omega^2)$ $[\because 1+\omega+\omega^2=0]$

$= (-3\omega)(-3\omega^2)$

$= 9\omega^3$

$= 9 \times 1$

$= 9$ RHS

$\therefore$ LHS = RHS proved.

i) $\frac{a+b\omega+c\omega^2}{a\omega+b\omega^2+c} + \frac{a+b\omega+c\omega^2}{a\omega^2+b+c\omega} = 1$

Solution

$$LHS = \frac{a+b\omega+c\omega^2}{a\omega+b\omega^2+c} + \frac{a+b\omega+c\omega^2}{a\omega^2+b+c\omega}$$

$$= \frac{(a+b\omega+c\omega^2)(a\omega^2+b+c\omega)+(a+b\omega+c\omega^2)(a\omega+b\omega^2+c)}{(a\omega+b\omega^2+c)(a\omega^2+b+c\omega)}$$

$$= \frac{(a+b\omega+c\omega^2)[a\omega^2+b+c\omega+a\omega+b\omega^2+c]}{a^2\omega^3+ab\omega+ac\omega^2+ab\omega^4+b^2\omega^2+bc\omega^3+ac\omega^2+bc+c^2\omega}$$

$$= \frac{(a+b\omega+c\omega^2)[a(\omega^2+\omega)+b(1+\omega^2)+c(\omega+1)]}{a^2+ab\omega+ac\omega^2+ab\omega+b^2\omega^2+bc+ac\omega^2+bc+c^2\omega} \quad [\because \omega^3=1]$$

$$= \frac{(a+b\omega+c\omega^2)(-a-b\omega-c\omega^2)}{a^2+2ab\omega+2ac\omega^2+b^2\omega^2+2bc+c^2\omega} \quad [\because 1+\omega+\omega^2=0]$$

$$= \frac{-[(a+b\omega+c\omega^2)(a+b\omega+c\omega^2)]}{(a^2+2ab\omega+2ac\omega^2+b^2\omega^2+2bc+c^2\omega)}$$

$$= \frac{-[a^2+ab\omega+ac\omega^2+ab\omega+b^2\omega^2+bc\omega^3+ac\omega^2+bc\omega^3+c^2\omega^4]}{(a^2+2ab\omega+2ac\omega^2+b^2\omega^2+2bc+c^2\omega)}$$

$$= \frac{-[a^2+2ab\omega+2ac\omega^2+b^2\omega^2+2bc+c^2\omega]}{[a^2+2ab\omega+2ac\omega^2+b^2\omega^2+2bc+c^2\omega]}$$

$= -1$ RHS

$\therefore$ LHS = RHS proved.

Date
Page

5. Prove that:

a) $\left(\frac{-1+\sqrt{-3}}{2}\right)^9 + \left(\frac{-1-\sqrt{-3}}{2}\right)^6 = 2$

Solution

Let $\omega = \left(\frac{-1+i\sqrt{3}}{2}\right)$ & $\omega^2 = \left(\frac{-1-i\sqrt{3}}{2}\right)$

$$LHS = \left(\frac{-1+\sqrt{-3}}{2}\right)^9 + \left(\frac{-1-\sqrt{-3}}{2}\right)^6$$

$$= \left(\frac{-1+\sqrt{3\times-1}}{2}\right)^9 + \left(\frac{-1-\sqrt{3\times-1}}{2}\right)^6$$

$$= \left(\frac{-1+\sqrt{3i^2}}{2}\right)^9 + \left(\frac{-1-\sqrt{3i^2}}{2}\right)^6$$

$$= \left(\frac{-1+i\sqrt{3}}{2}\right)^9 + \left(\frac{-1-i\sqrt{3}}{2}\right)^6$$

$$= \omega^9 + (\omega^2)^6$$

$$= (\omega^3)^3 + \omega^{12}$$

$$= (\omega^3)^3 + (\omega^3)^4$$

$$= (1)^3 + (1)^4 \quad [\because \omega^3 = 1]$$

$$= 1 + 1$$

$$= 2 \text{ RHS}$$

$\therefore$ LHS = RHS proved.

b) $\left(\frac{-1+\sqrt{-3}}{2}\right)^4 + \left(\frac{-1-\sqrt{-3}}{2}\right)^4 = -1$

Solution

Let $\omega = \frac{-1+i\sqrt{3}}{2}$ & $\omega^2 = \frac{-1-i\sqrt{3}}{2}$

Date
Page

$$LHS = \left(\frac{-1+\sqrt{-3}}{2}\right)^4 + \left(\frac{-1-\sqrt{-3}}{2}\right)^4$$

$$= \left(\frac{-1+\sqrt{3\times-1}}{2}\right)^4 + \left(\frac{-1-\sqrt{3\times-1}}{2}\right)^4$$

$$= \left(\frac{-1+\sqrt{3i^2}}{2}\right)^4 + \left(\frac{-1-\sqrt{3i^2}}{2}\right)^4$$

$$= \left(\frac{-1+i\sqrt{3}}{2}\right)^4 + \left(\frac{-1-i\sqrt{3}}{2}\right)^4$$

$$= \omega^4 + (\omega^2)^4$$

$$= \omega^4 + \omega^8$$

$$= \omega^3 \cdot \omega + \omega^3 \cdot \omega^3 \cdot \omega^2$$

$$= \omega + \omega^2 \quad [\because \omega^3 = 1]$$

$$= -1 \quad [\because 1+\omega+\omega^2 = 0,\ \omega+\omega^2 = -1]$$

$\therefore$ LHS = RHS proved.

---

c) $\left(\frac{-1+\sqrt{-3}}{2}\right)^6 + \left(\frac{-1-\sqrt{-3}}{2}\right)^9 = 2$

Solution

Let $\omega = \frac{-1+i\sqrt{3}}{2}$ & $\omega^2 = \frac{-1-i\sqrt{3}}{2}$

$$LHS = \left(\frac{-1+\sqrt{-3}}{2}\right)^6 + \left(\frac{-1-\sqrt{-3}}{2}\right)^9$$

$$= \left(\frac{-1+\sqrt{3\times-1}}{2}\right)^6 + \left(\frac{-1-\sqrt{3\times-1}}{2}\right)^9$$

$$= \left(\frac{-1+\sqrt{3i^2}}{2}\right)^6 + \left(\frac{-1-\sqrt{3i^2}}{2}\right)^9$$

$$= \left(\frac{-1+i\sqrt{3}}{2}\right)^6 + \left(\frac{-1-i\sqrt{3}}{2}\right)^9$$

$$= \omega^6 + (\omega^2)^9$$

$$= \omega^6 + \omega^{18}$$

Date
Page

$= (\omega^3)^2 + (\omega^3)^6$

$= (1)^2 + (1)^6 \quad [\because \omega^3 = 1]$

$= 1 + 1$

$= 2$ RHS

$\therefore$ LHS = RHS proved.

d) $\left(\frac{-1+\sqrt{-3}}{2}\right)^5 + \left(\frac{-1-\sqrt{-3}}{2}\right)^5 = -1$

Solution

let $\omega = \frac{-1+i\sqrt{3}}{2}$ & $\omega^2 = \frac{-1-i\sqrt{3}}{2}$

LHS $= \left(\frac{-1+\sqrt{-3}}{2}\right)^5 + \left(\frac{-1-\sqrt{-3}}{2}\right)^5$

$= \left(\frac{-1+\sqrt{3\times-1}}{2}\right)^5 + \left(\frac{-1-\sqrt{3\times-1}}{2}\right)^5$

$= \left(\frac{-1+\sqrt{3i^2}}{2}\right)^5 + \left(\frac{-1-\sqrt{3i^2}}{2}\right)^5$

$= \left(\frac{-1+i\sqrt{3}}{2}\right)^5 + \left(\frac{-1-i\sqrt{3}}{2}\right)^5$

$= \omega^5 + (\omega^2)^5$

$= \omega^5 + \omega^{10}$

$= \omega^3 . \omega^2 + \omega^3 . \omega^3 . \omega^3 . \omega$

$= \omega^2 + \omega \quad [\because \omega^3 = 1]$

$= -1 \quad [\because 1+\omega+\omega^2 = 0, \ \omega+\omega^2 = -1]$

$\therefore$ LHS = RHS proved.

Signature of Subject Teacher:

Signature of Director: