Chapter-2

Complex Numbers

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Homework

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Exercise 2(A)

1) Simplify:

a) $\frac{3-\sqrt{-4}}{2+\sqrt{-1}}$

Here,

let $z = \frac{3-\sqrt{-4}}{2+\sqrt{-1}}$

$= \frac{3-\sqrt{-1\times 4}}{2+\sqrt{-1\times 1}}$

$= \frac{3-\sqrt{4i^2}}{2+\sqrt{i^2}}$ $[\because i^2 = -1]$

$= \frac{3-2i}{2+i}$

$= \frac{3-2i}{2+i} \times \frac{2-i}{2-i}$

$= \frac{6-3i-4i+2i^2}{(2)^2-(i)^2}$

$= \frac{6-7i-2}{4+1}$

$= \frac{4-7i}{5}$

$= \frac{4}{5} + i\left(-\frac{7}{5}\right)$ which is in the form of $A+iB$

where, $A = \frac{4}{5}$ and $B = -\frac{7}{5}$

b) $\dfrac{2-\sqrt{-25}}{1-\sqrt{-16}}$

Here,

let $z = \dfrac{2-\sqrt{-25}}{1-\sqrt{-16}}$

$= \dfrac{2-\sqrt{-1\times25}}{1-\sqrt{-1\times16}}$

$= \dfrac{2-\sqrt{25i^2}}{1-\sqrt{16i^2}}$ $[\because i^2=-1]$

$= \dfrac{2-5i}{1-4i}$

$= \dfrac{2-5i}{1-4i} \times \dfrac{1+4i}{1+4i}$

$= \dfrac{2+8i-5i-20i^2}{(1)^2-(4i)^2}$

$= \dfrac{2+3i+20}{1-16i^2}$

$= \dfrac{22+3i}{1+16}$

$= \dfrac{22}{17} + i.\dfrac{3}{16}$ which is in the form of $A+iB$

where $A = \dfrac{22}{17}$ & $B = \dfrac{3}{16}$

c) $\sqrt{\dfrac{1+i}{1-i}}$

Here,

Let $z = \sqrt{\dfrac{1+i}{1-i}}$

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$$= \sqrt{\frac{1+i}{1-i} \times \frac{1+i}{1+i}}$$

$$= \sqrt{\frac{(1+i)^2}{(1)^2-(i)^2}}$$

$$= \frac{1+i}{\sqrt{1-i^2}}$$

$$= \frac{1+i}{\sqrt{1+1}}$$

$$= \frac{1+i}{\sqrt{2}}$$

$= \frac{1}{\sqrt{2}} + i.\frac{1}{\sqrt{2}}$, which is in the form of $A+iB$

Where $A = \frac{1}{\sqrt{2}}$ & $B = \frac{3}{16}$

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2. Express each of the following in the form $A+iB$.

a) $(3-4i)(2-i)$

Here,

let $z = (3-4i)(2-i)$

$= 6-3i-8i+4i^2$

$= 6-11i+4\times(-1)$ $[\because i^2=-1]$

$= 6-11i-4$

$= 2-11i$, which is in the form of $A+iB$

where, $A=2$, & $B=-11$

b) $\frac{1+2i}{3+2i}$

Here,

let $z = \frac{1+2i}{3+2i}$

$= \frac{1+2i}{3+2i} \times \frac{3-2i}{3-2i}$

$= \frac{3-2i+6i-4i^2}{(3)^2-(2i)^2}$

$= \frac{3+4i-4\times(-1)}{9-4i^2}$ $[\because i^2 = -1]$

$= \frac{3+4i+4}{9+4}$

$= \frac{7+4i}{13}$

$= \frac{7}{13} + i.\frac{4}{13}$ which is in the form $A+iB$

where, $A = \frac{7}{13}$ & $B = \frac{4}{13}$

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c) $\frac{i}{2+i}$

Here,

let $z = \frac{i}{2+i}$

$= \frac{i}{2+i} \times \frac{2-i}{2-i}$

$= \frac{2i-i^2}{4-i^2}$

$= \frac{2i+1}{4+1}$

$= \frac{2i+1}{5}$

$= \frac{1}{5} + i \cdot \frac{2}{5}$ which is in the form $A + iB$

where, $A = \frac{1}{5}$ & $B = \frac{2}{5}$.

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d) $\frac{(1,1)^3}{(3,4)}$

Here,

Let $z = \frac{(1,1)^3}{(3,4)}$

$= \frac{(1+i)^3}{3+4i}$

$= \frac{1^3 + i^3 + 3.1^2.i + 3.1.i^2}{3+4i}$

$= \frac{1 - i + 3i + 3i^2}{3+4i}$

$= \frac{1 + 2i + 3i^2}{3+4i}$

$= \frac{1 + 2i - 3}{3+4i}$

$= \frac{-2 + 2i}{3+4i}$

$= \frac{-2+2i}{3+4i} \times \frac{3-4i}{3-4i}$

$= \frac{-6 + 8i + 6i - 8i^2}{9 - 16i^2}$

$= \frac{-6 + 14i + 8}{9 + 16}$

$= \frac{2 + 14i}{25}$

$= \frac{2}{25} + i \cdot \frac{14}{25}$ which is in the form $A + iB$

Where, $A = \frac{2}{25}$ & $B = \frac{14}{25}$.

3. Find the conjugates and multiplicative inverse of the following complex numbers:

a) $(-2+i)^2$

Let $z = (-2+i)^2$

$z = (-2+i)^2$
$= 4-4i+i^2$
$= 4-4i-1 \; [\because i^2=-1]$
$= 3-4i$

Conjugate of z i.e. $\bar{z} = \overline{3-4i}$
$= 3+4i$

Also,
The multiplicative inverse of z i.e. $z^{-1} = \frac{1}{z}$

$\cancel{\frac{1}{3+4i}}$

$= \frac{1}{(-2+i)^2}$

$= \frac{1}{(-2+i)^2} \times \frac{(-2-i)^2}{(-2-i)^2}$

$= \frac{(-2-i)^2}{[(-2)^2-(i)^2]^2}$

$= \frac{4+4i+i^2}{(4-i^2)^2}$

$= \frac{3+4i}{25}$

$= \frac{3}{25} + \frac{4}{25}i$

b) $\frac{1}{(2-3i)^2}$

Let $z = \frac{1}{(2-3i)^2}$

$$z = \frac{1}{(2-3i)^2} \times \frac{(2+3i)^2}{(2+3i)^2}$$

$$= \frac{(2+3i)^2}{[(2)^2-(3i)^2]^2}$$

$$= \frac{4+12i+9i^2}{[4-9i^2]^2}$$

$$= \frac{-5+12i}{169}$$

Conjugate of z i.e. $\bar{z} = \overline{\frac{-5}{169} + \frac{12i}{169}}$

$$= -\frac{5}{169} - \frac{12i}{169}$$

Also,

The multiplicative inverse of z i.e. $z^{-1} = \frac{1}{z}$

$$= \frac{1}{\frac{1}{(2-3i)^2}}$$

$$= (2-3i)^2$$

$$= 4-12i+9i^2$$

$$= -5-12i$$

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c) $\frac{3(2+i)}{(1+2i)(1-i)}$

let $z = \frac{3(2+i)}{(1+2i)(1-i)}$

$z = \frac{6+3i}{1-i+2i-2i^2}$

$= \frac{6+3i}{3+i}$

$= \frac{6+3i}{3+i} \times \frac{3-i}{3-i}$

$= \frac{18-6i+9i-3i^2}{9-i^2}$

$= \frac{18+3i+3}{9+1}$

$= \frac{21+3i}{10}$

$= \frac{21}{10} + \frac{3i}{10}$

Conjugate of z i.e. $\bar{z} = \overline{\frac{21}{10} + \frac{3i}{10}}$

$= \frac{21}{10} - \frac{3i}{10}$

Also,
The multiplicative inverse of z i.e. $z^{-1} = \frac{1}{z}$

$= \frac{1}{\frac{6+3i}{3+i}}$

$= \frac{3+i}{6+3i}$

$= \frac{3+i}{6+3i} \times \frac{6-3i}{6-3i}$

$= \frac{18-9i+6i-3i^2}{(6)^2-(3i)^2}$

$= \frac{18-3i+3}{36-9i^2}$

$= \frac{21-3i}{36+9}$

$= \frac{21}{45} - \frac{3i}{45}$

4. Find the absolute value of the following :

a) $1+i\sqrt{3}$

Let $z = 1+i\sqrt{3}$

The modulus or absolute value of z i.e. $|z| = |1+i\sqrt{3}|$

$= \sqrt{(1)^2+(\sqrt{3})^2}$

$= \sqrt{1+3}$

$= 2$

b) $(3-2i)(2-3i)$

Let $z = (3-2i)(2-3i)$

$= 6-9i-4i+6i^2$

$= 6-9i-4i-6$

$= -13i$

$= 0-13i$

The modulus or absolute value of z i.e. $|z| = |0-13i|$

$= \sqrt{(0)^2+(-13)^2}$

$= \sqrt{169}$

$= 13$

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c) $\frac{(3+i)(1+2i)}{(2-i)(2+3i)}$

Here,

Let $z = \frac{(3+i)(1+2i)}{(2-i)(2+3i)}$

$= \frac{3+6i+i+2i^2}{4+6i-2i-3i^2}$

$= \frac{3+7i-2}{4+4i+3}$

$= \frac{1+7i}{7+4i}$

$= \frac{1+7i}{7+4i} \times \frac{7-4i}{7-4i}$

$= \frac{7-4i+49i-28i^2}{49-16i^2}$

$= \frac{7+45i+28}{49+16}$

$= \frac{35+45i}{65}$

$= \frac{35}{65} + \frac{45i}{65}$

The modulus or absolute value of z i.e. $|z| = \left|\frac{35}{65} + \frac{45}{65}i\right|$

$= \sqrt{\left(\frac{35}{65}\right)^2 + \left(\frac{45}{65}\right)^2}$

$= \sqrt{\frac{49}{169} + \frac{81}{169}}$

$= \sqrt{\frac{130}{169}}$

$= \sqrt{\frac{10}{13}}$

5. If $x = 3-2i$, $y = 2+3i$, find the value of

$5(x^2+xy+y^2)$

Here,

$x = 3-2i$

$y = 2+3i$

$$5(x^2+xy+y^2) = 5(x^2+2xy+y^2-xy)$$

$$x^2 = (3-2i)^2 = 9-12i+4i^2$$
$$= 9-12i-4 \quad [\because i^2=-1]$$
$$= 5-12i$$

$$y^2 = (2+3i)^2 = 4+12i+9i^2$$
$$= 4+12i-9 \quad [\because i^2=-1]$$
$$= -5+12i$$

$$xy = (3-2i)(2+3i)$$
$$= 6+9i-4i-6i^2$$
$$= 6+5i+6$$
$$= 12+5i$$

Then,

$$5(x^2+xy+y^2) = 5(\cancel{5}-\cancel{12i}+12+5i-\cancel{5}+\cancel{12i})$$
$$= 5(12+5i)$$
$$= 60+25i$$

$$\therefore 5(x^2+xy+y^2) = 5(12+5i)$$

6. If $\sqrt{a-ib} = x-iy$, prove that, $\sqrt{a+ib} = x+iy$.

Solution

Here,

$$\sqrt{a-ib} = x-iy$$

Squaring on both sides

$$(\sqrt{a-ib})^2 = (x-iy)^2$$

or, $a-ib = x^2 - 2ixy + i^2y^2$

or, $a-ib = x^2 - 2ixy - y^2$

or, $a-ib = x^2 - y^2 - 2ixy$.

Equating real & imaginary parts

$a = x^2 - y^2$, $b = 2xy$

Now,

$$\begin{aligned} LHS &= \sqrt{a+ib} \\ &= \sqrt{x^2-y^2+2ixy} \\ &= \sqrt{x^2+2ixy-y^2} \\ &= \sqrt{x^2+2ixy+i^2y^2} \\ &= \sqrt{(x+iy)^2} \\ &= x+iy \\ &= RHS \text{ proved} \end{aligned}$$

7. If $x-iy = \frac{2+3i}{2-3i}$, prove that, $x^2+y^2 = 1$.

Solution

Here,

$$\begin{aligned} x-iy &= \frac{2+3i}{2-3i} \\ &= \frac{2+3i}{2-3i} \times \frac{2+3i}{2+3i} \\ &= \frac{4+12i+9i^2}{4-9i^2} \end{aligned}$$

$$= \frac{4+12i-9}{4+9}$$

$$= \frac{-5+12i}{13}$$

Comparing real and imaginary part

$$x = -\frac{5}{13}, \ y = -\frac{12}{13}$$

Now,

$$LHS = x^2 + y^2$$

$$= \left(-\frac{5}{13}\right)^2 + \left(-\frac{12}{13}\right)^2$$

$$= \left(\frac{25}{169}\right) + \left(\frac{144}{169}\right)$$

$$= \frac{169}{169}$$

$$= 1$$

= RHS proved

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8. If $\frac{1-ix}{1+ix} = a-ib$, prove that, $a^2+b^2 = 1$.

Solution

Here,

$$a-ib = \frac{1-ix}{1+ix}$$

$$= \frac{1-ix}{1+ix} \times \frac{1-ix}{1-ix}$$

$$= \frac{1-2ix+i^2x^2}{1-i^2x^2}$$

$$= \frac{1-2ix-x^2}{1+x^2}$$

$$= \frac{1-x^2}{1+x^2} - \frac{2ix}{1+x^2}$$

Comparing real and imaginary parts,

$$a = \frac{1-x^2}{1+x^2} \quad \& \quad b = \frac{2x}{1+x^2}$$

Now,

$$LHS = a^2 + b^2$$
$$= \frac{(1-x^2)^2}{(1+x^2)^2} + \frac{(2x)^2}{(1+x^2)^2}$$
$$= \frac{(1-x^2)^2 + (2x)^2}{(1+x^2)^2}$$
$$= \frac{1-2x^2+x^4+4x^2}{1+2x^2+x^4}$$
$$= \frac{1+2x^2+x^4}{1+2x^2+x^4}$$
$$= 1$$
$= RHS$ proved.

9. If $x+iy = \sqrt{\frac{1+i}{1-i}}$, prove that $x^2+y^2 = 1$

Solution

Here,

$$x+iy = \sqrt{\frac{1+i}{1-i}}$$
$$= \sqrt{\frac{1+i}{1-i} \times \frac{1+i}{1+i}}$$
$$= \sqrt{\frac{(1+i)^2}{1-i^2}}$$

$= \sqrt{\dfrac{1+2i+i^2}{1+1}}$

$= \sqrt{\dfrac{2i}{2}}$

$= \sqrt{i}$

$\therefore x+iy = \sqrt{i}$

Squaring both sides,

$(x+iy)^2 = i$

or $x^2 + 2x.iy. + i^2y^2 = i$

$\therefore x+iy = \dfrac{\sqrt{2i}}{\sqrt{2}}$

<u>Equating real & imaginary parts</u>

$x = 0,\ y = \dfrac{\sqrt{2}}{\sqrt{2}} = 1.$

Now,

LHS $= x^2 + y^2$

$= 0^2 + 1^2$

$= 1$

$=$ RHS proved.

10. If $x - iy = \dfrac{a-bi}{a+bi}$, prove that, $x^2 + y^2 = 1$

Solution

Here,

$$x - iy = \frac{a-bi}{a+bi}$$

$$= \frac{a-bi}{a+bi} \times \frac{a-bi}{a-bi}$$

$$= \frac{(a-bi)^2}{(a)^2 - (bi)^2}$$

$$= \frac{a^2 - 2abi + b^2 i^2}{a^2 - b^2 i^2}$$

$$= \frac{a^2 - 2abi - b^2}{a^2 + b^2}$$

$$= \frac{a^2 - b^2 - 2abi}{a^2 + b^2}$$

~~Ea~~ Comparing real & imaginary parts

$$x = \frac{a^2 - b^2}{a^2 + b^2}, \quad y = \frac{2ab}{a^2 + b^2}$$

Now,

$$LHS = x^2 + y^2$$

$$= \left(\frac{a^2 - b^2}{a^2 + b^2}\right)^2 + \left(\frac{2ab}{a^2 + b^2}\right)^2$$

$$\cancel{= \frac{a^2 + 2ab + b^2}{a^2 + b^2}}$$

$$= \frac{(a^2 - b^2)^2}{(a^2 + b^2)^2} + \frac{(2ab)^2}{(a^2 + b^2)^2}$$

$$= \frac{(a^2 - b^2)^2 + (2ab)^2}{(a^2 + b^2)^2}$$

$$= \frac{(a^4 - 2a^2b^2 + b^4) + 4a^2b^2}{(a^2 + b^2)^2}$$

$$= \frac{a^4 + 2a^2b^2 + b^4}{(a^2+b^2)^2}$$

$$= \frac{(a^2)^2 + 2a^2b^2 + (b^2)^2}{(a^2+b^2)^2}$$

$$= \frac{(a^2+b^2)^2}{(a^2+b^2)^2}$$

$$= 1$$

= RHS proved.

11. If $\sqrt[3]{x+iy} = a+ib$, prove that, $\frac{x}{a} + \frac{y}{b} = 4(a^2-b^2)$

Solution

Here,

$$\sqrt[3]{x+iy} = a+ib$$

Cobing on both sides,

$$\left(\sqrt[3]{x+iy}\right)^3 = (a+ib)^3$$

or, $x+iy = a^3 + 3a^2ib + 3ai^2b^2 + b^3$

or, $x+iy = a^3 + 3a^2ib \cancel{+} - 3ab^2 + b^3$ $[\because i^2 = -1]$

or, $x+iy = a^3 - 3ab^2 + b^3 + 3a^2ib$

Comparing real and imaginary parts

$$x = a^3 - 3ab^2 + b^3, \quad y = 3a^2b$$

Now,

$$LHS = \frac{x}{a} + \frac{y}{b}$$

$$= \frac{a^3 - 3ab^2 + b^3}{a} + \frac{3a^2b}{b}$$

$$= \frac{a^3 - 3ab^2 + b^3 + 3a^3b}{ab}$$

$$= \frac{a^3 - 3ab^2 + b^3 + 3a^3}{a}$$

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11. If $\sqrt[3]{x+iy} = a+ib$, prove that, $\frac{x}{a} + \frac{y}{b} = 4(a^2-b^2)$

Solution

Here,

$\sqrt[3]{x+iy} = a+ib$

cubing on both sides,

$$\left(\sqrt[3]{x+iy}\right)^3 = (a+ib)^3$$

Or, $x+iy = a^3 + 3a^2.ib + 3a.i^2.b^2 + i^3b^3$

On $x+iy = a^3 + 3a^2 \cdot ib - 3ab^2 - ib^3$ $[\because i^2 = -1]$

or, $x+iy = a^3 - 3ab^2 + 3a^2 \cdot ib - ib^3$

Comparing real and imaginary parts

$x = a^3 - 3ab^2$, $y = 3a^2b - b^3$

Now,

$$LHS = \frac{x}{a} + \frac{y}{b}$$

$$= \frac{a^3 - 3ab^2}{a} + \frac{3a^2b - b^3}{b}$$

$$= \frac{a^3b - 3ab^3 + 3a^3b - ab^3}{ab}$$

$$= \frac{4a^3b - 4ab^3}{ab}$$

$$= \frac{4ab(a^2-b^2)}{ab}$$

$$= 4(a^2-b^2)$$

$=$ RHS proved.

12. a) Show that $\frac{a+ib}{c+id}$ may be real if ad = dc.

Solution

$\frac{a+ib}{c+id}$

$= \frac{a+ib}{c+id} \times \frac{c-id}{c-id}$

$= \frac{ac - iad + ibc - i^2 bd}{c^2 - i^2 d^2}$

$= \frac{ac - ibc + ibc - (-1)bd}{c^2 - (-1)d^2}$ [$\because$ ad = bc]

$= \frac{ac+bd}{c^2+d^2}$ —— (1)

Comparing (1) with the form A + iB where A is real and B is imaginary part of complex number

$A = \frac{ac+bd}{c^2+d^2}$, $iB = 0$

Hence, $\frac{a+ib}{c+id}$ is real number.

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b) Show that $\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$ is a real number.

Solution

$\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$

$= \frac{(3+2i)(2+5i) + (3-2i)(2-5i)}{(2)^2 - (5i)^2}$

$= \frac{6+15i+4i+10i^2 + 6 - \cancel{5i}\ 15i - 4i + 10i^2}{4-25i^2}$

$= \frac{12+20i^2}{4-25i^2}$

$$= \frac{12+20(-1)}{4-25(-1)}$$

$$= -\frac{8}{9} \quad \text{—} \quad (1)$$

Comparing (1) with the form A+iB where, A is real part & B is imaginary part of complex number, we get

$$A = -\frac{8}{29}$$

Hence, $\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$ is a real number.

13. If $(3-4i)(x+iy) = 3\sqrt{5}$, find $5x^2+5y^2$.

Solution

Here,

$(3-4i)(x+iy) = 3\sqrt{5}$

or, $3x+3iy-4ix-4i^2y = 3\sqrt{5}$

or, $3x+3iy-4ix+4y = 3\sqrt{5}$

or, $3x+4y+3iy-4ix = 3\sqrt{5}$

or, $3x+4y+i(3y-4x) = 3\sqrt{5}$

$(3-4i)(x+iy) = 3\sqrt{5}$

or, $(x+iy) = \frac{3\sqrt{5}}{3-4i}$

or, $(x+iy) = \frac{3\sqrt{5}}{3-4i} \times \frac{3+4i}{3+4i}$

or, $x+iy = \frac{9\sqrt{5}+i.12\sqrt{5}}{(3)^2-(4i)^2}$

or, $x+iy = \frac{9\sqrt{5}}{25} + i.\frac{12\sqrt{5}}{25}$

Comparing real & imaginary parts, we get,

$x = \frac{9\sqrt{5}}{25}$ & $y = \frac{12\sqrt{5}}{25}$

Now,

$5x^2 + 5y^2 = 5\left(\frac{9\sqrt{5}}{25}\right)^2 + 5\left(\frac{12\sqrt{5}}{25}\right)^2$

$= 5\left(\frac{81 \times 5}{625}\right) + 5\left(\frac{144 \times 5}{625}\right)$

$= 5\left(\frac{81}{125}\right) + 5\left(\frac{144}{125}\right)$

$= \frac{81}{25} + \frac{144}{25}$

$= \frac{225}{25}$

$= 9$

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14. Express the following complex number in polar form:

a) $\sqrt{3} + i$

Solution

Let $z = \sqrt{3} + i = r(\cos\theta + i\sin\theta)$

Now, $z = \sqrt{3} + i$ --- (1)

comparing (1) with $x + iy$,

$x = \sqrt{3}$, $y = 1$

We know,

$\tan\theta = \frac{y}{x}$

or, $\tan\theta = \frac{1}{\sqrt{3}}$

or, $\tan\theta = \tan 30°$

$\therefore \theta = 30°$

$r = \sqrt{x^2 + y^2}$

$= \sqrt{(\sqrt{3})^2 + (1)^2}$

$= \sqrt{4}$

$= 2$

Hence, $\sqrt{3} + i = 2(\cos 30^\circ + i \sin 30^\circ)$

b) $-\sqrt{3} + i$

Solution

Let $z = -\sqrt{3} + i = r(\cos\theta + i\sin\theta)$

Now, $z = -\sqrt{3} + i$ --- (1)

Comparing (1) with $x + iy$,

$x = -\sqrt{3}$, $y = 1$

We know,

$\tan\theta = \frac{y}{x}$

or, $\tan\theta = -\frac{1}{\sqrt{3}}$

or, $\tan\theta = \tan 150^\circ$

$\therefore \theta = 150^\circ$

$r = \sqrt{x^2 + y^2}$

$= \sqrt{(-\sqrt{3})^2 + (1)^2}$

$= \sqrt{4}$

$= 2$

Hence, $-\sqrt{3} + i = 2(\cos 150^\circ + i \sin 150^\circ)$

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c) $\frac{i}{1+i}$

Solution

Let $z = \frac{i}{1+i}$

$= \frac{i}{1+i} \times \frac{1-i}{1-i}$

$= \frac{\cancel{1}\, i - i^2}{1 - i^2}$

$= \frac{i+1}{1+1}$

$= \frac{1}{2} + \frac{i}{2}$ —— (1)

Comparing (1) with $x + iy$,

$x = \frac{1}{2}$, $y = \frac{1}{2}$

We know,

$\tan\theta = \frac{y}{x}$

or, $\tan\theta = \frac{\frac{1}{2}}{\frac{1}{2}}$

or, $\tan\theta = 1$

or, $\tan\theta = \tan 45°$

$\therefore \theta = 45°$

$r = \sqrt{x^2 + y^2}$

$= \sqrt{(1/2)^2 + (1/2)^2}$

$= \frac{1}{\sqrt{2}}$

Hence, $\frac{i}{1+i} = \frac{1}{\sqrt{2}}\left(\cos 45° + i \sin 45°\right)$

d) $\frac{1+i}{1-i}$

Solution

Let $z = \frac{1+i}{1-i}$

$= \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$= \frac{(1+i)^2}{1-i^2}$

$= \frac{1+2i+i^2}{1+1}$

$= \frac{2i}{2}$ — (1)

Comparing (1) with $x+iy$, we get

$x=0, y=1$

we know,

$\tan\theta = \frac{y}{x}$

or, $\tan\theta = \frac{1}{0}$

or, $\tan\theta = \infty$

or, $\tan\theta = \tan 90^\circ$

$\therefore \theta = 90^\circ$

Also,

$r = \sqrt{x^2+y^2}$

$= \sqrt{0^2+1^2}$

$= 1$

Hence, $\frac{1+i}{1-i} = 1(\cos 90^\circ + i\sin 90^\circ)$

15. Express each of the following in the form $A + iB$.

a) $3(\cos 60^\circ + i \sin 60^\circ)$

$= 3\left(\frac{1}{2} + i.\frac{\sqrt{3}}{2}\right)$

$= \frac{3}{2} + i.\frac{3\sqrt{3}}{2}$

b) $\sqrt{2}(\cos 135^\circ + i \sin 135^\circ)$

$= \sqrt{2}\left(\frac{-1}{\sqrt{2}} + i.\frac{1}{\sqrt{2}}\right)$

$= -1 + i$

c) $2(\cos(-45^\circ) + i \sin(-45^\circ))$

$= 2(\cos 45^\circ - i \sin 45^\circ)$

$= 2\left(\frac{1}{\sqrt{2}} - i.\frac{1}{\sqrt{2}}\right)$

$= \sqrt{2} - i\sqrt{2}$