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Homework

Exercise 1(C)

1. Write the following in set-builder form:

a) $A = [2,3]$

~~Here~~ Here,
$A = [2,3] = \{x : 2 \le x \le 3\}$

b) $B = [-5,3)$

Here,
$B = [-5,3) = \{x : -5 \le x < 3\}$

c) $C = (-1,2)$

Here,
$C = (-1,2) = \{x : -1 < x < 2\}$

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2. Find two rational numbers between $\frac{1}{4}$ and $\frac{1}{5}$.

Let the two rational numbers be R_1 and R_2.
Here, $a = \frac{1}{4}$, $b = \frac{1}{5}$

first rational number $(R_1) = \dfrac{a+b}{2}$

$= \dfrac{\frac{1}{4} + \frac{1}{5}}{2}$

$= \dfrac{\frac{5+4}{20}}{2}$

$= \frac{9}{20} \times \frac{1}{2}$

$= \frac{9}{40}$

$\therefore R_1 = \dfrac{9}{40}$

Second rational numbers $(R_2) = \frac{a+b}{2}$

$$= \frac{\frac{1}{5} + \frac{9}{40}}{2}$$

$$= \frac{\frac{8+9}{40}}{2}$$

$$= \frac{17}{40} \times \frac{1}{2}$$

$$= \frac{17}{80}$$

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3. Solve:

a) $3x - 5 < 6 - 2x$

Here,

The given inequality is $3x - 5 < 6 - 2x$

$\Rightarrow 3x + 2x - 5 + 5 < 6 - 2x + 2x + 5$

$\Rightarrow 5x < 11$

$\Rightarrow \frac{5x}{5} < \frac{11}{5}$

$\Rightarrow x < \frac{11}{5}$

i.e. $x \in (-\infty, \frac{11}{5})$

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b) $13-7x \geq 10x-4$

Here,

The given inequality is $13-7x \geq 10x-4$

$\Rightarrow 13-7x+7x+4 \geq 10x-4+7x+4$

$\Rightarrow 17 \geq 17x$

$\Rightarrow \frac{17}{17} \geq \frac{17x}{17}$

$\Rightarrow 1 \geq x$

i.e. $x \in (-\infty, 1]$

c) $-3 < 2x+5 \leq 7$

Here,

The given inequality is $-3 < 2x+5 \leq 7$

$\Rightarrow -3-5 < 2x+5-5 \leq 7-5$

$\Rightarrow -8 < 2x \leq 2$

$\Rightarrow \frac{-8}{2} < \frac{2x}{2} \leq \frac{2}{2}$

$\Rightarrow -4 < x \leq 1$

i.e. $x \in \cancel{[-4,1]}\ (-4,1]$

d) $-5 \leq 2x-3 \leq 5$

Here,

The given inequality is $-5 \leq 2x-3 \leq 5$

$\Rightarrow -5+3 \leq 2x-3+3 \leq 5+3$

$\Rightarrow -2 \leq 2x \leq 8$

$\Rightarrow \frac{-2}{2} \leq \frac{2x}{2} \leq \frac{8}{2}$

$\Rightarrow -1 \leq x \leq 4$

i.e. $x \in [-1, 4]$

e) $-6 \leq \frac{3x}{2} + 4 \leq 9$

Here,

The given inequality is $-6 \leq \frac{3x}{2} + 4 \leq 9$

$\Rightarrow -6-4 \leq \frac{3x}{2} + 4 - 4 \leq 9-4$

$\Rightarrow -10 \leq \frac{3x}{2} \leq 5$

$\Rightarrow -10 \times 2 \leq \frac{3x}{2} \times 2 \leq 5 \times 2$

$\Rightarrow -20 \leq 3x \leq 10$

$\Rightarrow -\frac{20}{3} \leq x \leq \frac{10}{3}$

i.e. $x \in \left[-\frac{20}{3}, \frac{10}{3}\right]$

f) $-2 \leq 3x+7 \leq 8$

Here,

The given inequality is $-2 \leq 3x+7 \leq 8$

$\Rightarrow -2-7 \leq 3x+7-7 \leq 8-7$

$\Rightarrow -9 \leq 3x \leq 1$

$\Rightarrow \frac{-9}{3} \leq \frac{3x}{3} \leq \frac{1}{3}$

$\Rightarrow -3 \leq x \leq \frac{1}{3}$

i.e. $x \in \left[-3, \frac{1}{3}\right]$

g) $\frac{-3}{2} \le 4x+1 \le \frac{5}{2}$

Here,

The given inequality is $\frac{-3}{2} \le 4x+1 \le \frac{5}{2}$

$\Rightarrow \frac{-3}{2} - 1 \le 4x+1-1 \le \frac{5}{2} - 1$

$\Rightarrow \frac{-5}{2} \le 4x \le \frac{3}{2}$

$\Rightarrow \frac{-5}{2} \times \frac{1}{4} \le \frac{4x}{4} \le \frac{3}{2} \times \frac{1}{4}$

$\Rightarrow -\frac{5}{8} \le x \le \frac{3}{8}$

i.e. $x \in \left[-\frac{5}{8}, \frac{3}{8}\right]$

4.

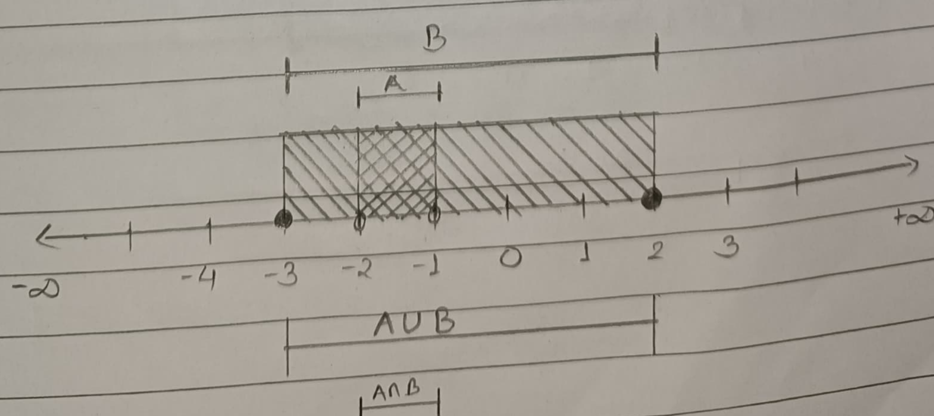
a) If $A = (-2, -1)$ and $B = [-3, 2]$, find $A \cup B$, $A \cap B$, $A - B$ and $B - A$.

Solution

Given,

$A = (-2, -1)$

$B = [-3, 2]$



$A \cup B = (-2,-1) \cup [-3,2]$
$= [-3,2]$

$A \cap B = (-2,-1) \cap [-3,2]$
$= (-2,-1)$

$A - B = (-2,-1) - [-3,2]$
$= \phi$

$B - A = [-3,2] - (-2,-1)$
$= [-3,-2] \cup [-1,2]$

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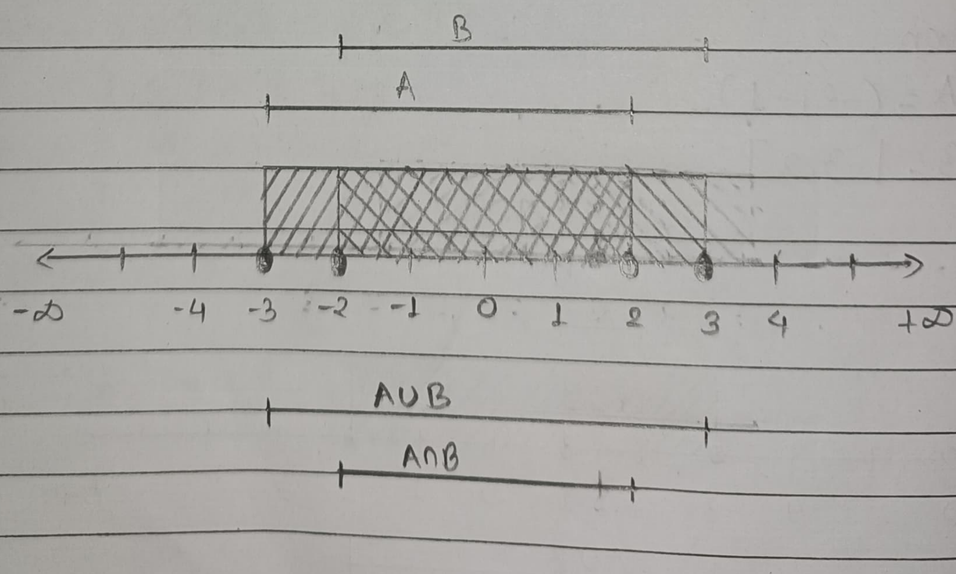
b) If $A = [-3,2)$ and $B = [-2,3]$, find $A \cup B$, $A \cap B$, $A - B$ and $B - A$.

Solution

Here,

$A = [-3,2)$

$B = [-2,3]$



$A \cup B = [-3,2) \cup [-2,3]$
$= [-3,3]$

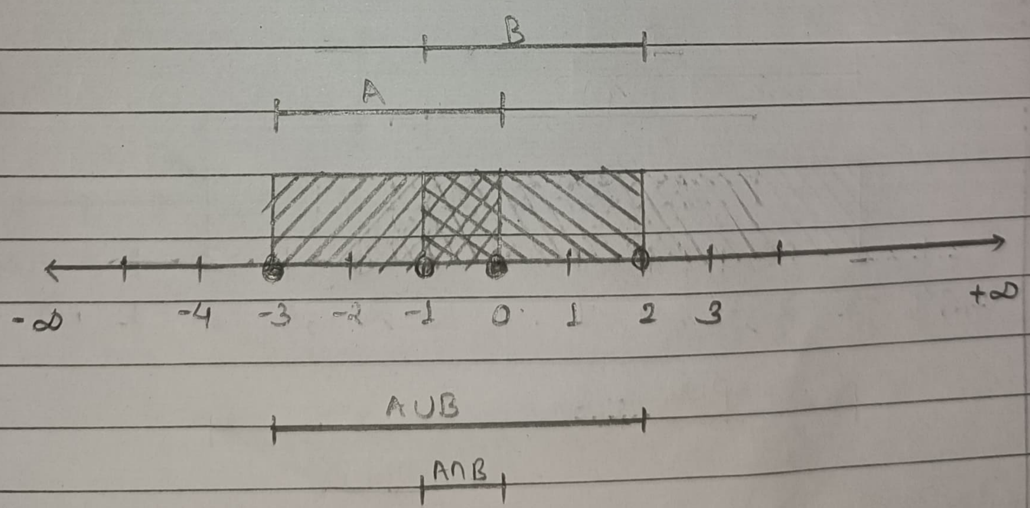
$A \cap B = [-3,2) \cap [-2,3]$
$= [-2,2)$

$A - B = [-3,2) - [-2,3]$
$= [-3,-2)$

$B - A = [-2,3] - [-3,2)$
$= [2,3]$

c) If $A = [-3,0]$ and $B = (-1,2)$ find $A \cup B$, $A \cap B$, $A - B$ and $B - A$.

Solution
Here,
$A = [-3,0]$
$B = (-1,2)$



$A \cup B = [-3, 0] \cup (-1, 2)$
$= [-3, 2)$

$A \cap B = [-3, 0] \cap (-1, 2)$
$= (-1, 0]$

$A - B = [-3, 0] - (-1, 2)$
$= [-3, -1]$

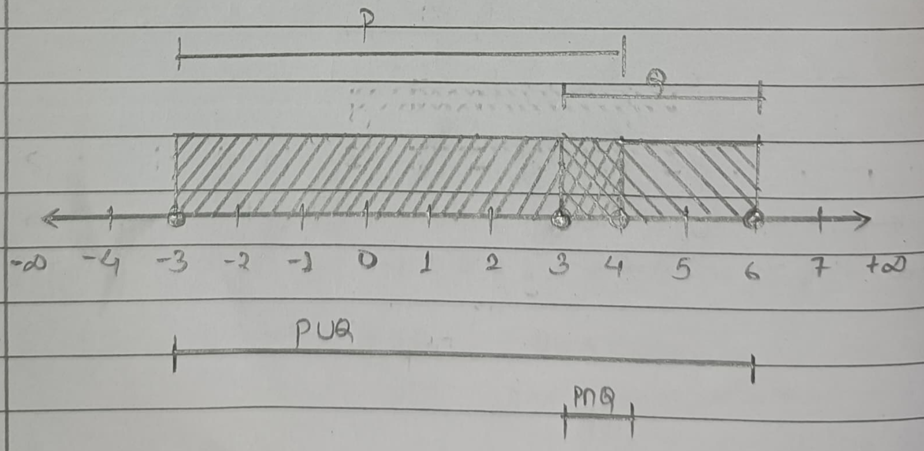
$B - A = (-1, 2) - [-3, 0]$
$= (0, 2)$

5. If $P = [-3, 4)$ and $Q = [3, 6]$. Find $P \cup Q$, $P \cap Q$, $P - Q$, $Q - P$.

Solution

Here,

$P = [-3, 4)$
$Q = [3, 6]$



$P \cup Q = [-3,4) \cup [3,6]$
$= [-3,6]$

$P \cap Q = [-3,4) \cap [3,6]$
$= [3,4)$

$P - Q = [-3,4) - [3,6]$
$= [-3,3)$

$Q - P = [3,6] - [-3,4)$
$= [4,6]$

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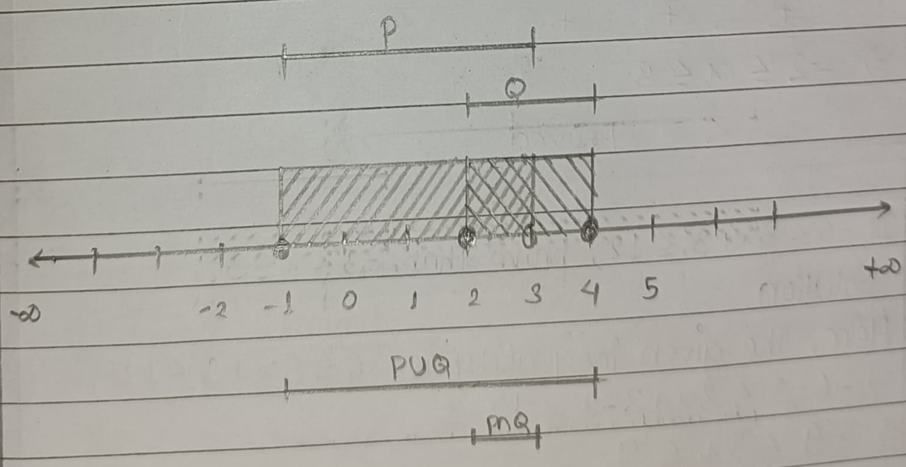
6. If $P = [-1,3)$, $Q = [2,4]$. Find $P \cup Q$, $P \cap Q$, $P-Q$, $Q-P$.

Solution

Here,

$P = [-1,3)$

$Q = [2,4]$



$P \cup Q = [-1,3) \cup [2,4]$
$= [-1,4]$

$P \cap Q = [-1,3) \cap [2,4]$
$= [2,3)$

$P - Q = [-1,3) - [2,4]$
$= [-1,2)$

$Q - P = [2,4] - [-1,3)$
$= \cancel{(2,4]}\ [3,4]$

7.
a) If $-5 \leq 7x+9 < 30$, prove that, $-2 \leq x \leq 3$.

Solution

Here, The given inequality is $-5 \leq 7x+9 \leq 30$

$\Rightarrow -5-9 \leq 7x+9-9 \leq 30-9$

$\Rightarrow -14 \leq 7x \leq 21$

$\Rightarrow \frac{-14}{7} \leq \frac{7x}{7} \leq \frac{21}{7}$

$\Rightarrow -2 \leq x \leq 3$

proved

b) If $-6 \leq 3x+3 \leq 12$, prove that, $-3 \leq x \leq 3$.

Solution

Here, The given inequality is $-6 \leq 3x+3 \leq 12$

$\Rightarrow -6-3 \leq 3x+3-3 \leq 12-3$

$\Rightarrow -9 \leq 3x \leq 9$

$\Rightarrow \frac{-9}{3} \leq \frac{3x}{3} \leq \frac{9}{3}$

$\Rightarrow -3 \leq x \leq 3$

proved

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c) If $-14 \le 3x-8 \le -2$, prove that, $-2 \le x \le 2$.

Solution

Here, The given inequality is $-14 \le 3x-8 \le -2$

$\Rightarrow -14+8 \le 3x-8+8 \le -2+8$

$\Rightarrow -6 \le 3x \le 6$

$\Rightarrow \frac{-6}{3} \le \frac{3x}{3} \le \frac{6}{3}$

$\Rightarrow -2 \le x \le 2$

proved.

8. If $x = 2$ and $y = 3$, verify that,

a) $|x+y| \le |x| + |y|$

Here,

$\Rightarrow \cancel{|2+3|}\ |x+y| \le |x| + |y|$

$\Rightarrow |2+3| \le |2| + |3|$

$\Rightarrow |5| \le 2+3$

$\Rightarrow 5 \le 5$ proved

b) $|x-y| \ge |x| - |y|$

Here,

$|x-y| \ge |x| - |y|$

$\Rightarrow |2-3| \ge |2| - |3|$

$\Rightarrow |-1| \ge 2-3$

$= 1 \ge -1$ proved.

c) $|xy| = |x| . |y|$

Here,

$|xy| = |x| . |y|$

$\Rightarrow |2 \times 3| = |2| . |3|$

$\Rightarrow |6| = 2 \times 3$

$\Rightarrow 6 = 6$ proved.

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d) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

Here,

$\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

$\Rightarrow \left|\frac{2}{3}\right| = \frac{|2|}{|3|}$

$\Rightarrow \frac{2}{3} = \frac{2}{3}$ proved

9. Solve the following inequalities.

a) $|4x-3| < 6$

The given inequality is $|4x-3| < 6$

$\Rightarrow \cancel{-6 < |4x-3}$

$\Rightarrow -6 < 4x-3 < 6$ [$\because |x|$]

$\Rightarrow -6+3 < 4x-3+3 < 6+3$

$\Rightarrow -3 < 4x < 9$

$\Rightarrow \frac{-3}{4} < \frac{4x}{4} < \frac{9}{4}$

$\Rightarrow \frac{-3}{4} < x < \frac{9}{4}$

b) $|3x-15| \leq \frac{3}{2}$

The given inequality is $|3x-15| \leq \frac{3}{2}$

$\Rightarrow \frac{-3}{2} \leq 3x-15 \leq \frac{3}{2}$ [$\because |x| \leq a = -a \leq x \leq a$]

$\Rightarrow -\frac{3}{2} + 15 \leq 3x-15+15 \leq \frac{3}{2} + 15$

$\Rightarrow \frac{-3+30}{2} \leq 3x \leq \frac{3+30}{2}$

$\Rightarrow \frac{27}{2} \leq 3x < \frac{33}{2}$

$\Rightarrow \frac{27}{2} \times \frac{1}{3} \leq \frac{3x}{3} < \frac{33}{2} \times \frac{1}{3}$

$\Rightarrow \frac{9}{2} \leq x < \frac{11}{2}$

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c) $|x-3| < 2$

The given inequality is $|x-3| < 2$

$\Rightarrow -2 < x-3 < 2$

$\Rightarrow -2+3 < x-3+3 < 2+3$

$\Rightarrow 1 < x < 5$

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d) $|3x-5| \leq 5$

The given inequality is $|3x-5| \leq 5$

$\Rightarrow -5 \leq 3x-5 \leq 5$

$\Rightarrow -5+5 \leq 3x-5+5 \leq 5+5$

$\Rightarrow 0 \leq 3x \leq 10$

$\Rightarrow \frac{0}{3} \leq \frac{3x}{3} \leq \frac{10}{3}$

$\Rightarrow 0 \leq x \leq \frac{10}{3}$

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e) $|2x-5| \geq 1$

The given inequality is $|2x-5| \geq 1$

$\Rightarrow 2x-5 \leq -1$ / ~~or~~ or $2x-5 \geq 1$ [$\because |x| \geq a \Rightarrow x \leq -a$ or $x \geq a$]

$\Rightarrow \cancel{+5}\ 2x-5+5 \leq -1+5$ or $2x -$

$\Rightarrow |2x-5| \leq -1$

$\Rightarrow -(-1) \leq 2x-5 \leq -1$

$\Rightarrow 1 \leq 2x-5 \leq -1$

$\Rightarrow 1+5 \leq 2x-5+5 \leq -1+5$

$\Rightarrow 6 \leq 2x \leq 4$

$\Rightarrow \frac{6}{2} \leq \frac{2x}{2} \leq \frac{4}{2}$

$\Rightarrow 3 \leq x < 2$

10. Rewrite the following relations using absolute value sign.

a) $-3 < x < 5$

The given inequality is $-3 < x < 5$

$\Rightarrow -3-1 < x-1 < 5-1$

$\Rightarrow -4 < x-1 < 4$

$\Rightarrow |x-1| < 4$

Rough worked out

Here,

$a = -3, b = 5$

$= -\left(\frac{a+b}{2}\right)$

$= -\left(\frac{-3+5}{2}\right)$

$= -1$

b) $-1 < x < 7$

The given inequality is $-1 < x < 7$

$\Rightarrow -1-3 < x-3 < 7-3$

$\Rightarrow -4 < x-3 < 4$

$\Rightarrow |x-3| < 4$

Rough worked out

Here, $a = -1, b = 7$

$= -\left(\frac{a+b}{2}\right)$

$= -\left(\frac{-1+7}{2}\right)$

$= -3$

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c) $-3 \le x \le 8$

The given inequality is $-3 \le x \le 8$

$\Rightarrow -3 - \frac{5}{2} \le x - \frac{5}{2} \le 8 - \frac{5}{2}$

$\Rightarrow \frac{-6-5}{2} \le \frac{2x-5}{2} \le \frac{16-5}{2}$

$\Rightarrow -\frac{11}{2} \le \frac{2x-5}{2} \le \frac{11}{2}$

$\Rightarrow -11 \le 2x-5 \le 11$

$\Rightarrow |2x-5| \le 11$

Rough worked out

Here, $a=-3$, $b=8$

$= -\left(\frac{a+b}{2}\right)$

$= -\left(\frac{-3+8}{2}\right)$

$= -\frac{5}{2}$

d) $-5 \le x \le -2$

The given inequality is $-5 \le x \le -2$

$\Rightarrow$ ~~$\frac{-5+7}{2}$~~

$\Rightarrow -5 + \frac{7}{2} \le x + \frac{7}{2} \le -2 + \frac{7}{2}$

$\Rightarrow \frac{-10+7}{2} \le \frac{2x+7}{2} \le \frac{-4+7}{2}$

$\Rightarrow -\frac{3}{2} \le \frac{2x+7}{2} \le \frac{3}{2}$

$\Rightarrow -3 \le 2x+7 \le 3$

$\Rightarrow |2x+7| \le 3$

Rough worked out

Here, $a=-5$, $b=-2$

$= -\left(\frac{a+b}{2}\right)$

$= -\left(\frac{-5-2}{2}\right)$

$= \frac{7}{2}$

e) $-3 \le x \le -2$

Here, The given inequality is $-3 \le x \le -2$

$\Rightarrow -3 + \frac{5}{2} \le x + \frac{5}{2} \le -2 + \frac{5}{2}$

$\Rightarrow \frac{-6+5}{2} \le \frac{2x+5}{2} \le \frac{-4+5}{2}$

$\Rightarrow \frac{-1}{2} \le \frac{2x+5}{2} \le \frac{1}{2}$

$\Rightarrow -1 \le 2x+5 \le 1$

$\Rightarrow |2x+5| \le 1$

Rough worked out

Here, $a = -3$, $b = -2$

$= -\left(\frac{a+b}{2}\right)$

$= -\left(\frac{-3-2}{2}\right)$

$= \frac{5}{2}$

f) $-5 < x < 7$

Here, The given inequality is $-5 < x < 7$

$\Rightarrow -5-1 < x-1 < 7-1$

$\Rightarrow -6 < x-1 < 6$

$\Rightarrow |x-1| < 6$

Rough worked out

Here, $a = -5$, $b = 7$

$= -\left(\frac{a+b}{2}\right)$

$= -\left(\frac{-5+7}{2}\right)$

$= -1$

11. a) If 'a' and 'b' are two positive rational number, show that, $\frac{5a+6b}{a+b}$ is a rational number lying between 5 and 6.

Proof:

To show: $\frac{5a+6b}{a+b}$ is a rational number lying between 5 and 6, it is essential to show that,

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$$\frac{5a+6b}{a+b} > 5 \quad \text{—— (i) \&}$$

$$\frac{5a+6b}{a+b} < 6 \quad \text{—— (ii)}$$

Now,

$$\frac{5a+6b}{a+b} = \frac{5a+5b+b}{a+b}$$

$$= \frac{5a+5b}{a+b} + \frac{b}{a+b}$$

$$= \frac{5(a+b)}{a+b} + \frac{b}{a+b}$$

$$= 5 + \frac{b}{a+b}$$

$= 5 +$ (a +ve number)

$$\therefore \frac{5a+6b}{a+b} > 5.$$

Again,

$$\frac{5a+6b}{a+b} = \frac{6a-a+6b}{a+b}$$

$$= \frac{6a+6b}{a+b} - \frac{a}{a+b}$$

$$= \frac{6(a+b)}{a+b} - \frac{a}{a+b}$$

$$= 6 - \frac{a}{a+b}$$

$= 6 -$ (a positive number)

$$\therefore \frac{5a+6b}{a+b} < 6.$$

proved

b) If 'a' and 'b' are two positive rational number, show that, $\frac{3a+4b}{a+b}$ is a rational number lying between 3 and 4.

Proof:

To show: $\frac{3a+4b}{a+b}$ is a rational number lying between 3 and 4, it is essential to show that,

$$\frac{3a+4b}{a+b} > 3 \text{ --- (I) } \&$$

$$\frac{3a+4b}{a+b} < 4 \text{ --- (II)}$$

Now,

$$\frac{3a+4b}{a+b} = \frac{3a+3b+b}{a+b}$$

$$= \frac{3a+3b}{a+b} + \frac{b}{a+b}$$

$$= \frac{3(a+b)}{a+b} + \frac{b}{a+b}$$

$$= 3 + \frac{b}{a+b}$$

$$= 3 + (\text{a +ve number})$$

$\therefore \frac{3a+4b}{a+b} > 3$

Again,

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$$\frac{3a+4b}{a+b} = \frac{4a-a+4b}{a+b}$$

$$= \frac{4a+4b}{a+b} - \frac{a}{a+b}$$

$$= \frac{4(a+b)}{a+b} - \frac{a}{a+b}$$

$$= 4 - \frac{a}{a+b}$$

$= 4 -$ (a +ve number)

$\therefore \frac{3a+4b}{a+b} < 4$

proved

c) Show that $\frac{6a+7b}{a+b}$ is a rational number lying between 6 and 7 where 'a' and 'b' are positive rational numbers.

Proof:

To show: $\frac{6a+7b}{a+b}$ is a rational number lying between 6 and 7, it is essential to show that,

$$\frac{6a+7b}{a+b} > 6 \quad \text{--- (I)} \quad \&$$

$$\frac{6a+7b}{a+b} < 7 \quad \text{--- (II)}$$

Now,

$$\frac{6a+7b}{a+b} = \frac{6a+6b+b}{a+b}$$

$$= \frac{6a+6b}{a+b} + \frac{b}{a+b}$$

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$$= \frac{6(a+b)}{a+b} + \frac{b}{a+b}$$

$$= 6 + \frac{b}{a+b}$$

$= 6 +$ (a +ve number)

$$\therefore \frac{6a+7b}{a+b} > 6$$

Again,

$$\frac{6a+7b}{a+b} = \frac{7a+7b-a}{a+b}$$

$$= \frac{7a+7b}{a+b} - \frac{a}{a+b}$$

$$= \frac{7(a+b)}{a+b} - \frac{a}{a+b}$$

$$= 7 - \frac{a}{a+b}$$

$= 7 -$ (a +ve number)

$$\therefore \frac{6a+7b}{a+b} < 7$$

proved