Homework

Date: 2080/08/25

Chapter-1

Set Theory and Real Number System

Exercise 1(A)

1. Let $U = \{0,1,2,3,4,5,6,7\}$, $A = \{0,2,4,6\}$, $B = \{1,3,5,7\}$ and $C = \{0,3,6\}$. Find
   a) $A \cup B$ b) $B \cap C$ c) B^c d) $A - B$

Solution

Given,

$U = \{0,1,2,3,4,5,6,7\}$

$A = \{0,2,4,6\}$

$B = \{1,3,5,7\}$

$C = \{0,3,6\}$

Now,

a) $A \cup B = \{0,2,4,6\} \cup \{1,3,5,7\}$
$= \{0,1,2,3,4,5,6,7\}$

b) $B \cap C = \{1,3,5,7\} \cap \{0,3,6\}$
$= \{3\}$

c) $B^c = \cancel{\{} U - B$
$= \{0,1,2,3,4,5,6,7\} - \{1,3,5,7\}$
$= \{0,2,4,6\}$

d) $A - B = \{0,2,4,6\} - \{1,3,5,7\}$
$= \{0,2,4,6\}$

2. If $A = \{1,2,3\}$, $B = \{3,4,5\}$, find $A-B$, $B-A$ and $A \Delta B$.

Solution

Given,

$A = \{1,2,3\}$

$B = \{3,4,5\}$

$\therefore A - B = \{1,2,3\} - \{3,4,5\}$

$= \{1,2\}$

$\therefore B - A = \{3,4,5\} - \{1,2,3\}$

$= \{4,5\}$

Also,

$A \Delta B = (A-B) \cup (B-A)$

$= \{1,2\} \cup \{4,5\}$

$= \{1,2,4,5\}$

---

3. If A and B are two disjoint sets and $n(A \cup B) = 475$, $n(A) = 435$, find $n(B)$.

Solution

Given,

$n(A \cup B) = 475$

$n(A) = 435$

$n(A \cap B) = 0$ [since A and B are two disjoint sets]

$n(B) = ?$

We know,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

or, $475 = 435 + n(B) - 0$

or, $n(B) = 475 - 435$

$\therefore n(B) = 40$

Date ____
Page ____

4. If $U = \{x : \text{is a natural number up to } 20\}$, $A = \{x : x \geq 6\}$, $B = \{x : x \leq 8\}$, $C = \{x : 10 < x < 15\}$, find $B \cup C$, $A \cap B$, $A - C$ and $\overline{A \cup B}$.

Solution

Given,

$U = \{1, 2, 3, 4, 5, 6, 7, 8, ----, 20\}$

~~$A = \{1, 2, 3, 4, 5, 6\}$~~ $A = \{6, 7, 8, 9, 10, --- 20\}$

$B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$C = \{11, 12, 13, 14\}$

Now,

$B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\} \cup \{11, 12, 13, 14\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14\}$

~~$A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 5, 6, 7, 8\}$~~

~~$= \{1, 2, 3, 4, 5, 6\}$~~

$A \cap B = \{6, 7, 8, 9, 10, ---, 20\} \cap \{1, 2, 3, 4, 5, 6, 7, 8\}$

$= \{6, 7, 8\}$

$A - C = \{6, 7, 8, 9, 10, ----, 20\} - \{11, 12, 13, 14\}$

$= \{6, 7, 8, 9, 10, 15, 16, 17, 18, 19, 20\}$

$\overline{A \cup B} = U - (A \cup B)$

$= U - \{1, 2, 3, 4, 5, 6, ----, 20\}$

$= \{1, 2, 3, 4, 5, 6, ---, 20\} - \{1, 2, 3, 4, 5, 6, ---, 20\}$

$= \{\}$

$= \phi$

Date
Page

5. If $U = \{x : x \text{ is a +ve integer less than } 12\}$, $A = \{3, 5, 7, 9\}$, $B = \{1, 2, 3, 8, 9\}$, find $\overline{A \cup B}$ and $(A - B) \cap B$.

Solution

Given,

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

$A = \{3, 5, 7, 9\}$

$B = \{1, 2, 3, 8, 9\}$

$A \cup B = \{3, 5, 7, 9\} \cup \{1, 2, 3, 8, 9\}$
$= \{1, 2, 3, 5, 7, 8, 9\}$

$\therefore \overline{A \cup B} = U - (A \cup B)$
$= \{1, 2, 3, 4, 5, 6, ---, 11\} - \{1, 2, 3, 5, 7, 8, 9\}$
$= \{4, 6, 10, 11\}$

Also,

$A - B = \{3, 5, 7, 9\} - \{1, 2, 3, 8, 9\}$
$= \{5, 7\}$

$\therefore (A - B) \cap B = \{5, 7\} \cap \{1, 2, 3, 8, 9\}$
$= \{\}$
$= \phi$

Homework Date: 2080/08/26

7. In a recent survey of 400 students in a school it was found out that 100 students read magazine A and 150 read magazine B, 75 students read both the magazines. Find how many read either magazine.

Solution

Let the ~~stu~~ set of students ~~liked to~~ read magazine A be A, magazine B be B and both be $A \cap B$.

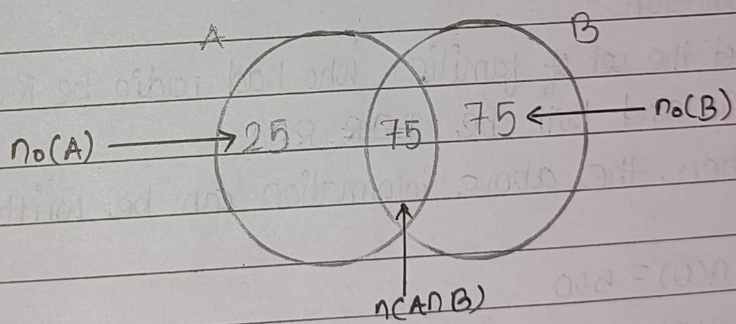
Then the above information can be written as:

$n(A) = 100$ $n(U) = 400$

$n(B) = 150$

$n(A \cap B) = 75$

Showing the above information in a venn-diagram as:



Now,

Number of student who ~~like to~~ read either magazine

$= n_o(A) + n_o(B) + n(A \cap B)$

$= 25 + 75 + 75$

$= 175$

$\therefore$ 175 students read either magazine.

Date
Page

8. In a statistical investigation of 500 families in a certain town, it was found that 40 families had neither a radio nor a TV, and 320 families had a radio and 190 a TV.

a) How many families in that group had both radio and TV?

b) How many families in that group had radio but not TV?

c) How many families in that group had TV but not radio?

d) How many families in that group had at least one type of ~~medal~~ media?

e) How many families in that group had exactly one type of media?

Solution

Let the set of families who had radio be R, TV be T and both be ~~TAR~~ $R \cap T$.
Then, the above information can be written as:

$n(U) = 500$

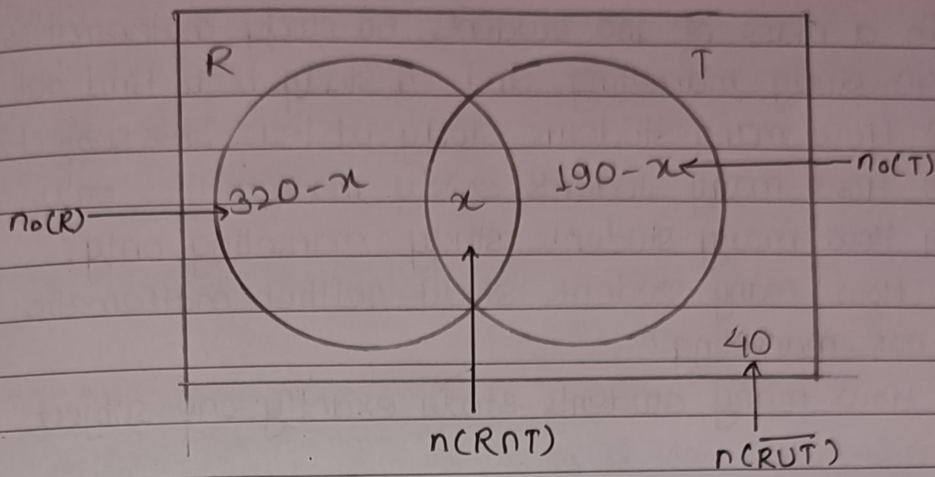
$n(R) = 320$

$n(T) = 190$

$n(\overline{R \cup T}) = 40$

$n(R \cap T) = x$ (let)

Showing the above information in venn diagram.



a) Now,

$n(U) = n_o(R) + n_o(T) + n(R\cap T) + n(\overline{R\cup T})$

or, $500 = 320 - x + 190 - x + \cancel{40 =} x + 40$

or, $500 = 550 - x$

$\therefore x = 50$

$\therefore n(R\cap T) = 50$

$\therefore$ 50 families had both radio and TV.

b) Number of families that had radio but not TV $= n_o(R)$

$= 320 - x$

$= 320 - 50$

$= 270$

c) Number of families that had TV but not radio $= n_o(T)$

$= 190 - x$

$= 190 - 50$

$= 140$

d) Number of families that had at least one type of media $= n_o(R) + n_o(T) + n(R\cap T)$

$= 270 + 140 + 50$

$= 460$

e) Number of families that had exactly one type of media

$= n_o(R) + n_o(T)$

$= 270 + 140 = 410$

9. In a class of 100 students, 50 study mathematics, 40 study marketing and 25 study both. Find out:

a) How many students study at least one subject?

b) How many students study mathematics only?

c) How many students study marketing only?

d) How many students study neither mathematic nor marketing?

e) How many students study exactly one subject?

Solution

Let the set of students who study mathematics be M_1, marketing be M_2 and both be $M_1 \cap M_2$. Then, the above information can be written as:

$n(U) = 100$

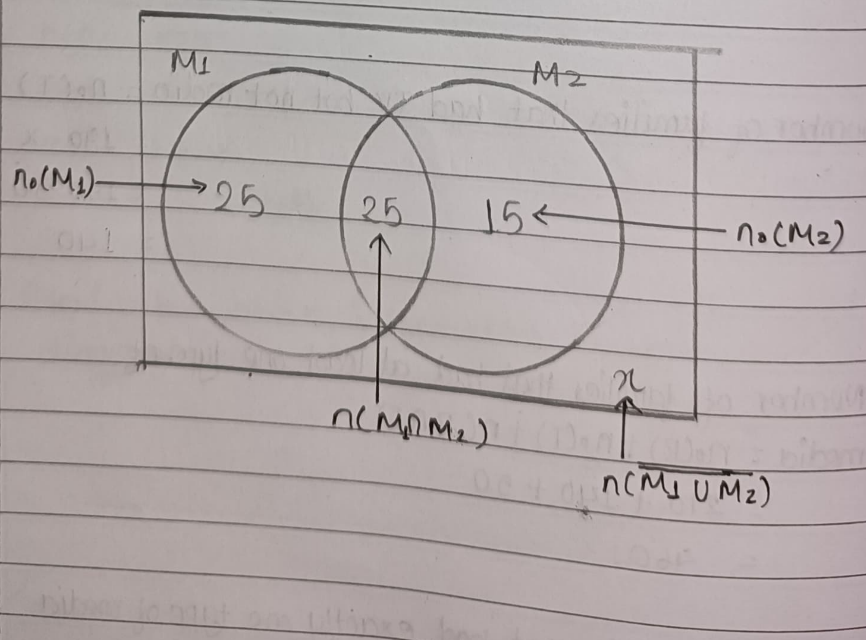
$n(M_1) = 50$

$n(M_2) = 40$

$n(M_1 \cap M_2) = 25$

Let $(\overline{M_1 \cup M_2})$ be x.

Showing the above information in a venn-diagram as:



Now,
$n(U) = n_o(M_1) + n_o(M_2) + n(M_1 \cap M_2) + n(\overline{M_1 \cup M_2})$
or, $100 = 25 + 15 + 25 + x$
or, $100 = 65 + x$
$\therefore x = 35$
$\therefore n(\overline{M_1 \cup M_2}) = 35$

Now,
a) Number of students who study at least one subject
$= \cancel{n(A \cup B)}\ n(M_1 \cup M_2)$
$= n_o(M_1) + n_o(M_2) + n(M_1 \cap M_2)$
$= 25 + 15 + 25$
$= 65$

b) Number of students who study mathematics only
$= n_o(M_1)$
$= n(M_1) - n(M_1 \cap M_2)$
$= 50 - 25$
$= 25$

c) Number of students who study marketing only
$= n_o(M_2)$
$= n(M_2) - n(M_1 \cap M_2)$
$= 40 - 25$
$= 15$

d) Number of students who study neither mathematic nor marketing $= n(\overline{M_1 \cup M_2})$
$= 35$

e) Number of students who study exactly one subject
$= n_o(M_1) + n_o(M_2)$
$= 25 + 15$
$= 40$

10. In a class consisting of 60 students, 25 students have taken MIS as a major, 17 have taken MIS but not operations and 22 have neither taken MIS nor operations as a major. Find the number of students who have taken

(i) MIS and Operations

(ii) Operations but not MIS.

(iii) At least one as major.

(iv) Exactly one as major.

Solution

Let the set of students who have taken MIS as major be M, operations be O and both be $M \cap O$.

Then, the above information can be written as:

$n(U) = 60$

$n(M) = 25$

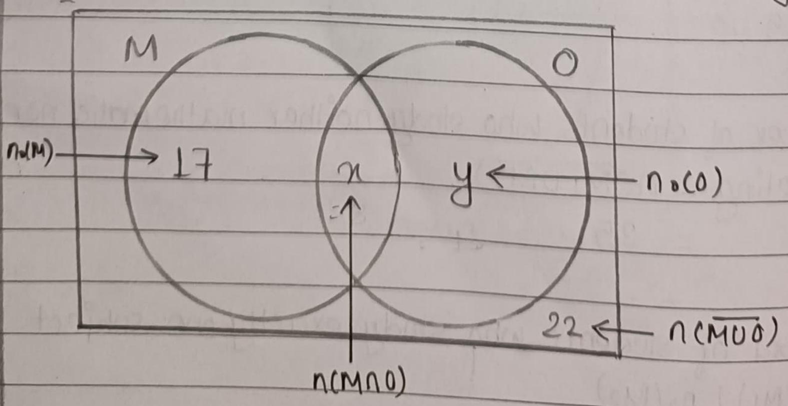
$n_o(M) = 17$

$n(\overline{M \cup O}) = 22$

Let $n(M \cap O) = x$

Let $n_o(O) = y$

Showing the above information in venn diagram:



Now,

~~A(O) = A~~

(i) Number of students who have taken both MIS and operations $= n(M \cap O)$

$= n(M) - n_o(M)$

$= 25 - 17$

$= 8$

~~No~~ Also,

~~(ii)~~ Number of students who have taken Operation

i.e. $n(O) = ?$

We have,

$n(U) = n_o(M) + n_o(O) + n(M \cap O) + n(\overline{M \cup O})$

or, $60 = 17 + y + 8 + 22$

or, $y = 60 - 47$

$\therefore y = 13$

$\therefore n_o(O) = 13$

ii) Number of students who have taken operations but not MIS $= n_o(O)$

$= 13$

iii) Number of students who have taken at least one as major $= n(M \cup O)$

$= n_o(M) + n_o(O) + n(M \cap O)$

$= 17 + 13 + 8$

$= 38$

iv) Number of students who have taken exactly one as major $= n_o(M) + n_o(O)$

$= 17 + 13$

$= 30$

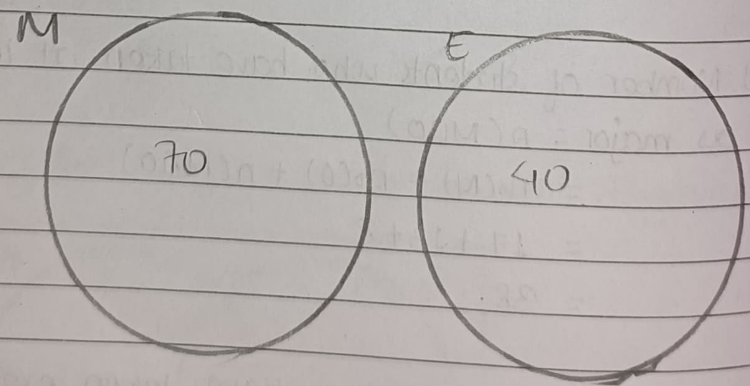
11. In a BIM program of Shanker Dev Campus, there are 40 students in English and 70 students in mathematics class. Find the total number of students if these two classes
a) run at the same time
b) run at the different time if 20 students are enrolled in both the courses.

Solution

Let the ~~student~~ set of students who read mathematics be M, English be E and both be $M \cap E$.
Then, the above information can be written as:

$\cancel{n(U) =}\ n(M) = 70$
$n(E) = 40$
$n(M \cap E) = 20$

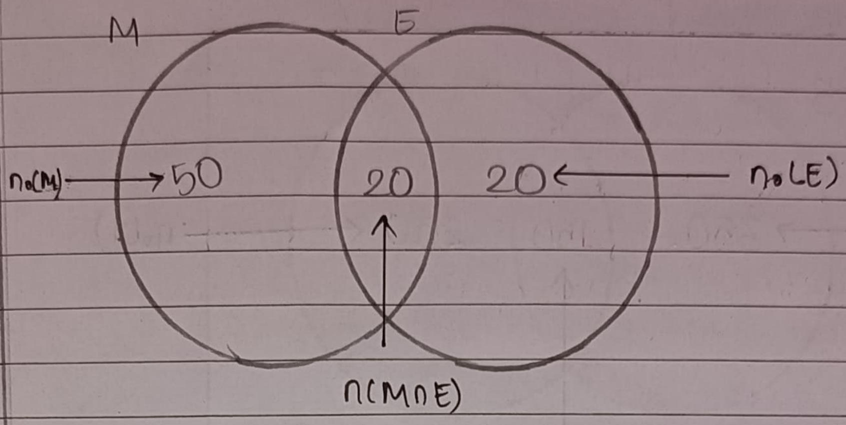
a) If the classes run at the same time.



Number of students $= n(M \cup E)$
$= n(M) + n(E)$
$= 70 + 40$
$= 110$

Date ______
Page ______

b) If the classes run at different time.



Number of students $= n(M\cup E)$

$= n_o(M) + n_o(E) + \cancel{Not}\ n(M\cap E)$

$= 50 + 20 + 20$

$= 90$

---

12. In a market survey of 1000 consumers of tea, it was found that 500 purchased Soktim tea, 400 purchased Tokla tea and 150 purchased both the brands. How many purchased

a) Soktim tea only

b) Tokla tea only

c) At least one type of tea

d) Neither of them.

Solution

Let the set of consumers who purchased Soktim tea be S, ~~and~~ Tokla tea be T and both be $S\cap T$.

Then, the above information can be written as:

$n(U) = 1000$

$n(S) = 500$

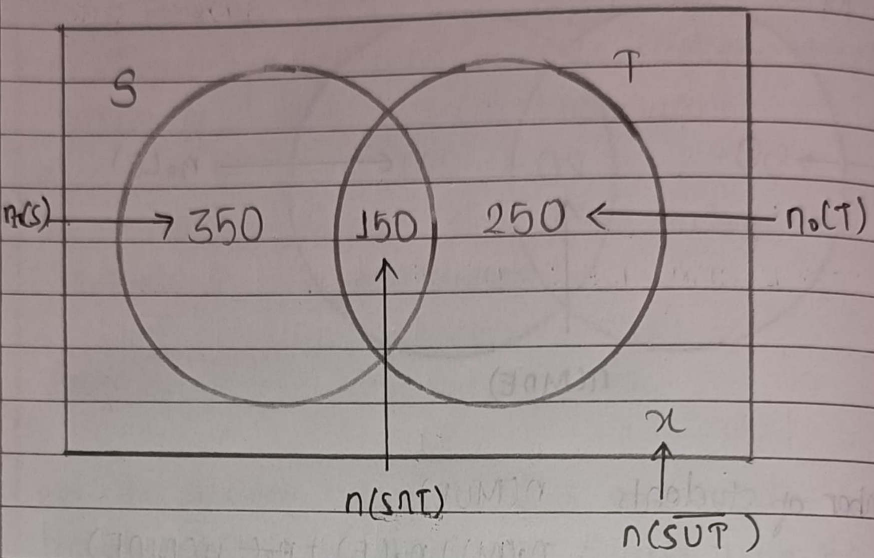
$n(T) = 400$

$n(S\cap T) = 150$

~~Let~~ ~~Let $n(S\cup T)$ be x~~. Let $n(\overline{S\cup T})$ be x.

Date
Page

Showing the information in venn-diagram:



Now,

a) Number of consumers who purchased Soktim tea only $= n_o(S)$

$= n(S) - n(S \cap T)$

$= 500 - 150$

$= 350$

b) Number of consumers who purchased Tokla tea only $= n_o(T)$

$= n(T) - n(S \cap T)$

$= 400 - 150$

$= 250$

c) Number of consumers who purchased at least one type of tea $= n(S \cup T)$

$= n_o(S) + n_o(T) + n(S \cap T)$

$= 350 + 250 + 150$

$= 750$

d) Number of consumers who purchased neither of them $= n(\overline{S \cup T})$

$= n(U) - n(S \cup T)$

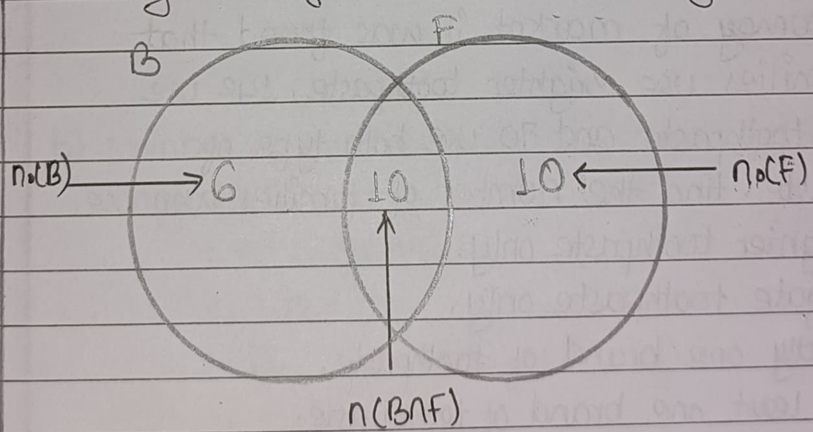
$= 1000 - 750 = 250$

13. 16 students play Basketball, 20 students play football and 10 play both. Find the number of students that liked to play
a) Basketball only
b) Football only
c) Exactly one type of games
d) At least one type of game

Solution
Let the set of students who play Basketball be B, Football be F and both be $B \cap F$.
Then, the above information can be written as;

$n(B) = 16$
$n(F) = 20$
$n(B \cap F) = 10$

Showing the information in venn-diagram:



Now,
a) Number of students who that liked to play basketball only $= n_o(B)$
$= n(B) - n(B \cap F)$
$= 16 - 10$
$= 6$

b) Number of students that liked to play football only $= n_o(F)$

$= n(F) - n(B \cap F)$

$= 20 - 10$

$= 10$

c) Number of students that liked exactly one type of games $= n_o(B) + n_o(F)$

$= 6 + 10$

$= 16$

d) Number of students that liked at least one type of game $= \cancel{n(A \cup B)}\ n(B \cup F)$

$= n_o(B) + n_o(F) + n(B \cap F)$

$= 6 + 10 + 10$

$= 26$

---

14. In a survey of market, it was found that 300 families use Brighter toothpaste, 142 use Colgate toothpaste and 70 use both type of toothpaste. Find the number of families who use
a) Brighter toothpaste only.
b) Colgate toothpaste only.
c) Exactly one brand of toothpaste.
d) At least one brand of toothpaste.

Solution

Let the set of families using brighter toothpaste be B, colgate toothpaste be C and both be $B \cap C$.

Then, the above information can be written as:

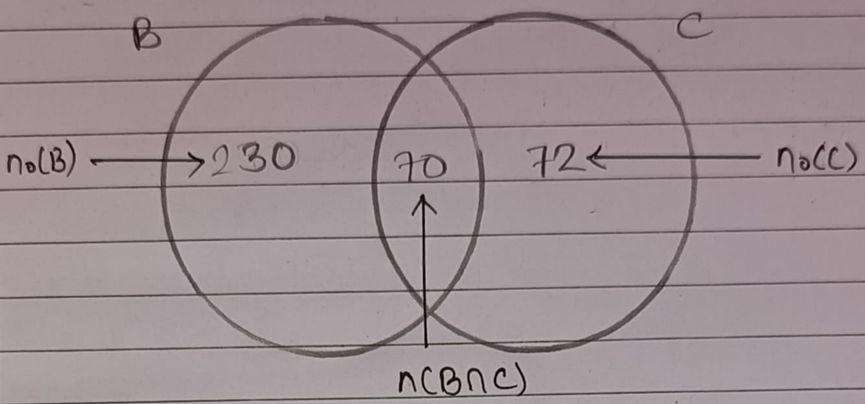
$\cancel{n(U) = 300}$

$n(B) = 300$

$n(C) = 142$

$n(B \cap C) = 70$

Showing the information in venn-diagram:



Now,

a) Number of families who use brighter toothpaste only $= n_o(B)$

$= n(B) - n(B \cap C)$

$= 300 - 70$

$= 230$

b) Number of families who use colgate toothpaste only $= n_o(C)$

$= n(C) - n(B \cap C)$

$= 142 - 70$

$= 72$

c) Number of families who use exactly one brand of toothpaste $= n_o(B) + n_o(C)$

$= 230 + 72$

$= 302$

d) Number of families who use atleast one brand of toothpaste $= n(B \cup C)$

$= n_o(B) + n_o(C) + n(B \cap C)$

$= 230 + 72 + 70$

$= 372$