Conic Section

\# Ellipse → deppiciency $e<1$
\# Hyperbola $e>1$

Defination (Ellipse)

The set of all points in a plane, the sum of whose distance from two fixed points in the plane is constant is an ellipse. These two fixed points are the foci (plural of focus) of the ellipse (fig 1). When a line segment is drawn joining the two focus points, then the mid-point of this line is the center of the ellipse

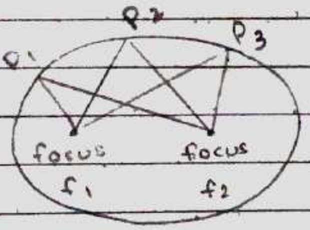


fig (1)

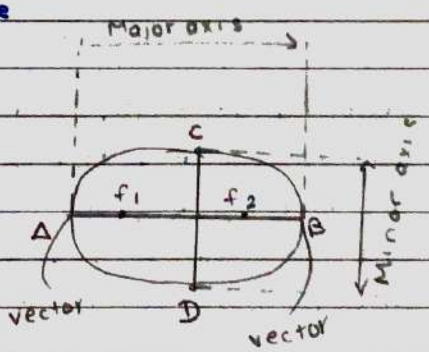
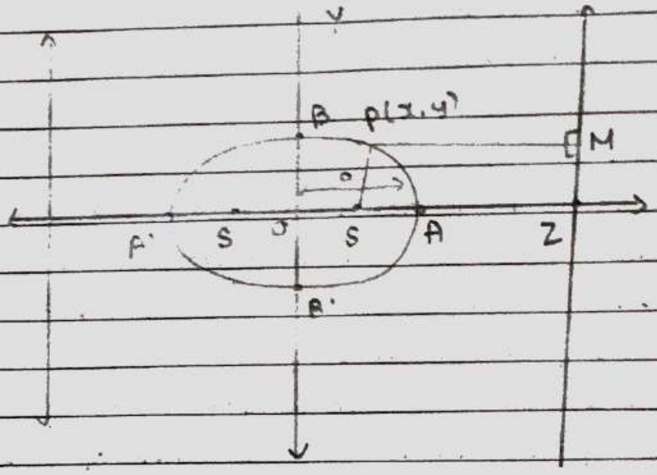


fig (2)

This line joining the two foci is called the major axis and a line drawn through the center and perpendicular to the major axis is the minor axis. The endpoints of the major axis are called vertices of the ellipse (fig 2)
Also we denote
The length of the major axis by '2a'
The length of the minor axis by '2b'

Standard Equation of Ellipse



let O be the origin, let us take an ellipse with centered at origin S and S' be the foci of the ellipse and A & A' be the vertices at a distance of a from center O. Also let ZM be the directrix of the ellipse. let A and A' divide SZ internally and externally in the ratio of e:1.

i.e $AS : AZ = e:1 \Rightarrow AS = eAZ$ —— (i)

Also $A'S : A'Z = e:1 \Rightarrow A'S = eA'Z$ —— (ii)

Now adding (i) and (ii) we get

$$AS + A'S = e(AZ + A'Z)$$

$$\Rightarrow AA' = e[OZ - OA + A'O + OZ]$$

$$\Rightarrow 2a = e[2OZ]$$

$$\Rightarrow OZ = \frac{a}{e}$$

Also subtracting (i) from (ii) we get

$$A'S - AS = e(A'Z - AZ)$$

$$\Rightarrow [A'O + OS - (OA - OS)] = e \cdot AA'$$

$$\Rightarrow 2 \cdot OS = e \cdot 2a$$

$$\Rightarrow OS = ae$$

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Therefore coordinate of foci are $(\pm ae, 0)$ and coordinates of Z and Z' are respectively $\left(\frac{a}{e}, 0\right)$ and $\left(-\frac{a}{e}, 0\right)$

Now, to find the equation of ellipse in standard form. let $P(x,y)$ be any point on the curve, then draw PM perpendicular to directrix ZM and also join PS. Then coordinates of M is $\left(\frac{a}{e}, y\right)$.

Now, by definition, $e = \frac{PS}{PM} \Rightarrow PS = e \cdot PM$

$\Rightarrow PS^2 = e^2 \cdot PM^2$

$\Rightarrow [(x-ae)^2 + (y-0)^2] = e^2\left[\left(x-\frac{a}{e}\right)^2 + 0^2\right]$

$\Rightarrow [x^2 + a^2e^2 - 2aex + y^2] = e^2\left[x^2 - \frac{2a}{e}x + \left(\frac{a}{e}\right)^2\right]$

$\Rightarrow [x^2 + a^2e^2 - 2aex + y^2] = e^2x^2 - 2aex + a^2$

$\Rightarrow x^2(1-e^2) + y^2 = a^2(1-e^2)$

dividing both sides by $a^2(1-e^2)$, we get

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$$

Now, setting $a^2(1-e^2) = b^2$ —— (#)

as $a^2(1-e^2)$ is positive, we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

which is equation of ellipse in the standard form.

Eccentricity

from (#) we have, $a^2(1-e^2) = b^2$

$$\Rightarrow 1 - e^2 = \frac{b^2}{a^2}$$

$$\Rightarrow e^2 = 1 - \frac{b^2}{a^2} \Rightarrow e = \sqrt{1 - \frac{b^2}{a^2}}$$

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Length of Latus Rectum

A focal chord perpendicular to the major axis called Latus Rectum. In the figure alongside LL' is latus rectum.

Now to find length of LL'

let co-ordinate of L be (ae, y')

then as L lies on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

we put $x = ae$ and $y = y'$

$$\frac{a^2e^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{y'^2}{b^2} = 1 - e^2$$

$$\Rightarrow y'^2 = b^2(1-e^2)$$

$$\Rightarrow y'^2 = b^2 \times \frac{b^2}{a^2} \quad \left[\because b^2 = a^2(1-e^2) \Rightarrow \frac{b^2}{a^2} = 1 - e^2\right.$$

$$\Rightarrow y'^2 = \frac{b^4}{a^2}$$

$$\Rightarrow y' = \pm\frac{b^2}{a}$$

So, co-ordinates of L and L' are $\left(ae, \frac{b^2}{a}\right)$ and $\left(ae, \frac{-b^2}{a}\right)$ respectively.

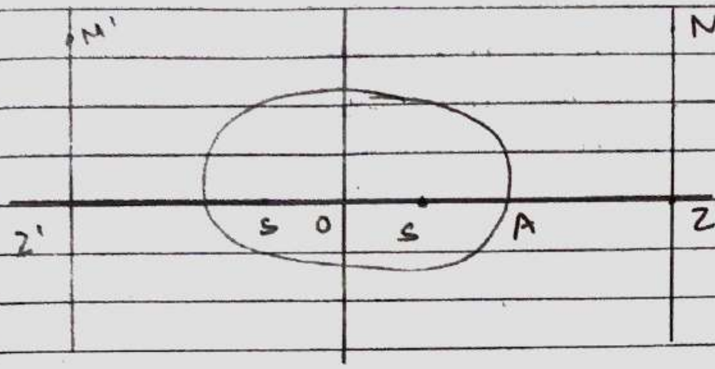
Now $LL' = LS + SL'$

$$= \frac{b^2}{a} + \frac{b^2}{a}$$

$$= \frac{2b^2}{a}$$

length of latus rectum $= \frac{2b^2}{a}$

Equation of directrices



We know coordinates of Z and Z' are $\left(\frac{a}{e}, 0\right)$ and $\left(\frac{-a}{e}, 0\right)$. So, eq^n of ZM is $x = \frac{a}{e}$

and eq^n of Z'M' is $x = \frac{-a}{e}$

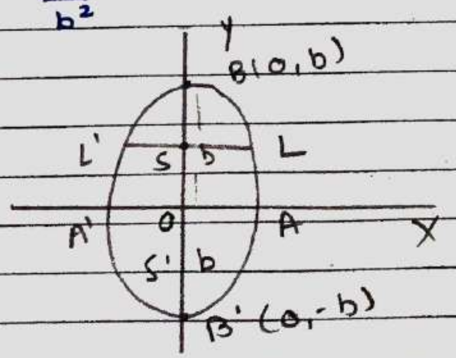
Combining both we got

$$x = \pm\frac{a}{e}$$

Equation of Ellipse with $a^2 < b^2$
(i.e vertical ellipse)

Standard eq^n of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ with } a^2 < b^2$$



vertices $(0, \pm b)$

foci : $(0, \pm be)$

eccentricity $e = \sqrt{1 - \frac{a^2}{b^2}}$

length of latus rectum $= \frac{2a^2}{b}$

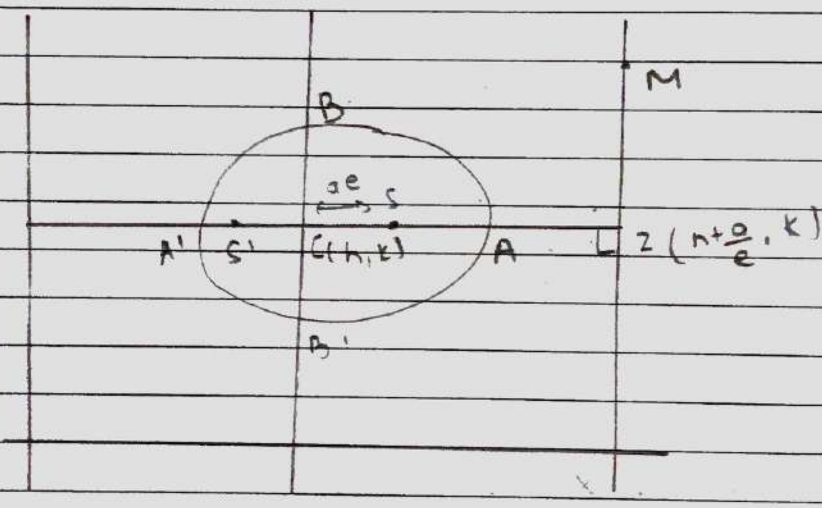
length of major axis $= 2b$

length of minor axis $= 2a$

eq^n of directrices $y = \pm \frac{b}{e}$

Equation of ellipse with centre at other than origin

a) When major axis is ∥ to x-axis (i.e $a^2 > b^2$)



Eq^n of ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

centre $= (h, k)$

vertices $= (h \pm a, k)$

foci $= (h \pm ae, k)$

Eccentricity, $e = \sqrt{1 - \frac{b^2}{a^2}}$

length of latus rectum $= \frac{2b^2}{a}$

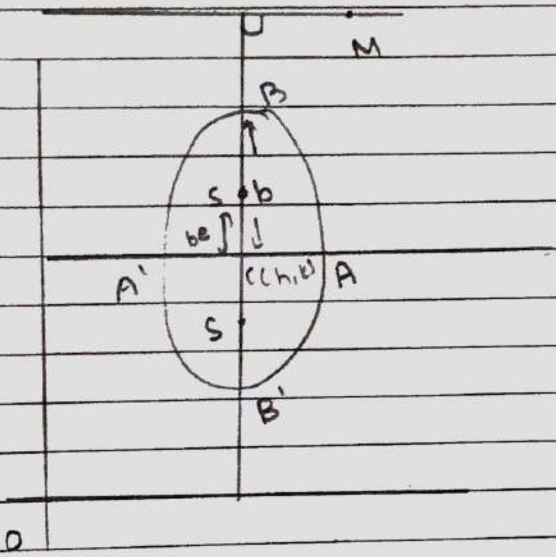
length of major axis $= 2a$

length of minor axis $= 2b$

Eqⁿ of directrices

$$x = h \pm \frac{a}{e}$$

b) When major axis is parallel to Y-axis



Eqⁿ of ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Centre = (h, k)

vertices = $(h, k \pm b)$

foci = $(h, k \pm be)$

eccentricity = $e = \sqrt{1 - \frac{b^2}{a^2}}$

length of latus rectum = $\frac{2a^2}{b}$

length of major axis = $2b$

length of minor axis = $2a$

Eqⁿ of directrices = $y = k \pm \frac{b}{e}$

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Exercise 10.1

1. Find the eccentricity, co-ordinates of the vertices and foci, the length of the latus rectum, major axis and minor axis of the following ellipse

a) $\frac{x^2}{16} + \frac{y^2}{4} = 1$

Sol^n

Given eq^n of ellipse

$\frac{x^2}{16} + \frac{y^2}{4} = 1$ —(1)

Comparing (1) with

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

we get $a^2 = 16$ $\quad b^2 = 4$

$\Rightarrow a = 4$ $\quad b = 2$

As $a^2 > b^2$, So, major axis is along x-axis.

So, we know, eccentricity $(e) = \sqrt{1 - \frac{b^2}{a^2}}$

$= \sqrt{1 - \frac{4}{16}}$

$= \sqrt{\frac{12}{16}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

~~vertice~~ coordinate of vertices $= (\pm a, 0)$

$= (\pm 4, 0)$

coordinate of foci $= (\pm ae, 0)$

$= (\pm 4 \cdot \frac{\sqrt{3}}{2}, 0)$

$= (\pm 2\sqrt{3}, 0)$

length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 4}{4} = 2$ units

length of major axis = $2a = 2\times 4 = 8$ units
length of minor axis = $2b = 2\times 2 = 4$ units

b) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

Sol^n

Given eq^n of ellipse

$$\frac{x^2}{9} + \frac{y^2}{25} = 1 \quad \text{--- (1)}$$

Comparing (1) with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

we get $a^2 = 9 \qquad b^2 = 25$

$\Rightarrow a = 3 \qquad b = 5$

As $a^2 < b^2$, So, major axis is along y-axis

So, we know eccentricity $(e) = \sqrt{1 - \frac{a^2}{b^2}}$

$$= \sqrt{1 - \frac{9}{25}}$$

$$= \sqrt{\frac{16}{25}} = \frac{4}{5}$$

coordinate of vertices $= (0, \pm b)$

$= (0, \pm 5)$

Coordinate of foci $: (0, \pm be)$

$= (0, \pm \cancel{5}\cdot\frac{4}{\cancel{5}})$

$= (0, \pm 4)$

length of latus rectum $= \frac{2a^2}{b} = \frac{2\times 9}{5} = \frac{18}{5}$

length of major axis $= 2b = 2\times 5 = 10$
length of minor axis $= 2a = 2\times 3 = 6$

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c) $3x^2+4y^2=36$

Sol^n

Given eq^n of ellipse is

$3x^2+4y^2=36$

Dividing both side by 36

$$\frac{3x^2}{36}+\frac{4y^2}{36}=1$$

$$\Rightarrow \frac{x^2}{12}+\frac{y^2}{9}=1 \quad \text{--- (1)}$$

Comparing (1) with

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

we get $a^2=12$ $\qquad b^2=9$

$a=2\sqrt{3}$ $\qquad \Rightarrow b=3$

As $a^2>b^2$. So, major axis is along x-axis

So, we know, eccentricity (e) $=\sqrt{1-\frac{b^2}{a^2}}$

$$=\sqrt{1-\frac{9}{12}}$$

$$=\sqrt{\frac{3}{12}}=\frac{\sqrt{3}}{2\sqrt{3}}=\frac{1}{2}$$

Coordinate of vertices $=(\pm a, 0)$

$=(\pm 2\sqrt{3}, 0)$

coordinate of foci $=(\pm ae, 0)$

$$=\left(\pm \cancel{2}\sqrt{3}\cdot\frac{1}{\cancel{2}}, 0\right)$$

$=(\pm\sqrt{3}, 0)$

length of latus rectum $=\frac{2b^2}{a}=\frac{\cancel{2}\cdot 9}{\cancel{2}\sqrt{3}}=\frac{3\sqrt{3}\cdot\sqrt{3}}{\sqrt{3}}=3\sqrt{3}$

length of major axis $=2a=2\times 2\sqrt{3}=4\sqrt{3}$

length of minor axis $=2b=2\times 3=6$

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d. $5x^2+9y^2=45$

Sol^n

Given eq^n of ellipse is

$5x^2+9y^2=45$

Dividing both side by 45

$$\frac{5x^2}{45}+\frac{9y^2}{45}=1$$

$$\Rightarrow \frac{x^2}{9}+\frac{y^2}{5}=1 \quad \text{---(1)}$$

we get $a^2=9$ $\quad b^2=5$

$\Rightarrow a=3 \quad b=\sqrt{5}$

As $a^2>b^2$. So, major axis is along x-axis.

So, we know eccentricity (e) $=\sqrt{1-\frac{b^2}{a^2}}$

$=\sqrt{1-\frac{5}{9}}$

$=\sqrt{\frac{4}{9}}=\frac{2}{3}$

Coordinate of vertices $=(\pm a,0)$

$=(\pm 3,0)$

coordinate of foci $=(\pm ae,0)$

$=\left(\pm \cancel{3}\cdot\frac{2}{\cancel{3}},0\right)$

$=(\pm 2,0)$

length of latus rectum $=\frac{2b^2}{a}=\frac{2.5}{3}=\frac{10}{3}$

length of major axis $=2a=2\times 3=6$

length of minor axis $=2b=2\times\sqrt{5}=2\sqrt{5}$

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e) $5x^2 + 4y^2 = 1$

Sol^n

Given eq^n of ellipse

$$5x^2 + 4y^2 = 1$$

$$\Rightarrow \frac{x^2}{\frac{1}{5}} + \frac{y^2}{\frac{1}{4}} = 1 \quad \text{---(1)}$$

Comparing (1) with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

we get $a^2 = \frac{1}{5}$ $\quad b^2 = \frac{1}{4}$

$$\Rightarrow a = \frac{1}{\sqrt{5}} \qquad \Rightarrow b = \frac{1}{2}$$

As $a^2 < b^2$: So, major axis is along Y-axis

So, we know eccentricity $(e) = \sqrt{1 - \frac{a^2}{b^2}}$

$$= \sqrt{1 - \frac{1/5}{1/4}}$$

$$= \sqrt{\frac{1/4 - 1/5}{1/4}} = \sqrt{\frac{1/20}{1/4}}$$

$$= \sqrt{\frac{1}{20} \times 4} = \frac{1}{\sqrt{5}}$$

Coordinate of vertices $= (0, \pm b)$

$$= (0, \pm 1/2)$$

coordinate of foci $= (0, \pm be)$

$$= (0, \pm 1/2 \cdot \frac{1}{\sqrt{5}})$$

$$= (0, \pm 1/2\sqrt{5})$$

length of latus rectum $= \frac{2a^2}{b} = \frac{2 \times 1/5}{1/2} = \frac{4}{5}$

length of major axis $= 2b = 2 \cdot \frac{1}{2} = 1$

length of minor axis $= 2a = 2 \times \frac{1}{\sqrt{5}} = \frac{2}{\sqrt{5}}$

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7. Deduce the equation of ellipse in standard position with the following data

a) A focus at (-2,0) and a vertex at (5,0)

Sol^n

Given,

focus at $(-2,0) = (\pm ae, 0)$

vertex at $(5,0) = (\pm a, 0)$

So, $ae = 2$

$a = 5$

We know, $b^2 = a^2(1-e^2)$

$= a^2 - a^2e^2$

$= 5^2 - 2^2$

$= 21$

So, Req^d eq^n of ellipse $\Rightarrow \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$\Rightarrow \dfrac{x^2}{5^2} + \dfrac{y^2}{21} = 1$

$\Rightarrow \dfrac{x^2}{25} + \dfrac{y^2}{21} = 1$

b) Vertex at (0,10) and eccentricity $= \dfrac{4}{5}$

Sol^n

Given vertex (0,10)

$e = \dfrac{4}{5}$

Since x- coordinate of vertex is zero. So, major axis is along y-axis

i.e $b > a$

Here, vertex $= (0,10) = (0, \pm b)$

So, $b = 10$

We know $a^2 = b^2(1-e^2)$

$= b^2 - b^2e^2$

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$= 10^2 - 10^2 \times \frac{4^2}{5^2}$

$= 36$

Required equation of ellipse $\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{36} + \frac{y^2}{10^2} = 1$

$\Rightarrow \frac{x^2}{36} + \frac{y^2}{100} = 1$

c) focus at $(\pm 2, 0)$ and eccentricity at ~~(3,0)~~ $\frac{1}{2}$

Sol^n

Given

focus at $(\pm 2, 0) = (\pm ae, 0)$

~~So~~ eccentricity $= \frac{1}{2}$

So, $ae = 2$

$e = \frac{1}{2}$

Now,

$ae = 2$

$\Rightarrow a \cdot \frac{1}{2} = 2 \quad [\because e = 1/2]$

$\Rightarrow a = 4$

We know $b^2 = a^2(1 - e^2)$

$= \cancel{(4)^2}\ a^2 - a^2 e^2$

$= \cancel{(4)^2 - \left(\frac{1}{2}\right)^2} \quad = (4)^2 - (2)^2$

$= 16 - 4$

$= \cancel{16 - \frac{1}{4}} \quad = 12$

$= \cancel{\frac{63}{4}}$

Required equation of ellipse $\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\Rightarrow \frac{x^2}{(4)^2} + \frac{y^2}{(12)} = 1$$

$$\Rightarrow \frac{x^2}{16} + \frac{y^2}{12} = 1$$

d) A vertex at (0, 8) and passing through $\left(3, \frac{32}{5}\right)$

Sol^n

Given vertex $= (0, 8) = (0, b) \Rightarrow b = 8$

and through $\left(3, \frac{32}{5}\right)$

let eq^n of ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{8^2} = 1 \quad \text{---(1)}$$

Since (1) passes through $\left(3, \frac{32}{5}\right)$

$$\Rightarrow \frac{(3)^2}{a^2} + \frac{\left(\frac{32}{5}\right)^2}{8^2} = 1$$

$$\Rightarrow \frac{9}{a^2} + \frac{1024}{25 \times 64} = 1$$

$$\Rightarrow \frac{9}{a^2} + \frac{16}{25} = 1$$

$$\Rightarrow \frac{9}{a^2} = 1 - \frac{16}{25}$$

$$\Rightarrow \frac{9}{a^2} = \frac{9}{25}$$

$$\Rightarrow a^2 = 25$$

putting the value of a^2 in eq^n (1)

$$\frac{x^2}{25} + \frac{y^2}{64} = 1$$

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length of transverse axis $= 2a = 2 \times 12 = 24$
length of conjugate axis $= 2b = 2 \times 5 = 10$

b) $5x^2 - 20y^2 - 20x = 0$
Soln
Given eqn of hyberbola
$5x^2 - 20y^2 - 20x = 0$
$\Rightarrow x^2 - 4y^2 - 4x = 0$
$\Rightarrow x^2 - 4x - 4y^2 = 0$
$\Rightarrow (x^2 - 2 \cdot 2x + 2^2) - 4y^2 = 2^2$
$\Rightarrow (x-2)^2 - 4(y-0)^2 = 4$
Dividing both side by
$$\frac{(x-2)^2}{4} - \frac{(y-0)^2}{1} = 1$$
Comparing with
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
we get $a^2 = 4$ $\quad$ $b^2 = 1$
$\Rightarrow a = 2$ $\quad$ $\Rightarrow b = 1$
$h = 2, k = 0$
Since coeff of x^2 is +ve. So, transverse axis is parallel to x-axis
We know
eccentricity $(e) = \sqrt{1 + \frac{b^2}{a^2}}$
$= \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2}$

coordinate of vertice $= (h \pm a, k)$
$= (2 \pm 2, 0)$
$= (4, 0)$ & $(0, 0)$
Coordinate of foci $= (h \pm ae, k)$
$= \left(2 \pm 2 \cdot \frac{\sqrt{5}}{2}, 0\right)$

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length of latus rectum $= \frac{2b^2}{a} = \frac{2\cdot 1}{2} = 1$

length of transverse axis $: 2a = 2\times 2 = 4$

length of conjugate axis $: 2b = 2\times 1 = 2$

c) $16x^2 - 9y^2 + 96x - 72y + 144 = 0$

Solⁿ

Given curve

$16x^2 - 9y^2 + 96x - 72y + 144 = 0$

$\Rightarrow 16x^2 - 9y^2 + 96x - 72y = -144$

$\Rightarrow 16x^2 + 96x - 9y^2 - 72y = -144$

$\Rightarrow 16(x^2 + 6x) - 9(y^2 + 8y) = -144$

$\Rightarrow 16(x^2 + 2\cdot 3x + 3^2) - 9(y^2 + 2\cdot 4y + 4^2) = -144 + 16\cdot 3^2 - 9\cdot 4^2$

$\Rightarrow 16(x+3)^2 - 9(y+4)^2 = -144$

dividing both side by -144

$\Rightarrow -\frac{(x+3)^2}{9} + \frac{(y+4)^2}{16} = 1$

Comparing (1) with

$-\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

we get $h = -3$, $k = -4$

$a^2 = 9 \quad b^2 = 16$

$\Rightarrow a = 3 \quad b = 4$

Since coefficient of y^2 is +ve. So, transverse axis is parallel to y-axis

eccentricity $e = \sqrt{1 + \frac{a^2}{b^2}}$

$= \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

centre $= (h, k) = (-3, -4)$

coordinate of vertices $= (h, k \pm b)$

$= (-3, -4 \pm 4)$

$= (-3, 0)$ & $(-3, -8)$

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coordinate of foci $= (h, k \pm be)$

$$= \left(-3, -4 \pm 4 \cdot \frac{5}{4}\right)$$

$$= (-3, -4 \pm 5)$$

$$= (-3, 1), (-3, -9)$$

length of latus rectum $= \frac{2a^2}{b} = \frac{2 \cdot 9}{4} = \frac{9}{2}$

length of transverse axis $= 2b = 2 \times 4 = 8$

length of conjugate axis $= 2a = 2 \times 3 = 6$

3. find the equation of hyperbola in standard position satisfying the following conditions

a) transverse and conjugate axis are respectively 4 and 5.

Sol^n

Given transverse axis $\Rightarrow 2a = 4$

$$\Rightarrow a = 2$$

conjugate axis $\Rightarrow 2b = 5$

$$\Rightarrow b = \frac{5}{2}$$

Now

we know standard eq^n of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{2^2} - \frac{y^2}{\left(\frac{5}{2}\right)^2} = 1$$

$$\Rightarrow \frac{x^2}{4} - \frac{4y^2}{25} = 1$$

which is required equation

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b) foci at $(\pm 3, 0)$ and eccentricity $\frac{3}{2}$

Sol^n

Given foci $= (\pm 3, 0) = (\pm ae, 0)$

$\Rightarrow ae = 3$

and eccentricity $(e) = \frac{3}{2}$

Here,

$ae = 3$

$\Rightarrow a \cdot \frac{\cancel{3}}{2} = \cancel{3}$

$\Rightarrow a = 2$

Now we know

$b^2 = a^2(e^2 - 1)$

$= (2)^2\left(\left(\frac{3}{2}\right)^2 - 1\right)$

$= 4\left(\frac{9}{4} - 1\right)$

$= \cancel{4}\frac{(9-4)}{\cancel{4}}$

$= 5$

Required equation is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{2^2} - \frac{y^2}{5} = 1$

$\Rightarrow \frac{x^2}{4} - \frac{y^2}{5} = 1$

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c) latus rectum is 4 and eccentricity is 3.

Sol^n

Given, latus rectum $\Rightarrow \frac{2b^2}{a} = 4 \Rightarrow b^2 = 2a$

eccentricity $(e) = \sqrt{1 + \frac{b^2}{a^2}}$

or, $3 = \sqrt{1 + \frac{2a}{a^2}}$

or, $3 = \sqrt{1 + \frac{2}{a}}$

$\Rightarrow 3 = \sqrt{\frac{a+2}{a}}$

$\Rightarrow 3^2 = \frac{a+2}{a}$

$\Rightarrow 9a = a+2$

$\Rightarrow 9a - a = 2$

$\Rightarrow 8a = 2$

$\Rightarrow a = \frac{1}{4}$

Then,

$b^2 = 2 \cdot \frac{1}{4} = \frac{1}{2}$

Now

Required equation is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{\left(\frac{1}{4}\right)^2} - \frac{y^2}{\left(\frac{1}{2}\right)} = 1$

$\Rightarrow 16x^2 - 2y^2 = 1$

d. vertex at (0, 8) and passing through $(4, 8\sqrt{2})$

Sol^n

vertex: $(0, \pm b) = (0, 8)$

i.e $b = 8$

passing point $(4, 8\sqrt{2})$

Now

Required equation is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{——} (1)$$

Since it passes through $(4, 8\sqrt{2})$

$$\frac{(4)^2}{a^2} - \frac{(8\sqrt{2})^2}{(8)^2} = -1$$

or, $\frac{16}{a^2} - \frac{\cancel{8^2} \times 2}{\cancel{8^2}} = -1$

or, $\frac{16}{a^2} - 2 = -1$

$\Rightarrow$ or, $\frac{16}{a^2} = -1 + 2$

or, $\frac{16}{a^2} = 1$

or $a^2 = 16$

Then,

from (1)

$$\frac{x^2}{16} - \frac{y^2}{8^2} = -1$$

$\Rightarrow \frac{x^2}{16} - \frac{y^2}{64} = -1$ which is the required equation

e) vertices at $(0, \pm 7)$, $e = \frac{4}{3}$

Soln

vertices $= (0, \pm 7) = (0, \pm b)$

i.e $b = 7$

$e = \frac{4}{3}$

We know

$a^2 = b^2(e^2 - 1)$

$= b^2e^2 - b^2$

$= 7^2 \cdot \left(\frac{4}{3}\right)^2 - (7)^2$

$= \frac{784}{9} - 49$

$= \frac{343}{9}$

Now

Required equation is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

or $\frac{x^2}{\frac{343}{9}} - \frac{y^2}{7^2} = -1$

or $\frac{9x^2}{343} - \frac{y^2}{49} = -1$

or $\frac{9x^2 - 7y^2}{343} = -1$

or $9x^2 - 7y^2 = -343$

or $9x^2 - 7y^2 + 343 = 0$

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f. focus at $(6, 0)$ and a vertex at $(4, 0)$

Sol^n

~~focus = (6, 0) = (±a, 0)~~

$\cancel{\Rightarrow a = 6}$

focus : $(6, 0) = (\pm ae, 0)$

$\Rightarrow ae = 6$

vertex : $(4, 0) = (\pm a, 0)$

$\Rightarrow a = 4$

Now

$ae = 6$

$\Rightarrow 4e = 6$ [$\because a = 4$

$\Rightarrow e = \frac{6}{4}$

$\Rightarrow e = \frac{3}{2}$

We know, $b^2 = a^2(e^2 - 1)$

$= \cancel{4} a^2e^2 - a^2$

$= (4)^2 \left(\frac{3}{2}\right)^2 - (4)^2$

$= \cancel{16}^{4} \cdot \frac{9}{\cancel{4}} - 16$

$= 36 - 16$

$= 20$

Required equation is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{(4)^2} - \frac{y^2}{20} = 1$

$\Rightarrow \frac{x^2}{16} - \frac{y^2}{20} = 1$

e) Passing through the points (1,4) and (-3,2).

Sol^n

let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ —(i)

Given points (1,4) & (-3,2)

taking (1,4)

$\frac{1^2}{a^2} + \frac{4^2}{b^2} = 1$

$\Rightarrow \frac{1}{a^2} + \frac{16}{b^2} = 1$ —(ii)

Also (-3,2) lies in (1)

$\frac{(-3)^2}{a^2} + \frac{2^2}{b^2} = 1$

$\Rightarrow \left(\frac{9}{a^2} + \frac{4}{b^2} = 1\right) \times 4$ ~~—(iii)~~

$\Rightarrow \frac{36}{a^2} + \frac{16}{b^2} = 4$ —(iii)

Solving (ii) and (iii)

$\frac{1}{a^2} + \cancel{\frac{16}{b^2}} = 1$

$\frac{36}{a^2} + \cancel{\frac{16}{b^2}} = 4$

$- \quad - \quad -$

$\frac{-35}{a^2} = -3$

$\Rightarrow \frac{35}{a^2} = 3$

$\Rightarrow \frac{35}{3} = a^2$

putting value of a^2 in (ii)

$\frac{1}{\frac{35}{3}} + \frac{16}{b^2} = 1$

$\Rightarrow \frac{3}{35} + \frac{16}{b^2} = 1$

$\Rightarrow \frac{563}{\cancel{300b^2}} \times 4 \quad 3b^2 + 16 = 35$

$\Rightarrow b^2 = \frac{563}{35}$

putting value of a^2 and b^2 in (1)

$$\frac{x^2}{\frac{35}{3}} + \frac{y^2}{\frac{563}{35}} = 1$$

$$\Rightarrow \frac{3x^2}{35} + \frac{35y}{563} = 1$$

3. find the eccentricity, the co-ordinates of the centre and foci of the following

a) $\frac{(x+2)^2}{16} + \frac{(y-5)}{9} = 1$

Sol^n

given eq^n of ellipse

$$\frac{(x+2)^2}{16} + \frac{(y-5)^2}{9} = 1$$

~~comparing~~

$$\Rightarrow \frac{(x+2)^2}{4^2} + \frac{(y-5)^2}{3^2} = 1 \quad -(1)$$

Comparing (1) with

$$\frac{(x-h)^2}{a^2} + \frac{(y-5)^2}{b^2}$$

we get $h = -2$, $k = 5$

$a^2 = 16 \qquad b^2 = 9$

$\Rightarrow a = 4 \qquad \Rightarrow b = 3$

As $a^2 > b^2$. So, major axis $\parallel$ to X-axis.

Now centre $(h, k) = (-2, 5)$

~~Foci = $(h, k \pm be)$~~, $e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

$= (\cancel{-2, 5 \pm \frac{\sqrt{7}}{3}})$

$= (\cancel{-2,}$

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foci: $(h \pm ae, k)$
$= \left(-2 \pm \frac{\sqrt{7}}{4}, 5\right)$

b) $\frac{(x-3)^2}{9} + \frac{(y-5)^2}{25} = 1$

Sol^n

Given ellipse

$\frac{(x-3)^2}{9} + \frac{(y-5)^2}{25} = 1$ — (1)

Comparing (1) with

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

we get $h = 3$ and $k = 5$

$a^2 = 9$ $\quad$ $b^2 = 25$

$\Rightarrow a = 3$ $\quad$ $\Rightarrow b = 5$

As $a^2 < b^2$ So major axis $\parallel$ to Y-axis

Now centre $(h, k) = (3, 5)$

eccentricity $(e) = \sqrt{1 - \frac{a^2}{b^2}}$

$= \sqrt{1 - \frac{9}{25}}$

$= \sqrt{\frac{16}{25}} = \frac{4}{5}$

foci: $(h, k \pm be)$

$= \left(3, 5 \pm 5 \times \frac{4}{5}\right)$

$= (3, 5 \pm 4)$

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c) $x^2+4y^2-4x+24y+24=0$

Sol^n

Given eq^n of ellipse

$x^2+4y^2-4x+24y+24=0$

$\Rightarrow x^2-4x+4y^2+24y=-24$

$\Rightarrow (x^2-2\cdot 2x+2^2)+4(y^2+6y)=-24+2^2$

$\Rightarrow (x-2)^2+4(y^2+2\cdot 3\cdot y+3^2)=-24+2^2+12^2$

$\Rightarrow (x-2)^2+4(y+3)^2=124$

Dividing both side by 124

$\Rightarrow \dfrac{(x-2)^2}{124}+\dfrac{(y+3)^2}{31}=1$

we get $h=2$, $y=-3$

$a^2=124$ $\quad$ $b^2=31$

$\Rightarrow a=2\sqrt{31}$ $\quad$ $\Rightarrow b=\sqrt{31}$

As $a^2>b^2$. So, major axis is $\parallel$ to x axis

Now, centre $(h,k)=(2,-3)$

eccentricity $(e)=\sqrt{1-\dfrac{b^2}{a^2}}$

$=\sqrt{1-\dfrac{31}{124}}$

$=\sqrt{\dfrac{3}{4}}=\dfrac{\sqrt{3}}{2}$

foci $=(h\pm ae,k)$

$=\left(2\pm 2\sqrt{31}\cdot\dfrac{\sqrt{3}}{2},k\right)$

$=\left(2\pm\sqrt{31}\cdot\sqrt{3},k\right)$

$=(2\pm\sqrt{93},k)$

d) $9x^2+5y^2-30y=0$

Sol^n

Given eq^n of ellipse

$9x^2+5y^2-30y=0$

$\Rightarrow 9x^2+5(y^2-6y)=0$

$\Rightarrow 9x^2+5(y^2-2\cdot y\cdot 3+3^2)=45$

$\Rightarrow 9x^2+5(y-3)^2$

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Dividing both side by 45

$$\Rightarrow \frac{9x^2}{45} + \frac{5(y-3)^2}{45} = 1$$

$$\Rightarrow \frac{(x-0)^2}{5} + \frac{(y-3)^2}{9} = 1 \quad \text{---(1)}$$

Comparing (1) with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$\Rightarrow$ we get $h=0$, $k=3$

$a^2 = 5$ $\quad$ $b^2 = 9$

$\Rightarrow a = \sqrt{5}$ $\quad \Rightarrow b = 3$

As $a^2 < b^2$, So major axis $\parallel$ to Y-axis

Now centre $(h,k) = (0,3)$

eccentricity $(e) = \sqrt{1 - \frac{a^2}{b^2}}$

$$= \sqrt{1-\frac{5}{9}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

foci $= (h, k \pm be)$

$= (0, 3 \pm 3 \cdot \frac{2}{3})$

$= (0, 3 \pm 2)$

$= (0,5) \& (0,1)$

e) $9x^2 + 4y^2 + 40y + 18x + 73 = 0$

Sol^n

Given eq^n of ellipse

$9x^2 + 4y^2 + 40y + 18x + 73 = 0$

$\Rightarrow 9x^2 + 18x + 4y^2 + 40y = -73$

$\Rightarrow 9(x^2 + 2x) + 4(y^2 + 10y) = -73$

$\Rightarrow 9[x^2 + 2 \cdot x \cdot 1 + 1^2] + 4(y^2 + 2 \cdot y \cdot 5 + 5^2] = -73 + 9 + 4 \times 25$

$\Rightarrow 9(x+1)^2 + 4(y+5)^2 = 36$

dividing both side by 36

$$\frac{(x+1)^2}{36} + \frac{(y+5)^2}{9} = 1 \quad \text{---(1)}$$

Comparing (1) with

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

we get $h = -1$, $k = -5$

$a^2 = 4$ $\quad$ $b^2 = 9$

$\Rightarrow a = 2$ $\quad$ $\Rightarrow b = 3$

As $a^2 < b^2$, so major axis is $\parallel$ to Y-axis

Now centre $(h, k) = (-1, -5)$

eccentricity $e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$

~~coord~~ foci $= (h, k \pm be)$

$$= \left(-1, -5 \pm 3 \cdot \frac{\sqrt{5}}{3}\right)$$

$$= (-1, -5 \pm \sqrt{5})$$

4. find the equation of ellipse whose

a) major axis is twice its minor axis and which passes through the point $(0, 1)$

Sol^n

Given major axis = 2 minor axis

$\Rightarrow 2a = 2(2b)$

$\Rightarrow a = 2b$ —— (1)

let eq^n of ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{(2b)^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2 + 4y^2}{4b^2} = 1$$

$\Rightarrow x^2 + 4y^2 = 4b^2$ —— (11)

Also given (11) passes through $(0, 1)$

$0^2 + 4 \times 1^2 = 4b^2$

$\Rightarrow 4b^2 = 4$

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So, from (ii)

$x^2+4y^2=4$ which is required equation of ellipse.

b) latus rectum 3 and eccentricity is $\frac{1}{\sqrt{2}}$.

Sol^n

latus rectum $= 3 = \frac{2b^2}{a} \Rightarrow 2b^2 = 3a \Rightarrow b^2 = \frac{3a}{2}$

~~we~~ $e = \frac{1}{\sqrt{2}}$

Now

~~latus rectum = 3~~

$e = \frac{1}{\sqrt{2}}$

$\Rightarrow e^2 = \frac{1}{2}$

$\Rightarrow 1 - \frac{b^2}{a^2} = \frac{1}{2}$

$\Rightarrow \frac{-b^2}{a^2} = \frac{1}{2} - 1$

$\Rightarrow \frac{b^2}{a^2} = \frac{1}{2}$

$\Rightarrow b^2 = \frac{a^2}{2}$ —— (1)

putting value of b^2 in (1)

$\Rightarrow \frac{3\cancel{a}}{\cancel{2}} = \frac{a^{\cancel{2}}}{\cancel{2}}$

$\Rightarrow a = 3$

when $a = 3$ from (1)

$b^2 = \frac{3^2}{2} = \frac{9}{2}$

Now required equation of ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{(9/2)} = 1$

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$\Rightarrow \frac{x^2}{9} + \frac{2y^2}{9} = 1$

$\Rightarrow x^2 + 2y^2 = 1$

c) distance between the two foci is 8 and the semi latus rectum is 6.

Sol^n

Given,

distance between two foci = 8

$\Rightarrow 2ae = 8$

$\Rightarrow ae = 4 \quad -(1)$

Also semi-latus rectum $= \frac{1}{\cancel{2}}\left(\frac{\cancel{2}b^2}{a}\right)$

$$6 = \frac{b^2}{a^2}$$

$$\Rightarrow b^2 = 6a \quad -(ii)$$

We know

$b^2 = a^2(1-e^2)$

$\Rightarrow b^2 = a^2 - a^2e^2$

$\Rightarrow 6a = a^2 - 4$ [using (1) & (ii)]

$\Rightarrow a^2 - 6a - 16 = 0$

$\Rightarrow (a-8)(a+2) = 0$

$\Rightarrow a = -2, 8$

Since a cannot be -ve so,

$a = 8$

Now putting the value of a in (ii)

$b^2 = 6 \times 8 = 48$

Now

Req^d equation of ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{(8)^2} + \frac{y^2}{48} = 1$

$\Rightarrow \frac{x^2}{64} + \frac{y^2}{48} = 1$

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d. latus rectum is equal to the half its major axis and which passes through the point (4,3).

Sol^n

latus rectum is equal to half its major axis

$\Rightarrow \dfrac{2b^2}{a} = \dfrac{1}{\cancel{2}} \cdot \cancel{2}a$

$\Rightarrow 2b^2 = a^2$

$\Rightarrow a^2 = 2b^2$ —— (i)

let the required equation of ellipse be

~~Since it passes through (4,3). So,~~

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{—— (ii)}$$

Since it passes through (4,3).

$\Rightarrow \dfrac{(4)^2}{2b^2} + \dfrac{3^2}{b^2} = 1$

$\Rightarrow \dfrac{16}{2b^2} + \dfrac{9}{b^2} = 1$

$\Rightarrow \dfrac{16+18}{2b^2} = 1$

$\Rightarrow 34 = 2b^2$

$\Rightarrow b^2 = 17$

Now

putting value of b^2 in (i)

$a^2 = 2 \times 17 = 34$

Then

putting the value of a^2 and b^2 in eq^n (ii) we get

$$\frac{x^2}{34} + \frac{y^2}{17} = 1$$

$\Rightarrow \dfrac{x^2 + 2y^2}{34} = 1$

$\Rightarrow x^2 + 2y^2 = 34$

which is the required equation

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e) foci are at $(\pm 2, 0)$ and length of latus rectum is 6.

Sol^n

foci at $(\pm 2, 0) = (ae, 0)$

i.e $ae = 2$

and

length of latus rectum $= 6$

$$\frac{2b^2}{a} = 6$$

or $b^2 = 3a$ —— (1)

~~we know~~ $\cancel{a^2 = b^2(1-ae}$

We know $b^2 = a^2(1-e^2)$

$\Rightarrow b^2 = a^2 - a^2e^2$

$\Rightarrow 3a = a^2 - 4$

$\Rightarrow a^2 - 3a - 4 = 0$

$\Rightarrow (a-4)(a+1) = 0$

$\Rightarrow a = 4, -1$

Since a cannot be -ve

$a = 4$

Now

~~The~~ putting value of a in (1)

$b^2 = 3 \times 4 = 12$

Then

The required equation ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{(4)^2} + \frac{y^2}{(12)} = 1$$

$$\Rightarrow \frac{x^2}{16} + \frac{y^2}{12} = 1$$

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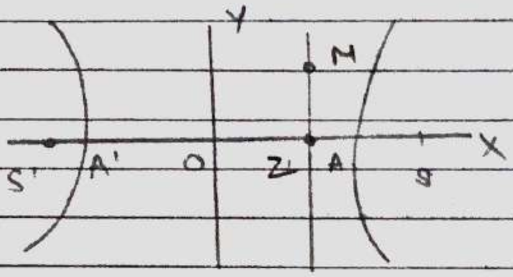
Hyberbola

Equation of Hyberbola in the standard position

a. When transverse axis is along X-axis.



Equation:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

1) Co-ordinate of vertices = $(\pm a, 0)$
2) Co-ordinate of foci = $(\pm ae, 0)$
3) Eccentricity $e = \sqrt{1 + \frac{b^2}{a^2}}$

4) length of latus rectum = $\frac{2b^2}{a}$

5) length of transverse axis = $2a$
6) length of conjugate axis = $2b$
7) Eq^n of directrices.

Note: To find transverse axis lies along which axis we need to make 1 in RHS +ve
Then (1) of coeff of x^2 is +ve then transverse axis is along X-axis.
(2) If coeff of y^2 is +ve, then transverse axis is along Y-axis

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Exercise 10.2

1. find the eccentricity, co-ordinates of the vertices and foci, length of latus rectum, length of latus rectum, length of transverse axis and conjugate axis of hyberbola

a) $\frac{x^2}{25} - \frac{y^2}{16} = 1$

Soln

Given equation of hyberbola

$$\frac{x^2}{25} - \frac{y^2}{16} = 1 \quad \text{— (1)}$$

Comparing (1) with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

we get $a^2 = 25$ $\quad b^2 = 16$

$\Rightarrow a = 5$ $\quad \Rightarrow b = 4$

Since coeff of x^2 in eqn (1) is +ve - So, transverse axis is along x-axis. So, we know

eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}}$

$$= \sqrt{1 + \frac{16}{25}}$$

$$= \frac{\sqrt{41}}{5}$$

coordinate of vertices $= (\pm a, 0)$

$= (\pm 5, 0)$

coordinate of foci: $(\pm ae, 0)$

$= (\pm 5 \cdot \frac{\sqrt{41}}{5}, 0)$

$= (\pm\sqrt{41}, 0)$

length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 16}{5} = \frac{32}{5}$

length of transverse axis = $2a = 2 \times 5 = 10$
length of conjugate axis = $2b = 2 \times 4 = 8$

b) $\dfrac{x^2}{9} - \dfrac{y^2}{25} = -1$

Sol^n

Given equation of hyperbola

$$\frac{x^2}{9} - \frac{y^2}{25} = -1 \quad -(1) \quad \text{i.e. } -\frac{x^2}{9} + \frac{y^2}{25} = 1$$

Comparing (1) with

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

we get

$a^2 = 9 \qquad b^2 = 25$

$\Rightarrow a = 3 \qquad \Rightarrow b = 5$

Since coeff of y^2 in eq^n (1) is +ve. So, transverse is long y-axis.

We know

eccentricity $= \sqrt{1 + \dfrac{a^2}{b^2}}$

$$= \sqrt{1 + \frac{9}{25}} = \sqrt{\frac{34}{25}} = \frac{\sqrt{34}}{5}$$

coordinate of vertices $= (0, \pm b)$
$= (0, \pm 5)$

coordinate of foci $= (0, \pm be)$
$= (0, \pm \cancel{5} \cdot \dfrac{\sqrt{34}}{\cancel{5}})$

$= (0, \pm\sqrt{34})$

length of latus rectum $= \dfrac{2a^2}{b} = \dfrac{2 \cdot 3^2}{5} = \dfrac{2 \times 9}{5} = \dfrac{18}{5}$

length of transverse axis $= 2b = 2 \times 5 = 10$
length of conjugate axis $= 2a = 2 \times 3 = 6$

c) $3x^2 - 4y^2 = 36$

Sol^n

Given equation

$3x^2 - 4y^2 = 3$

Dividing both

$\dfrac{3x^2}{36} - \dfrac{4y^2}{36}$

$\Rightarrow \dfrac{x^2}{12} - \dfrac{y^2}{9} = 1$

Comparing (1

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2}$

we get $a^2 = 1$

$\Rightarrow a = \sqrt{}$

i.e $a = 2$

Since coeff

transverse ax

We know

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c) $3x^2 - 4y^2 = 36$

Sol^n

Given equation of hyberbola

$$3x^2 - 4y^2 = 36 \quad \text{---(1)}$$

Dividing both side by 36

$$\frac{3x^2}{36} - \frac{4y^2}{36} = 1$$

$$\Rightarrow \frac{x^2}{12} - \frac{y^2}{9} = 1 \quad \text{---(1)}$$

Comparing (1) with^n

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

we get $a^2 = 12$ $\quad b^2 = 9$

$\Rightarrow a = \sqrt{12}$ $\quad \Rightarrow b = 3$

i.e $a = 2\sqrt{3}$

Since coeff of x^2 in eq^n (1) is +ve. So, transverse axis is along Y-axis

We know

$$\text{eccentricity} = \sqrt{1 + \frac{b^2}{a^2}}$$

$$= \sqrt{1 + \frac{9}{12}} = \sqrt{\frac{21}{12}} = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2}$$

coordinate of vertices $= (\pm a, 0)$

$= (\pm\sqrt{12}, 0)$

$= (\pm 2\sqrt{3}, 0)$

coordinate of foci $: (\pm ae, 0)$

$= (\pm \cancel{2}\sqrt{3} \cdot \frac{\sqrt{7}}{\cancel{2}}, 0)$

$= (\pm\sqrt{21}, 0)$

$\frac{18}{5}$

length of latus rectum $= \frac{2b^2}{a} = \frac{2 \cdot 3^2}{\sqrt{12}} = \frac{18}{2\sqrt{3}} = \frac{9}{\sqrt{3}} = \frac{3\sqrt{3} \times \sqrt{3}}{\sqrt{3}}$

$= 3\sqrt{3}$

length of transverse axis $: 2a = 2 \times 2\sqrt{3} = 4\sqrt{3}$

length of conjugate axis $: 2b = 2 \times 3 = 6$

2. find the eccentricity, coordinate of vertices and foci, length of latus rectum, length of transverse axis and conjugate axis of the hyberbola

a) $\dfrac{(x+1)^2}{144} - \dfrac{(y-1)^2}{25} = 1$

Sol^n

Given equation of hyberbola is

$$\frac{(x+1)^2}{144} - \frac{(y-1)^2}{25} = 1 \quad -(1)$$

Comparing (1) with

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

we get $\cdot a^2 = 144$ $\qquad b^2 = 25$

$\Rightarrow a = 12$ $\qquad \Rightarrow b = 5$

$h = -1$ & $k = 1$

Since coeff of x^2 is +ve. So, transverse axis is parallel to x-axis

We know

eccentricity (e) $= \sqrt{1 + \dfrac{b^2}{a^2}}$

$= \sqrt{1 + \dfrac{25}{144}} = \sqrt{\dfrac{169}{144}} = \dfrac{13}{12}$

coordinate of vertices $= (h \pm a, k)$

$= (-1 \pm 12, 1)$

$= (11, 1)$ & $(-13, 1)$

coordinate of foci $= (h \pm ae, k)$

$= \left(-1 \pm \dfrac{13}{\cancel{12}} \cdot \cancel{12}, K\right)$

$= (-1 \pm 13, 1)$

$= (-14, 1)$ & $(12, 1)$

length of latus rectum $= \dfrac{2b^2}{a} = \dfrac{2 \times 25}{12} = \dfrac{25}{6}$