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Group A

1. What is the amplitude of a complex number i
→ 90°

2 How many three digit numbers can be formed using the integer 4, 5, 6, 8 with repetition?
→ 64

3. Which of the following is equal to $\cot^{-1} x$?
→

4. Which one of the following expressions is a trigonometric equation?
→

5. What is the area of a triangle with vertices (1, -1, 0), (0, -1, 0) and (0, 0, 1)?
→ $1/\sqrt{2}$

6. In a school ground, the area is the set of all points in a plane. The sum of whose distances from two fixed points in constant. The enclosed area represented gives a
→ ellipse

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7. What is the probability of getting 53 Friday or Saturdays or both in a leap year?
→ $3/7$

8. If the degree of the differential equation
→ 4

9. What is the point on the curve $y = 2x^2 - 4x - 3$ at which the tangent of the curve is parallel to the line $4x - y + 2 = 0$?
→

10. While solving a system of three linear equations in the three variables x, y, z by using Gauss elimination method, what happens to the system if the third variable z is found to be a free variable?
→ The system becomes consistent with infinite solutions.

11. If X be the output vector, D be the demand vector, A be the input coefficient matrix and I be the unit matrix then the Leontif's technology matrix is given by
→

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Group B

12. Given, 1, 2, 3, 4, ... ∞ is set of natural num

sol', So,

expression for sum of (n+2) natural number

$$= \Sigma(n+2)$$

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13. Given;

Sol: $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$

To prove: $C_0C_3 + C_1C_4 + C_2C_5 + \dots + C_{n-3}C_n = \frac{(2n)!}{(n-3)!(n+3)!}$

We have,

$(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_{n-3}x^{n-3} + \dots + C_nx^n$ — (i)

and

$(x+1)^n = C_0x^n + C_1x^{n-1} + \dots + C_3x^{n-3} + \dots + C_{n-1}x + C_n$ — (ii)

Multiplying both (i) and (ii)

$(1+x)^{2n} = C_0 + C_1x + C_2x^2 + \dots + C_{n-3}x^{n-3} + \dots + C_nx^n \times$
$C_0x^n + C_1x^{n-1} + \dots + C_3x^{n-3} + \dots + C_{n-1}x + C_n$ — (iii)

By actual multiplication, the term containing x^{n-3} in the RHS of (iii) is

$(C_0C_3 + C_1C_4 + \dots + C_{n-3}C_n)\, x^{n-3}$

Also, the term containing x^{n-3} in the expansion of RHS of (iii) is

$^{2n}C_{n-3}\, x^{n-3}$

$\left[t_{r+1} = {}^{2n}C_r\, 1^{2n-r}\, x^r \right.$
$\left. \text{put } r = n-3 \right]$

Since, eq^n (iii) is an identity, equating the like terms we get,

$C_0C_3 + C_1C_4 + \dots + C_{n-3}C_n = {}^{2n}C_{n-3}$

$= \frac{2n!}{(2n-n+3)!\,(n-3)!}$

$= \frac{2n!}{(n+3)!\,(n-3)!}$ proved

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14a. $\sin\left(\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}\right)$

sol.

$= \sin\left(\cos^{-1}\left(\frac{4}{5}\times\frac{12}{13}-\sqrt{1-\left(\frac{4}{5}\right)^2}\cdot\sqrt{1-\left(\frac{12}{13}\right)^2}\right)\right)$

$= \sin\left(\cos^{-1}\left(\frac{48}{65}-\frac{3}{\cancel{5}}\cdot\frac{\cancel{5}}{13}\right)\right)$

$= \sin\cdot\cos^{-1}\left(\frac{48}{65}-\frac{3}{13}\right)$

$= \sin\cos^{-1}\left(\frac{33}{65}\right)$

Let $\cos^{-1}\left(\frac{33}{65}\right)=A$

$\Rightarrow \cos A=\frac{33}{65}=\frac{b}{h}$ $\left[\begin{array}{l} b=33,\ h=65 \\ p=\sqrt{h^2-b^2}=56 \end{array}\right]$

$\Rightarrow \sin A=\frac{56}{65}$

$\Rightarrow A=\sin^{-1}\left(\frac{56}{65}\right)=\cos^{-1}\left(\frac{33}{65}\right)$

Now,

$\sin\cdot\sin^{-1}\left(\frac{56}{65}\right)$

$=\frac{56}{65}$

14b. Given vector are :-

$\vec{a} = (1,0,2) = \vec{i} + 0\vec{j} + 2\vec{k}$

and

$\vec{b} = (2,0,1) = 2\vec{i} + 0\vec{j} + \vec{k}$

Now,

vector perpendicular to $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b}$

So, $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 2 \\ 2 & 0 & 1 \end{vmatrix}$

$= \vec{i}(0-0) - \vec{j}(1-4) + \vec{k}(0-0)$

$= +3\vec{j}$

$\therefore \vec{a} \times \vec{b} = 3\vec{j}$

Now,

$|\vec{a} \times \vec{b}| = \sqrt{(3)^2} = 3$

Now,

Unit vector $= \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

$= \dfrac{3\vec{j}}{3}$

$= \vec{j}$

$= (0,1,0)$

15.
Sol; calculation of co-rrelation

Ages (x)	Weights (y)	xy	x^2	y^2
13	45	585	169	2025
15	41	615	225	1681
14	32	448	196	1024
16	45	720	256	2025
18	50	900	324	2500
21	55	1155	441	3025
$\Sigma x = 97$	$\Sigma y = 268$	$\Sigma xy = 4423$	$\Sigma x^2 = 1611$	$\Sigma y^2 = 12$[illegible]

Here,

$N = 6$

Now, Regression equation of Y on X is given by

$$y - \overline{y} = b_{yx}(x - \overline{x}) \quad \text{— (1)}$$

where

$$\overline{y} = \frac{\Sigma y}{N} = \frac{268}{6} = 44.667$$

and $$\overline{x} = \frac{\Sigma x}{N} = \frac{97}{6} = 16.1667$$

$$b_{yx} = \frac{N\Sigma xy - \Sigma x \Sigma y}{N\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{6 \times 4423 - 97 \times 268}{6 \times 1611 - (97)^2}$$

$$= \frac{542}{257}$$

$$= 2.10$$

So, eqn ⓘ become,

$$y - 44.667 = 2.10(x - 16.1667)$$

$$\Rightarrow y - 44.667 = 2.10x - 34.094$$

$$\Rightarrow y = 2.10x + 10.573$$

Now,

when age $(x) = 17$ then weight is

$$y = 2.10 \times 17 + 10.573$$

$$y = 46.3 \text{ kg}$$

Now,

$$r = \frac{N\Sigma XY - \Sigma X \Sigma Y}{\sqrt{N\Sigma X^2 - (\Sigma X)^2}\sqrt{N\Sigma Y^2 - (\Sigma Y)^2}}$$

$$= \frac{6 \times 4423 - 97 \times 263}{\sqrt{6 \times 1611 - (97)^2}\sqrt{6 \times 12280 - (268)^2}}$$

$$= \frac{542}{\sqrt{257 \times 1856}}$$

$$r = 0.784$$

Thus,

There is positive correlation between two variable i.e age and weights.

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16a. What is the relation between continuity and differentiability of a function at a point?

→ If, A function at a point is differentiable then it is also continous at same point. But if a function is continuous at a point that it may not be differentiable at same point.
i.e

Differentiability ⇒ continuity.
But converse may not be true.

b. What is the first principle of differentiation

→

c. Write any one condition here we can use the derivative to find the limit of a function.

→ when function takes the form of $\left(\frac{\infty}{\infty}\right.$ or $\frac{0}{0}$, then we can use derivative to find the limit.

That is;

in L' Hospital rule we use derivative to find limit of a function.

d. Write one difference between differentiation and integration of a function.

→

e. In the expression $\int f(x)\,dx = F(x) + c$, what is the relation between $f(x)$ and $F(x)$ from differentiation point of view?

→

In the expression ; $f(x)\,dx = F(x) + c$

$$\frac{d\,F(x)}{dx} = f(x) \quad \text{or;}$$

$$\int f(x)\,dx = F(x)$$

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17 (a) Sol^n :- We have;

$$x^{\sin y} = y^{\cos x}$$

Taking 'ln' on the both side.

$$\ln x^{\sin y} = \ln y^{\cos x}$$

$$\Rightarrow \sin y \ln x = \cos x \ln y$$

Diff. both side w.r.t. x

$$\frac{d(\sin y \ln x)}{dx} = \frac{d(\cos x \ln y)}{dx}$$

$$\Rightarrow \sin y \frac{d\ln x}{dx} + \ln x \frac{d\sin y}{dx} = \cos x \frac{d\ln y}{dx} + \ln y \frac{d\cos x}{dx}$$

$$\Rightarrow \sin y \frac{1}{x} + \ln x \cos y \frac{dy}{dx} = \cos x \frac{1}{y}\frac{dy}{dx} - \ln y \sin x$$

$$\Rightarrow \frac{\sin y}{x} + \ln y \sin x = \frac{dy}{dx}\left(\frac{\cos x}{y} - \ln x \cos y\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{\sin y}{x} + \ln y \sin x}{\frac{\cos x}{y} - \ln x \cos y}$$

$$\therefore = \frac{y(\sin y + x \ln y \cdot \sin x)}{x(\cos x - y \ln x \cos y)}$$

$$\therefore \frac{dy}{dx} = \frac{-y[x \ln y \sin x + \sin y]}{x[y \ln x \cos y - \cos x]}$$

$H_{\frac{11}{5}}$

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17b. Sol;
$$\int \frac{dx}{x(1-x)}$$
$$= \int \left(\frac{1}{x} + \frac{1}{1-x}\right) dx$$
$$= [\ln x - \ln(1-x)] + c$$
$$= \ln\left(\frac{x}{1-x}\right) + c$$

18.
Sol; Given system of eqn are :-
$$x + 2y \leq 25$$
$$3x + 2y \leq 28$$
$$F = 18x + 8y$$
Introducing slack variable in above eqn.
$$x + 2y + s = 25$$
$$3x + 2y + t = 28$$
$$-18x - 8y + F = 0$$
writing above eqn in cannonical form:
$$x + 2y + 1s + 0t + 0F = 25$$
$$3x + 2y + 0s + t + 0F = 28$$
$$-18x - 8y + 0s + 0t + F = 0$$

Supplying simplex tableau for above cannonical form :-

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B.V	x	y	s	t	F	RHS	RHS
s	1	2	1	0	0	25	25
t	3	2	0	1	0	28	9.3 →
	-18	-8	0	0	1	0	

Here, pivoting elements is 3

performing $R_2 \rightarrow \frac{1}{3} R_2$

B.V	x	y	s	t	F	R.HS
s	1	2	1	0	0	25
t	1	2/3	0	1/3	0	28/3
	-18	-8	0	0	1	0

Performing $R_1 \rightarrow R_1 - R_2$ & $R_3 \rightarrow R_3 + 18R_2$

B.V	x	y	s	t	F	RHS.
s	0	-4/3	-1	1/3	0	-47/3
x	1	2/3	0	1/3	0	28/3
	0	4	0	6	1	168

Since in bottom row there are no negative entries, so, the tableau gives maximum value.

The maximum value is 168 at (28/3, 0)

∴ 168 at (28/3, 0) Ans.

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Group 'c'

20. Let x, y, z be the production by machines P, Q, R to produce in 18 hours.

Then,

Articles	Machines P(x)	Q(y)	R(z)
A	3	4	2
B	4	2	3
C	2	3	4

The above eqn can be written as

$$3x + 4y + 2z = 18$$
$$4x + 2y + 3z = 18$$
$$2x + 3y + 4z = 18$$

Coefficient of x	coeff of y	coeff of z	constant
3	4	2	18
4	2	3	18
2	3	4	18

Here,

$$D = \begin{vmatrix} 3 & 4 & 2 \\ 4 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$= 3(8-9) - 4(16-6) + 2(12-4)$

$= -3 - 40 + 16$

$= -27$

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$$D_x = \begin{vmatrix} 18 & 4 & 2 \\ 18 & 2 & 3 \\ 18 & 3 & 4 \end{vmatrix}$$

$$= 18(8-9) - 4(72-54) + 2(54-36)$$

$$= -54$$

$$D_y = \begin{vmatrix} 3 & 18 & 2 \\ 4 & 18 & 3 \\ 2 & 18 & 4 \end{vmatrix}$$

$$= 3(72-54) - 18(16-6) + 2(72-36)$$

$$= 54 - 180 + 72$$

$$= -54$$

$$D_z = \begin{vmatrix} 3 & 4 & 18 \\ 4 & 2 & 18 \\ 2 & 3 & 18 \end{vmatrix}$$

$$= 3(36-54) - 4(72-36) + 18(12-4)$$

$$= -54 - 144 + 144$$

$$= -54$$

Now,

$$x = \frac{D_x}{D} = \frac{-54}{-27} = 2 \qquad y = \frac{D_y}{D} = \frac{-54}{-27} = 2 \qquad z = \frac{D_z}{D} = \frac{-54}{-27} = 2$$

ii when per unit cost of production of A, B and C are Rs 200, Rs 160 and Rs 120
then

Total cost = Rs $(200 \times 2 + 160 \times 2 + 120 \times 2)$

= Rs 960

21a.