Set III

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1. Third term in the expansion of $\left(x^2 - \frac{1}{x^3}\right)^n$ is independent of x, then $n = \ldots$?

→ 5

2. In how many ways letters of the word "ARRANGE" be arranged so that two R's come together

→ 360

3. Amplitude of $2-\sqrt{7}$ is ....?

→ π

4. If the roots of the equation $lx^2+nx+n=0$ be in the ratio of $p:q$ then $\sqrt{p/q}+\sqrt{q/p}+\sqrt{n/l}\ldots$?

→ 0

5. $2\tan^{-1}x = \ldots$?

→ all of above

6. The parametric equation $x = a\cos\theta$ and $y = b\sin\theta$ represents.....

→ ellipse

7. $\int \frac{|x|}{x}\,dx$ is equal to

→

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8. The probability of having exactly two tails in 6 tosses of coin is...

→ $15/64$

9. If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ and $\vec{a}$ is non-zero vector then

→ either $\vec{b} = \vec{c}$ or $\vec{a}$ is parallel to $\vec{b} - \vec{c}$

10. When two regression lines are perpendicular to eachother, then there is

→ no correlation

11. The equation of normal of the curve $y = 2x^3 - 5x^2 + 8$ at $(2,4)$ is

→

Group B

12a. Given word is 'SUNDAY'

There are all together 6 letter

So, the word can be arranged in $= 6!$ ways

$= 720$ ways

$$\underline{S}\ \underbrace{\_\ \_\ \_\ \_\ \_}_{5!}$$

The arrangement which begins with S canb

$= 1(S) \times 5!$ (UNDAY)

$= 5!$

$= 120$ ways

Thus, the arrangement that donot begin with S
= 720 − 120
= 600 ways

$\frac{\cancel{S}}{1!}$ __ __ __ __ __

Again, for the arrangement which begin with S and ends with a.

$\underset{(1)}{S}$ $\underbrace{P(4,4) = 4!}_{4!}$ $\underset{(1)}{A}$

It can be done in $= 1 \times 1 \times 4!$ ways
$= 24$ ways

Hence,
The arrangement which begin with S and doesnot ends with a $= (120 - 24)$ ways
$= 96$ ways

12b.

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13b. Given,
distance between two foci = 8 $= 2ae = 8 \therefore ae = 4$ — ①
semi - latus rectum = 6

$$\frac{1}{2}\,\frac{2b^2}{a} = 6$$

$$\frac{b^2}{a} = 6 \text{ — ⑪} \qquad b^2 = 6a$$

Where

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$ae = \sqrt{a^2 - b^2}$$

$$4^2 = \sqrt{a^2 - b^2}$$

$$4^2 = a^2 - b^2$$

$$4^2 = a^2 - 6a$$

$$0 = a^2 - 6a - (4)^2$$

$$0 = a^2 - 6a - 16$$

$$0 = a^2 - (8-2)a - 16$$

$$0 = a^2 - 8a + 2a - 16$$

$$0 = a(a-8) + 2(a-8)$$

$\therefore a = 8, -2$ (false)

$$b^2 = 6a$$

$$b = \sqrt{6 \times 8}$$

$$b = 48$$

Now,
Required eqn is

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$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\therefore \frac{x^2}{64} + \frac{y^2}{48} = 1 \qquad \therefore 3x^2 + 4y^2 = 192$$

14a.

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14b. Given : $x + y = 2$ —— (i)

$x - y = 0$ —— (ii)

It can be written as....

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$AX = B$

$\Rightarrow \quad x = A^{-1} B$

Now,

$$|A| = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

$= -1 - 1 \quad = -2 \neq 0$

So, A^{-1} exists

cofactor of $a_{11} = |-1| = -1$

cofactor of $a_{12} = -|1| = -1$

Cofactor of $a_{21} = -|1| = 1$

Cofactor of $a_{22} = |1| = 1$

$\therefore \; A^{-1} = \begin{vmatrix} -1 & -1 \\ -1 & 1 \end{vmatrix} \left(\frac{1}{-2}\right)$

Now,

$X = A^{-1} B$

$= \frac{1}{-2} \begin{vmatrix} -1 & -1 \\ -1 & 1 \end{vmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$= \frac{1}{-2} \begin{pmatrix} -1 \times 2 + (-1 \times 0) \\ -1 \times 2 + 1 \times 0 \end{pmatrix}$

$= \frac{1}{-2} \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

$= \frac{1}{1} \qquad \therefore x = 1$ and $y = 1$

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15.

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16a. Given series is

$$1+3+6+10+\cdots\cdots$$

Let S_n and t_n be the sum and n^{th} term of given serie. Then,

$$S_n = 1+3+6+10+\cdots\cdots+t_n$$

$$\underline{\cancel{S_n} = 1+3+6+10+\cdots+t_n-t_{n-1}-t_n}$$

$$- \quad - \quad - \quad - \; - \quad - \qquad +$$

$$0 = 1+2+3+4+\cdots+t_n-t_{n-1}-t_n$$

or, $t_n = 1+2+3+4+\cdots+n^{th}$ term

$$= \frac{n}{2}(2\times1+(n-1)d) \quad \left[S_n=\frac{n}{2}(2a+(n-1)d\right]$$

or, $t_n = \frac{n}{2}(n+1)$ ---- (*)

again,

$$S_n = \Sigma t_n$$

$$= \Sigma\left(\frac{n}{2}(n+1)\right) = \Sigma\left(\frac{n^2}{2}+\frac{n}{2}\right)$$

$$= \frac{1}{2}\Sigma n^2 + \frac{1}{2}\Sigma n$$

$$= \frac{1}{2}\frac{(n(n+1)(2n+1))}{6} + \frac{1}{2}\left(\frac{n(n+1)}{2}\right)$$

$$= \frac{n(n+1)}{\cancel{4}2}\left[\frac{2n+1}{3}+1\right]$$

$$\therefore S_n = \frac{n(n+1)(n+2)}{6}$$

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(18) Sol^n:-

Vector Product:

Let $\vec{a}$ and $\vec{b}$ be two vector in a space. Th the vector product of $\vec{a}$ and $\vec{b}$ is denoted defined as

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \, \hat{n}$$

where θ is the angle between $\vec{a}$ and $\vec{b}$ an $\hat{n}$ is the unit normal vectors perpendular to plane of $\vec{a}$ and $\vec{b}$.

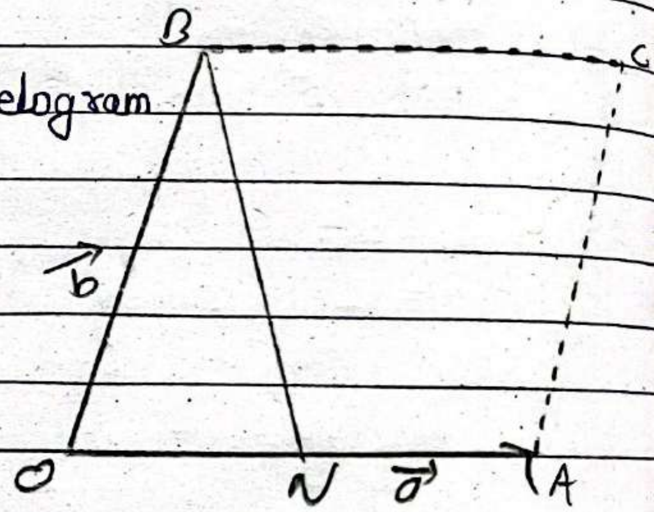
Geometrical meaning

Let OACB be a parallelogram having adjacent sides

$\vec{OA} = \vec{a}$ and $\vec{OB} = \vec{b}$

$\therefore |\vec{OA}| = |\vec{a}| = a$ and

$|\vec{OB}| = |\vec{b}| = b$



Let θ be the angle between $\vec{a}$ and $\vec{b}$ from B draw BN perpendicular to OA.

From figure,

$$\sin\theta = \frac{BN}{OB}$$

$$\Rightarrow BN = (OB)\sin\theta \longrightarrow (*)$$

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Now, we know that

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$
$$= (OA)(OB) \sin\theta$$
$$= (OA)(BN) \quad [\because \text{using } \circledast]$$
$$= (\text{Base}) \times (\text{Height})$$
$$= \text{Area of parallelogram } \square ABC.$$

$\therefore$ Geometrically magnitude of $\vec{a} \times \vec{b}$ is the area of the parallelogram having adjacent side $\vec{a}$ and $\vec{b}$.

2nd part:

Given adjacent sides are

$$AB = \vec{u} = 2\vec{i} + 3\vec{j} = 2\vec{i} + 3\vec{j} + 0\vec{k}$$
$$BC = \vec{v} = \vec{i} + 4\vec{j} = \vec{i} + 4\vec{j} + 0\vec{k}$$

Now,

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 0 \\ 1 & 4 & 0 \end{vmatrix}$$
$$= \vec{i}(0-0) - \vec{j}(0-0) + \vec{k}(8-3)$$
$$= 5\vec{k}$$

Now, $|\vec{u} \times \vec{v}| = \sqrt{0^2+0^2+5^2}$
$$= 5$$

Now,

Area of parallelogram $= |\vec{u} \times \vec{v}|$
$= 5$ sq. unit

19a. Given: $f(x) = \frac{1}{x^2-1}$ in $[-2,2]$

i. $f(x)$ is a being polynomial function, is continuous on $[-2,2]$

ii Diff. (i) w. r. to x, we get

$$f'(x) = \frac{-2x}{(x^2-1)^2} \text{ ------ (ii)}$$

which is not defined on $(-2,2)$.

So, $f(x)$ is not differentiable on the interval $(-$

Thus,

$f(x)$ doesnot satisfies all the condition of Rolle's theorem.

So, Rolle's theorem is not applicable on $f(x)$ at $[-2,2]$ ##

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19b.

sol: Let $I = \int \frac{\sinh x \, dx}{4 \tanh x - \operatorname{cosech} x \ \operatorname{sech} x}$

$= \int \frac{\sinh x \cosh x \, dx}{4 \sinh^2 x - 1}$

put $\sinh x = t$

$\Rightarrow \cosh x \, dx = dt$

$= \int \frac{t^2}{4t^2 - 1} dt$

$= \frac{1}{4} \int \frac{4t^2 - 1 + 1}{4t^2 - 1} dt$

$= \frac{1}{4} \left[\int \frac{4t^2 - 1}{4t^2 - 1} dt + \frac{1}{4} \int \frac{1}{t^2 - (1/2)^2} dt \right]$

$= \frac{1}{4} t + \frac{1}{16} \times \frac{1}{1/2} \cdot \ln \left| \frac{t - 1/2}{t + 1/2} \right| + c$

$= \frac{1}{4} \sinh x + \frac{1}{8} \ln \left| \frac{2 \sinh x - 1}{2 \sinh x + 1} \right| + c$

Group C

20 a.

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20b. To prove :- $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \dots \infty = 2e$

$$LHS = 1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \dots \infty = \sum_{n=1}^{\infty} t_n$$

Let, $t_n = \frac{n^2}{n!}$

$$= \frac{n \times n}{n(n-1)!} = \frac{n}{n-1\ !}$$

$$= \frac{(n-1)+1}{(n-1)!}$$

$$= \frac{\cancel{(n-1)}}{\cancel{(n-1)}(n-2)!} + \frac{1}{(n-1)!}$$

$$= \frac{1}{(n-2)!} + \frac{1}{(n-1)!}$$

21a.

sol', Given α and β are complex cube roots of unity.
So, Let $\omega = \alpha$ and $\omega^2 = \beta$

To prove : $\alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1} = 0$

LHS

$$\alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1}$$

$$= \omega^4 + (\omega^2)^4 + \omega^{-1}(\omega^2)^{-1}$$

$$= \omega \cdot \omega^3 + \omega^6 \cdot \omega^2 + \frac{1}{\omega \cdot \omega^2} \quad [\omega^3 = 1]$$

$$= \omega + \omega^2 + 1 \quad [\omega^2 + \omega + 1 = 0]$$

$$= 0 \quad \text{RHS proved}$$

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21 b. De-moivre's theorem :-

For any positive integer n,

$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

and 2nd part

$z^4 + 1 = 0$

$\Rightarrow z^4 = -1$

Let $z^4 = -1 = r(\cos\theta + i\sin\theta)$

Here, $r = \sqrt{(-1)^2 + 0^2} = 1$

and $\theta = 180°$ [since $z^4 = -1$ is purely -ve real So, $\theta = 180°$]

Now,

$z^4 = (\cos 180° + i\sin 180°)$

$z = (\cos 180° \times n + i\sin 180° \times n)^{1/4}$

$= \left(\cos\frac{180n}{4} + i\sin\frac{180n}{4}\right)$

$z = (\cos 45° \times n + i\sin 45° \times n)$

when n = 0, 1, 2, 3

when n = 0

$z = \cos 0° + i\sin 0° = 1$

when n = 1

$z = \cos 45° + i\sin 45° = \frac{1+i}{\sqrt{2}}$

when n = 2

$z = \cos 90° + i\sin 90° = 0 + i1 = i$

when n = 3

$z = \cos 135° + i\sin 135° = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{-1+i}{\sqrt{2}}$

$\therefore\ z = 1,\ \frac{1+i}{\sqrt{2}},\ i,\ \frac{-1+i}{\sqrt{2}}$

22a. sol: Given system of equation is

$$x + y \le 10$$
$$x \le 4$$
$$y \le 8$$
$$-3x - 4y + z = 0$$

Writing in canonical form;

$$x + y + s + 0t + 0u + 0z = 10$$
$$x + 0y + 0s + t + 0u + 0z = 4$$
$$0x + y + 0s + 0t + u + 0z = 8$$
$$-3x - 4y + 0s + 0t + 0u + z = 0$$

Tableau :

BV	x	y	s	t	u	z	RHS	Ratio
s	1	1	1	0	0	0	10	10
t	1	0	0	1	0	0	4	∞
u	0	1	0	0	1	0	8	8 →
	−3	−4	0	0	0	1	0	

Here, −4 is most negative entry in last row and 8 is least positive entry in ratio column. so, 1 is pivoting element.

Performing $R_1 \to R_1 - R_3$ and $R_4 \to R_4 + 4R_3$

B.V	x	y	s	t	u	z	RHS	Ratio
s	1	0	1	0	-1	0	2	2 →
t	1	0	0	1	0	0	4	4
y	0	1	0	0	1	0	8	∞
	-3	0	0	0	4	1	32	
	↑							

Again, here, -3 is most negative entry and 2 is least entry in ratio column. So, 1 is pivoting element.

Performing $R_2 \to R_2 - R_1$, $R_4 \to R_4 + 3R_1$

BV	x	y	s	t	u	z	RHS
	1	0	1	0	-1	0	2
	0	0	-1	1	1	0	2
	0	1	0	0	1	0	8
	0	0	3	0	1	1	38

Since, all element of bottom row is the positive. So, solution of given equation is given by tableau is

$\therefore$ Maximum = 38 at $x = 2$, $y = 8$.

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22b.

sol; Lagrange's Mean Value Theorem :-

If $y = f(x)$ be a function

i continuous on the closed interval $[a, b]$

ii differentiable on the open interval (a, b)

Then, there exists at least one point $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

2^{nd} part.

Given $f(x) = x - x^3$ —— (i) on $[-2, 1]$

i Here, $f(x)$ is defined on $[-2, 1]$. so, it is continuous on $[-2, 1]$

ii Diff (i) w. r. to x, we get;

$f'(x) = 1 - 3x^2$, which is defined on $[-2, 1]$

So, $f(x)$ is differential on $(-2, 1)$

Since, $f(x)$ holds both condition of Lagrang mean value theorem. so, we can find atlea one point $c \in (-2, 1) = (a, b)$

such that; $f'(c) = \frac{f(b) - f(a)}{b - a}$

$$\Rightarrow 1 - 3c^2 = \frac{(1 - 1^3) - (-2 + 8)}{1 + 2}$$

$$\Rightarrow -3c^2 = \frac{0 - 6}{3} - 1$$

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$$\Rightarrow -3c^2 = \frac{-6-3}{3}$$

$$\Rightarrow -3c^2 = \frac{-9}{3}$$

$$\Rightarrow c = \pm 1 \in [-2, 1)$$

Hence, the theorem is proved.