SET – II

Group 'A'

1. How many numbers are there between 100 and 1000 such that every digit is either 2 or 9?

→ 8

2. If a student has to pass each of the five subjects to get through. In how many ways the candidate fail?

→ 31

3. If the coefficient of x^{-1} in the ~~basis~~ expansion $\left(x + \frac{k}{x^2}\right)^5$ is 90, then the value of k is..

→ both a and b

4. The set of all integers is not group under multiplication because

→ inverse does not exist

5. Conjugate of a complex number is $1/(i-1)$ The the complex number is...

→ $\frac{-1}{i+1}$

6. If $1-p$ is a root of $x^2 + px + 1-p = 0$ then its roots are

→

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7. The sum of the focal distances from any point on the ellipse $9x^2+16y^2=144$ is
→ 8

8. $\int \tan^3 x \sec^2 x \, dx$ equals
→ $\frac{1}{4} \tan^4 x + c$

9. Which of the following is not true?
→ cross product of any vector $\vec{a}$ with itself is]

10. The foot of perpendicular drawn from point (a, b, c upon the yz-plane is ...
→

11. The derivative of $\tanh^{-1} x$ with respect to x is
→ $\cancel{\frac{1}{x^2-1}}$ $\frac{1}{1-x^2}$

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Group B

12a.

Given,

Total no. of boys = 6

Total no of girls = 4

No. of member in community = 5

i exactly 1 boy

$= {}^{n}c(6,1) \times {}^{n}c(4,4)$

$= 1 \times 6 = 6$

ii atleast two boy

$= {}^{n}c(6,2) \times {}^{n}c(4,3) + {}^{n}c(6,3) \times {}^{n}c(4,2) + {}^{n}c(6,4) \times {}^{n}c(4,$
$+ {}^{n}c(6,5) \times {}^{n}c(4,0)$

$= 246$

12 b. Given,

p = probability of hitting a target $= \frac{1}{4}$

Since 5 hitting (trial) is done

So, $n = 5$.

$q = 1 - p$

$= \frac{3}{4}$

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Now,
We know
$P(r) = {}^nC_r\, p^r\, q^{n-r}$

i P(none will strick target) i.e $r=0$
$= {}^5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{5-0}$

$= \frac{243}{1024}$

(ii) Exactly one will strick the target = ?
$P(\text{Exactly one}) = P(r=1)$
$= {}^5C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^{5-1}$

$= \frac{405}{1024}$

13.
sol'

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14. sol; Let a, b, c be the number of truck of types A, B, C respectively required to carry 17 boxes of x, 33 boxes of sizes Y and 23 boxes of size Z.

So, the given information can be written in equation form as:

$$2a + 3b + c = 17 \quad \text{— (I)}$$
$$5a + 12b + 6c = 33 \quad \text{— (II)}$$
$$0a + 5b + 4c = 23 \quad \text{— (III)}$$

Writing above equation in matrix form;

$$\begin{pmatrix} 2 & 3 & 1 \\ 5 & 2 & 6 \\ 0 & 5 & 4 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 17 \\ 33 \\ 23 \end{pmatrix}$$

$$\Rightarrow PA = Q$$
$$\Rightarrow A = P^{-1} Q$$

So, $|P| = \begin{vmatrix} 2 & 3 & 1 \\ 5 & 2 & 6 \\ 0 & 5 & 4 \end{vmatrix}$

$= -5(12-5) + 4(4-15) = -35-44$

$= -79 \neq 0$ So, P^{-1} exist

Now, cofactors are;

$A_{11} = \begin{vmatrix} 2 & 6 \\ 5 & 4 \end{vmatrix} = (8-30) = -22$

$A_{12} = -\begin{vmatrix} 5 & 6 \\ 0 & 4 \end{vmatrix} = -(20-0) = -20$

$A_{13} = \begin{vmatrix} 5 & 2 \\ 0 & 5 \end{vmatrix} = (25-0) = 25$

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$$A_{21} = -\begin{vmatrix} 3 & 1 \\ 5 & 4 \end{vmatrix} = -(12-5) = -7$$

$$A_{22} = \begin{vmatrix} 2 & 1 \\ 0 & 4 \end{vmatrix} = (8-0) = 8$$

$$A_{23} = -\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = -(10-0) = -10$$

$$A_{31} = \begin{vmatrix} 3 & 1 \\ 2 & 6 \end{vmatrix} = (18-2) = 16$$

$$A_{32} = -\begin{vmatrix} 2 & 1 \\ 5 & 6 \end{vmatrix} = -(12-5) = -7$$

$$A_{33} = \begin{vmatrix} 2 & 3 \\ 5 & 2 \end{vmatrix} = (4-15) = -11$$

So,

$$\text{Cofactor matrix} = \begin{pmatrix} -22 & -20 & 25 \\ -7 & 8 & -10 \\ 16 & -7 & -11 \end{pmatrix}^T$$

$$= \begin{pmatrix} -22 & -7 & 16 \\ -20 & 8 & -7 \\ 25 & -10 & -11 \end{pmatrix}$$

Now, $P^{-1} = \frac{1}{-79} \begin{pmatrix} 22 & -7 & 16 \\ -20 & 8 & -7 \\ 25 & -10 & -11 \end{pmatrix}$

Now, $A = P^{-1} Q$

$$= \frac{1}{-79} \begin{pmatrix} -22 & -7 & 16 \\ -20 & 8 & -7 \\ 25 & -10 & -11 \end{pmatrix} \begin{pmatrix} 17 \\ 33 \\ 23 \end{pmatrix}$$

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$$= \frac{1}{-79}\begin{pmatrix} -1237 \\ -237 \\ 158 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix}$$

$$\therefore \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix}$$

So, $a = 3$, $b = 3$ and $c = 2$

Also,

Carrying charge by A, B, C are 1200, Rs 1500 and Rs 1350

So, the total income made by transport company = Rs $(1200 \times 3 + 1500 \times 3 + 1350 \times 2)$

= Rs 10800

15 a. Given series;

$1.2 + 2.3 + 3.4 + \ldots$

Now,

n^{th} term of $1, 2, 3, \ldots = n$

n^{th} term of $2, 3, 4, \ldots = n+1$

So,

n^{th} term of given series is;

$t_n = n(n+1)$

$= n^2 + n$

Now, $S_n = \Sigma t_n$

$= \Sigma(n^2 + n) = \Sigma n^2 + \Sigma n$

$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$$= \frac{n(n+1)}{2}\left[\frac{2n+1}{3}+1\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{2n+4}{3}\right]$$

$$= \frac{n(n+1)\cancel{2}(n+2)}{\cancel{2}\times 3}$$

$$S_n = \frac{n(n+1)(n+2)}{3}$$

15b. Given statement ; $3^{2n}-1$ is divisible by 8

$P(n): 3^{2n}-1$ is divisible by 8

$P(1): 3^2-1 = 8$ is divisible by 8

$\therefore P(1)$ is true

Assume that $P(n)$ is true for $n=k$

i.e $P(k): 3^{2k}-1$ is divisible by 8.

We will show that $P(k+1)$ is true whenever $P(k)$ is true,

i.e $P(k+1)$ is $3^{2k+2}-1$ is divisible by 8

Now,

$$3^{2k+2}-1$$

$$= 3^{2k}\cdot 3^2-1$$

$$= (3^{2k}-1)\,3^2+8$$

Since,

$(3^{2k}-1)$ is divisible by 8 So, $(3^{2k}-1)3^2$ is also divisible by 8 and 8 is also divisible by 8.

So, $(3^{2k}-1)3^2+8$ is divisible by 8.

This shows that $P(k+1)$ is true whenever $P(k)$ is true.

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16 b. Given eq^n of hyperbola ;

$$\frac{x^2}{9} - \frac{y^2}{16} = 1 \quad \text{——— (i)}$$

Comparing eq^n (i) with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$$a^2 = 9 \qquad , \quad b^2 = 16$$
$$\Rightarrow a = 3 \qquad \quad b = 4$$

Now,

$$\text{eccentricity } (e) = \sqrt{1 + \frac{b^2}{a^2}}$$
$$= \sqrt{1 + \frac{16}{9}}$$
$$\therefore e = \frac{5}{3}$$

Again,

coordinate of foci $= (\pm ae, 0)$
$= (\pm 3 \times 5/3, 0)$
$= (\pm 5, 0)$

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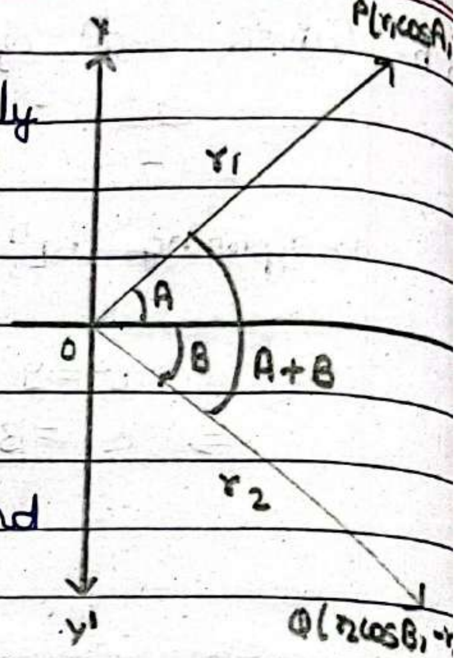
17.

Sol; Let OX and YOX be two mutually perpendicular axises and also,

$\angle POX = A$, $\angle QOX = B$ and

$\angle POQ = A + B$

Also $OP = r_1$ and $OQ = r_2$



Here,

$(A+B)$ is the angle between $\vec{OP}$ and $\vec{OQ}$.

Coordinate of P and Q are:

$\vec{OP} = (r_1 \cos A, r_1 \sin A, 0)$

$\vec{OQ} = (r_2 \cos B, -r_2 \sin B, 0)$

Now,

$$\vec{OP} \times \vec{OQ} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_1 \cos A & r_1 \sin A & 0 \\ r_2 \cos B & -r_2 \sin B & 0 \end{vmatrix}$$

$$= \vec{i}(0-0) - \vec{j}(0-0) + \vec{k}(-r_1 r_2 \sin B \cos A - r_2 r_1 \sin A \cos B$$

$$= ((-r_1 r_2)(\sin A \cos B + \sin B \cos A))\vec{k}$$

So,

$$|\vec{OP} \times \vec{OQ}| = \sqrt{[(-r_1 r_2)(\sin A \cos B + \sin B \cos A)]^2 + 0 + 0}$$

$$= r_1 r_2 (\sin A \cos B + \sin B \cos A).$$

Now, since $(A+B)$ is angle between $\vec{OP}$ and $\vec{OQ}$

$$\sin(A+B) = \frac{|\vec{OP} \times \vec{OQ}|}{OP \cdot OQ}$$

$$= \frac{\cancel{r_1 r_2} (\sin A \cos B + \sin B \cdot \cos A)}{\cancel{r_1 r_2}}$$

$\therefore \sin(A+B) = \sin A \cos B + \cos A \sin B$

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19\. Given inequality are :-

$$U = 25x + 45y$$
$$x + 3y \leq 21$$
$$2x + 3y \leq 24$$

Introducing slack variable in given inequality

$$x + 3y + s = 21$$
$$2x + 3y + t = 24$$
$$-25x - 45y + U = 0$$

Writing above equation in cannocial form;

$$x + 3y + s + 0t + 0u = 21$$
$$2x + 3y + 0s + t + 0u = 24$$
$$-25x - 45y + 0s + 0t + U = 0$$

Supplying tableau for above cannonical form.

BV	x	y	s	t	u	RHS	Ratio
s	1	3	1	0	0	21	7 ←
t	2	3	0	1	0	24	8
	-25	-45	0	0	1	0	
		↑					

The in the bottom row (-45) in most negative value so y-column is pivot column and in ratio column (7) is least positive number so R_1 is pivot row. Hence pivot element is 3

Performing $R_1 \rightarrow R_1 \times \frac{1}{3}$

BV	x	y	s	t	u	RHS
s	1/3	1	1/3	0	0	7
t	2	3	0	1	0	24
	-25	-45	0	0	1	0

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Performing $R_2 \to R_2 - 3R_1$ and $R_3 \to R_3 + 45R_1$

BV	x	y	s	t	u	RHS
y	1/3	1	1/3	0	0	27
t	1	0	-1	1	0	3 ←
	-10	0	15	0	1	315

↑

In the bottom row (-10) is most negative value. so, x-column is pivot column and in ratio column 3 is the least positive number so, R_2 is pivot row and hence pivot element is 1.

Performing $R_1 \to R_1 - \frac{1}{3} R_2$ and $R_3 \to R_3 + 10R_2$

BV	x	y	s	t	u	RHS
y	0	1	2/3	-1/3	0	6
x	1	0	-1	1	0	3
	0	0	5	10	1	345

Since, the bottom column does not have any negative entries, so,
maximum value is 345 at (3,6) #

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Group C

20a State binomial theorem, then write down the formula for finding general term of binomial theorem. Also show that sum of all binomial coefficient is 2^n.

→

Binomial theorem (statement):

$$(a+x)^n = {}^nc_0 a^n + {}^nc_1 a^{n-1} x + {}^nc_2 a^{n-2} x^2 + \dots + {}^nc_n x^n$$

where, $^nc_r = C(n,r) = \frac{n!}{(n-r)!\, r!}$

General term:

$(r+1)^{th}$ term i.e $t_{r+1} = {}^nc_r a^{n-r} x^r$

To show: sum of all binomial coefficient is 2^n

We have

$$(1+x)^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots + c_n x^n$$

when $x=1$

$$(1+1)^n = c_0 + c_1 \cdot 1 + c_2 \cdot 1^2 + c_3 \cdot 1^3 + \dots + c_1$$

$$\Rightarrow 2^n = c_0 + c_1 + c_2 + c_3 + c_n$$

$$\therefore c_0 + c_1 + c_2 + c_3 + \dots + c_n = 2^n$$

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b. Given,

$$\left(x - \frac{1}{x}\right)^{2n} \text{ is } \frac{1.3.5 \dots (2n-1)}{n!} (-2)^n$$

Proof: We know, for every positive integer n, than 2n is even. So there is only one (single) middle term in the expansion of $\left(x - \frac{1}{x}\right)^{2n}$. Namely, the single middle term is $\left(\frac{2n}{2} + 1\right)^{th}$ term i.e $(n+1)^{th}$ term.

$$\therefore t_{n+1} = (-1)^n \, C(2n, n) \, x^n \times \frac{1}{x^n}$$

Now, $C(2n, n) = \dfrac{(2n)!}{(2n-n)!\, n!}$

$$= \frac{2n.(2n-1)(2n-2)(2n-3)\dots 5.4.3.2.1}{(n!)^2}$$

$$= \frac{\{2n(2n-2)(2n-4)\dots 6.4.2\}.\{(2n-1)(2n-3)\dots 5.3.1\}}{(n!)^2}$$

$$= \frac{2^n\{n.(n-1)(n-2)\dots 3.2.1\}.\{(2n-1)(2n-3)\dots 5.3.1\}}{(n!)^2}$$

$$= \frac{2^n\, n!\, \{(2n-1)(2n-3)\dots 5.3.1\}}{(n!)^2}$$

$$= \frac{2^n\{(2n-1)(2n-3)\dots 5.3.1\}}{n!}$$

$\therefore$ middle term is $(-1)^n \times \dfrac{2^n\{(2n-1)(2n-3)\dots 5.3.1\}}{n!}$

$$= \frac{\{(2n-1)(2n-3)\dots 5.3.1\}}{n!}(-2)^n$$

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21a. Given: α and β are complex curbe root

To show; $\alpha^4 + \beta^4 + \alpha^2\beta^2 = 0$

Let $\alpha = w$ and $\beta = w^2$

Now,

$(w)^4 + (w^2)^4 + (w)^2 (w^2)^2$

$= w^3 \cdot w + w^6 \cdot w^2 + w^2 \cdot w^3 \cdot w \quad [w^3 = 1]$

$= w + w^2 + 1 \cdot 1$

$= w + w^2 + 1 \quad [\because w + w^2 + 1 = 0]$

$= 0$ RHS prond.

21b. Given,

$\Sigma x = 24$, $\Sigma y = 12$, $\Sigma x^2 = 374$, $\Sigma y^2 = 97$

and $\Sigma xy = 157$ and $n = 7$.

So,

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{7 \times 157 - 24 \times 12}{7 \times 374 - (24)^2} = \frac{811}{2092}$$

$\therefore b_{yx} = 0.397$

Now,

$$b_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

$$= \frac{7 \times 15 - 24 \times 12}{7 \times 97 - (12)^2} = \frac{811}{535}$$

$\therefore b_{xy} = 1.5158$

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21c. Again,
Regression line of x on y is
$$x-\bar{x} = b_{xy}(y-\bar{y}) \quad \text{--- (i)}$$

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22a. sol;

Let $f(x) = e^{\cos x}$

Then, $f(x+h) = e^{\cos(x+h)}$

By definition we know that,

$$\frac{dy}{dx} = \lim_{x \to h} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{x \to h} \frac{e^{\cos(x+h)} - e^{\cos x}}{h}$$

put $\cos x = u$ and $\cos(x+h) = u+k$

$$\Rightarrow k = \cos(x+h) - \cos x$$

$$\Rightarrow k = 2\sin\frac{x+h+x}{2} \cdot \sin\frac{x-x-h}{2}$$

$$= 2\sin\frac{2x+h}{2} \cdot \sin -\frac{h}{2}$$

when $h \to 0$, then $k \to 0$

Now,

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{e^{u+k} - e^{u}}{h}$$

$$= \lim_{k \to 0} \frac{e^{u}(e^{k}-1)}{k} \cdot \frac{k}{h}$$

$$= e^{u} 1 \left(\lim_{h \to 0} \frac{2\sin\frac{2x+h}{2} \cdot \sin -h/2}{h} \right)$$

$$= e^{u} \cdot \sin x \left(-\lim_{h \to 0} \frac{\sin -h/2}{h/2} \right)$$

$$= e^{\cos x} - (\sin hx)$$

$$= -\sin x \; e^{\cos x}$$

$$\therefore \frac{d(e^{\cos x})}{dx} = -\sin x \, (e^{\cos x})$$

at $x=0$,

$$\frac{de^{\cos x}}{dx} = -\sin 0^\circ \left(e^{\cos 0}\right)$$

$$= 0$$

22b. Given,

Let $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

put $x = \tan\theta \Leftrightarrow \theta = \tan^{-1} x$

Then

$$y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

or; $y = \tan^{-1} \tan 2\theta$

or; $y = 2\theta = 2\tan^{-1} x$

Diff. w. r. to x,

$$\frac{dy}{dx} = \frac{d(2\tan^{-1}x)}{dx}$$

$$= 2\frac{d\tan^{-1}x}{dx}$$

$$= 2\frac{1}{1+x^2}$$

$$= \left(\frac{2}{1+x^2}\right)$$

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22c. Given:

$$\int \frac{5x^3}{3x^4+x^2-2}\,dx$$