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Group A

Rewrite the correct option in your answer sheet.

1. Which of the following is true?

→ d. All of above

2. Sum of all binomial coefficient is

→ a. 2^n

3. Product of fourth root of unity is

→ b. -1

4. If the roots of the equation $(b-c)x^2+(c-a)x+(a-b)=0$ are equal, then a, b, c are in

→

5. Which of the statement is incorrect?

→

6. The sum of the roots of a quadratic equation is 1 and sum of their square is 13, then the equation is

→ a. $x^2-x-6=0$

7. $\int \frac{e^x}{1+e^x}\,dx$ is equal to

→ $\ln(1+e^x)+c$

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8: The equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ represents a horizontal ellipse if

→ a. $a^2 > b^2$

9. What is the value of $\vec{i} \cdot (\vec{j} \times \vec{k}) + \vec{j} (\vec{k} \times \vec{i}) + \vec{k} (\vec{i} \times \vec{j})$

→ d. 3

10. The demand function for a certain commodity is $P = 60 - 2x - x^2$, Where P is price and x the quantity. Then the consumer surplus when dema is 6 is

→

11. The derivative of $2 \tanh^{-1} \tan x/2$ with respect to x is

→ b. $\sec x$

Group-B

12a.
sol", No. of boys(6) No. of girls(4)

i exactly 1 girl
$= C(6,4) \times C(4,1)$
$= 60$

ii at least two girl
$= C(6,3) \times C(4,2) + C(6,2) \times C(4,3) + C(6,1) \times C(4,4)$
$= 186$

12b. Given: $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$
To show: $x+y+z = xyz$
Let,
$\tan^{-1}x = A \Leftrightarrow x = \tan A$
$\tan^{-1}y = B \Leftrightarrow y = \tan B$
$\tan^{-1}z = C \Leftrightarrow z = \tan C$
We know that,

$$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan D}$$

$$\tan(\tan^{-1}x + \tan^{-1}y + \tan^{-1}z) = \frac{x+y+z-xyz}{1-xy-yz-zy}$$

$$\tan \pi = \frac{x+y+z-xyz}{1-xy-yz-zy}$$

$$0 \times (1-xy-yz-zy) = x+y+z-xyz$$

$\therefore x+y+z = xyz$ Proved

13a

13b.

no. of trial = 3

$n = 3$

probability of getting success 5 or 6

$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

$p = \frac{1}{3}$

then $q = 1 - p$

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

We know, $P(r) = C(n,r)\, p^r\, q^{n-r}$

(i) 2 success

$r = 2$

$= C(3,2) \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{3-2}$

$= 3 \times \frac{1}{9} \times \frac{2}{3} = \frac{2}{9}$

(ii) 3 success

$r = 3$

$= C(3,3) \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{3-3}$

$= 1 \times \frac{1}{27} = \frac{1}{27}$

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14a.

Sol; Given eq^n are

$$ax^2 + bx + c = 0$$
$$a'x^2 + b'x + c' = 0$$

Let α be the common root in eq^n (i) and (ii):

$$a\alpha^2 + b\alpha + c = 0$$
$$a'\alpha^2 + b'\alpha + c' = 0$$

Using cross multiplication

$$\frac{\alpha^2}{bc'-b'c} = \frac{\alpha}{a'c-ac'} = \frac{1}{ab'-a'b}$$

Taking first two ratios

$$\frac{\alpha^2}{bc'-b'c} = \frac{\alpha}{a'c-ac'}$$

$$\alpha = \frac{bc'-b'c}{a'c-ac'} \quad \text{——} (*)$$

Taking second and third ratio

$$\alpha = \frac{a'c-ac'}{ab'-a'b} \quad \text{——} (*)(*)$$

Equating (*) and (**)

$$\frac{bc'-b'c}{a'c-ac'} = \frac{a'c-ac'}{ab'-a'b}$$

$$(ab'-a'b)(bc'-b'c) = (a'c-ac')^2$$

The condition in which one root is common is

$\therefore$ $(ab'-a'b)(bc'-b'c) = (a'c-ac')^2$

14b.
Sol;

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15.
Sol; Let x, y, z be the quantity of A, B, C to m[ake] a mixture of nutritents mixed in the final outp[ut] respectively. Then by question
So,

Food	P	Q	R
A(x)	$2x$	$2x$	$4x$
B(y)	$3y$	$5y$	$0y$
C(z)	$4z$	$3z$	$5z$

The above eq^n can be reduced in following

$$2x+3y+4z=45$$
$$2x+5y+3z=54$$
$$4x+0y+5z=45$$

The above eq^n can be written in matrix for $AX=B$ as

$$\begin{bmatrix}2 & 3 & 4\\2 & 5 & 3\\4 & 0 & 5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}45\\54\\45\end{bmatrix}$$

$AX=B$ where $A=\begin{bmatrix}2 & 3 & 4\\2 & 5 & 3\\4 & 0 & 5\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$, $B=\begin{bmatrix}45\\54\\45\end{bmatrix}$

Now, $|A|=\begin{vmatrix}2 & 3 & 4\\2 & 5 & 3\\4 & 0 & 5\end{vmatrix}$

Expanding along R_1

$$=2\begin{vmatrix}5 & 3\\0 & 5\end{vmatrix}-3\begin{vmatrix}2 & 3\\4 & 5\end{vmatrix}+4\begin{vmatrix}2 & 5\\4 & 0\end{vmatrix}$$

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$= 2(25-0) - 3(10-12) + 4(0-20)$
$= -24 \neq 0$

Hence, an inverse exists,

$A_{11} = \begin{vmatrix} 5 & 3 \\ 0 & 5 \end{vmatrix} = 25$

$A_{12} = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = -(10-12) = +2$

$A_{13} = \begin{vmatrix} 2 & 5 \\ 4 & 0 \end{vmatrix} = -20$

$A_{21} = \begin{vmatrix} 3 & 4 \\ 0 & 5 \end{vmatrix} = -15$

$A_{22} = \begin{vmatrix} 2 & 4 \\ 4 & 5 \end{vmatrix} = 10-16 = -6$

$A_{23} = \begin{vmatrix} 2 & 3 \\ 4 & 0 \end{vmatrix} = 0-12 = -12$

$A_{31} = \begin{vmatrix} 3 & 4 \\ 5 & 3 \end{vmatrix} = 9-20 = -11$

$A_{32} = \begin{vmatrix} 2 & 4 \\ 2 & 3 \end{vmatrix} = -(6-8) = 2$

$A_{33} = \begin{vmatrix} 2 & 3 \\ 2 & 5 \end{vmatrix} = 10-6 = 4$

Let C = matrix of cofactors $= \begin{bmatrix} 25 & 2 & -20 \\ -15 & -6 & -12 \\ -11 & 2 & 4 \end{bmatrix}$

$\therefore C^T = \begin{bmatrix} 25 & -15 & -11 \\ 2 & -6 & 2 \\ -20 & -12 & 4 \end{bmatrix}$

$$A^{-1} = \frac{\text{Adj. } A}{|A|}$$

$$= \frac{1}{-54}\begin{bmatrix} 25 & -15 & -11 \\ 2 & -6 & 2 \\ -20 & -12 & 4 \end{bmatrix}$$

$\therefore$ The solution is $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-24}\begin{bmatrix} 25 & -15 & -11 \\ 2 & -6 & 2 \\ -20 & -12 & 4 \end{bmatrix}\begin{bmatrix} 45 \\ 54 \\ 45 \end{bmatrix}$$

$$= \frac{1}{-24}\begin{bmatrix} -180 \\ -144 \\ -72 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7.5 \\ 6 \\ 3 \end{bmatrix}$$

$\therefore x = 7.5$

$y = 6$

$z = 3$

Also, cost per kg of A, B, C are Rs. 40, 60 and 80 respectively.

So,

The total cost $=$ Rs. $(7.5 \times 40 + 6 \times 60 + 3 \times 80)$

$=$ Rs. $(300 + 360 + 240)$

$=$ Rs. 900 //

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16 Correlation :-

soli The relation between two dependent and independent variable is called correlation.

Eg:- age of husband and wife
weight of husband and wife

Given,

x	6	2	10	4	8
y	9	11	–	8	7

x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$
6	9	0	1	0	0	1
2	11	−4	3	−12	16	9
10	– x (Let) [5]	4	−3	−12	16	9
4	8	−2	0	0	4	0
8	7	2	−1	−2	4	1
	$\Sigma y = 35 + x$			$\boxed{-26}$	$\boxed{40}$	$\boxed{20}$

$$(\bar{y}) = \frac{\Sigma y}{N}$$

$$8 = \frac{35+x}{5}$$

$$\therefore x = 5$$

We know,

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2 \Sigma(y-\bar{y})^2}} = \frac{-26}{\sqrt{40 \times 20}} = -0.91$$

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17a. Let, the eq^n of ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{—(i) with}$$

vertex at (0,8) and passing through (3, 32/5)

Here,

Vertex = (0, 8) = (0, b)

$\therefore b = 8$

Since, (i) passes through (3, 32/5). So

$$\Rightarrow \frac{3^2}{a^2} + \frac{\left(\frac{32}{5}\right)^2}{8^2} = 1 \qquad [\because b = 8]$$

$$\Rightarrow \frac{9}{a^2} + \frac{16}{25} = 1$$

$$\Rightarrow \frac{\cancel{9}}{a^2} = \frac{\cancel{9}}{25}$$

$$\therefore a^2 = 25$$

So, eq^n of ellipse is

$$\frac{x^2}{(5)^2} + \frac{y^2}{(8)^2} = 1$$

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17b.

Given limit is :-

$$\lim_{x \to 0} = \frac{x - \tan x}{x^2}$$

when $x = 0$, function takes the form 0/0. So,
Using L-Hospital rule

$$\lim_{x \to 0} = \frac{1 - \sec^2 x}{2x} \quad \left[\frac{0}{0}\right]$$

$$\Rightarrow \lim_{x \to 0} = \frac{0 - (\sec x \tan x) 2\sec x)}{2} \Rightarrow \lim_{x \to 0} = \frac{-2\sin x}{2\cos^3 x}$$

$$\Rightarrow \cancel{\left(\frac{0 - \sec 0^{\circ} \times \tan 0}{2}\right)} \Rightarrow \frac{\sin 0}{\cos^3 0}$$

$$\Rightarrow \cancel{1/0} \qquad = 0$$ RHS proved//

18.

Sol; The vector product of $\vec{a}$ and $\vec{b}$ is defined as

$$\vec{a} \times \vec{b} = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1)$$

To Prove : $\sin(A - B) = \sin A \cos B - \cos A \sin B$

Proof :- Let O be the origin, OX and OY be two mutually perpendicular lines (axes)

Let $\angle POX = B$ and $\angle QOX = A$

$OP = r_1$, $OQ = r_2$

Then $\angle POQ = A - B$

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$$\boxed{|\vec{OP}| = OP}$$

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i.e $(A-B)$ is the angle between OP and OQ. Th
the coordinates of P and Q are
$(r_1 \cos B, r_1 \sin B)$ and $(r_2 \cos A, r_2 \sin A)$ respectively.

$\therefore \vec{OP} = (r_1 \cos B, r_1 \sin B) = (r_1 \cos B, r_1 \sin B, 0)$

$\vec{OQ} = (r_2 \cos A, r_2 \sin A) = (r_2 \cos A, r_2 \sin A, 0)$

Now

$$\vec{OP} \times \vec{OQ} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_1 \cos B & r_1 \sin B & 0 \\ r_2 \cos A & r_2 \sin A & 0 \end{vmatrix}$$

$$= \vec{i}(0-0) - \vec{j}(0-0) + \vec{k}(r_1 r_2 \sin A \cos B - r_1 r_2 \cos A \sin B)$$

$$= (0, 0, r_1 r_2 (\cos B \sin A - \cos A \sin B))$$

and

$$|\vec{OP} \times \vec{OQ}| = \sqrt{0^2 + 0^2 + [r_1 r_2 (\sin A \cos B - \cos A \sin B)]^2}$$

$$= r_1 r_2 (\sin A \cos B - \cos A \sin B)$$

since $(A-B)$ is the angle between $\vec{OP}$ and $\vec{OQ}$

So,

$$\sin(A-B) = \frac{|\vec{OP} \times \vec{OQ}|}{|\vec{OP}||\vec{OQ}|}$$

$$\sin(A-B) = \frac{\cancel{r_1 r_2} (\sin A \cos B - \cos A \sin B)}{\cancel{r_1 r_2}}$$

$\therefore \sin(A-B) = \sin A \cos B - \cos A \sin B$ proved

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19a

Rational fraction :-

Rational fraction is a fraction that can be expressed as the ratio of two polynomial such that $f(x) = \frac{P(x)}{Q(x)}$

where, $P(x)$ and $Q(x)$ are polynomial and $Q(x) \neq 0$.

For eg:- $f(x) = \frac{x^2 - 2x + 1}{x - 1}$

Proper fraction :-

A proper fraction is a fraction in which the degree of numerator is less than degree of denomenator. Eg:- $\frac{x^2 + 3x + 2}{x^3 + x^2 + x + 1}$

Improper fraction :-

A improper fraction is a fraction in which the degree of numeraitor is greater or equal to degree of denomerator. Eg:- $\frac{x^3 + x^2 + x + 1}{x^2 + 3x + 2}$

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19 b.
sol;

Given, $\int \frac{dx}{a+b\cos x}$ $(a>b)$

Let $I = \int \frac{dx}{a+b\cos x}$

$= \int \frac{dx}{a(\cos^2 x/2 + \sin^2 x/2) + b(\cos^2 x/2 - \sin^2 x/2)}$

$= \int \frac{dx}{a\cos^2 x/2 + a\sin^2 x/2 + b\cos^2 x/2 - b\sin^2 x/2}$

$= \int \frac{dx}{(a+b)\cos^2 x/2 + (a-b)\sin^2 x/2}$

$= \int \frac{dx}{\cos^2 x/2\{(a+b) + a(a-b)\tan^2 x/2\}}$ —— (*)

If, $a>b$:

$= \int \frac{dx \sec^2 x/2}{(\sqrt{a+b})^2 + (\sqrt{a-b}\cdot\tan x/2)^2}$

put $\sqrt{a-b}\tan\frac{x}{2} = t$

$\Rightarrow \sec^2 x/2\, dx = \frac{2dt}{\sqrt{a-b}}$

Now,

$I = \int \frac{2dt}{\sqrt{a-b}\,(\sqrt{a+b})^2 + t^2}$

$= \frac{2}{\sqrt{a-b}} \int \frac{dt}{(\sqrt{a+b})^2 + t^2}$

$= \frac{2}{\sqrt{a-b}} \left[\frac{1}{\sqrt{a+b}} \tan^{-1}\left(\frac{t}{\sqrt{a+b}}\right)\right] + c$

$= \frac{2}{\sqrt{(a-b)(a+b)}} \tan^{-1}\left(\frac{\sqrt{a-b}\tan x/2}{\sqrt{a+b}}\right) + c$

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$$= \frac{2}{\sqrt{a^2-b^2}} \tan^{-1}\left(\sqrt{\frac{a-b}{a+b}} \cdot \tan\left(\frac{x}{2}\right)\right) + c$$

Group C

20a To show: function $f(x) = |x-1|$ is continuous but not differential at $x = 1$.

So,

$$f(x) = |x-1| = \begin{cases} x-1 & \text{if } (x-1) \geq 0 \\ -x+1 & \text{if } (x-1) < 0 \end{cases}$$

Left hand limit is;

$$\lim_{x \to 1^-} \left(\frac{-x+1}{1}\right)$$

$$= \frac{-1+1}{1} = 0$$

Right hand limit is;

$$\lim_{x \to 1^+} \left(\frac{x-1}{1}\right)$$

$$= \frac{1-1}{1} = 0$$

$f(1) = 1-1 = 0$

Here

LHL = RHL = $f(1)$

i.e $\lim_{x \to 1^-} = \lim_{x \to 1^+} = f(1)$

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So, $f(x) = |x-1|$ is continuous at $x=1$

To show differential;

$$f(x) = |x-1| = \left.\begin{cases} x-1 & \text{if } (x-1) \geq 0 \\ -x+1 & \text{if } (x-1) < 0 \end{cases}\right\} \text{ at } x=1$$

Right hand derivative:

$$\lim_{x \to 1^+} = \frac{f(x+h) - f(x)}{h} \qquad [f(x+h) = (x+h-1)]$$

$$\Rightarrow \lim_{x \to 1^+} = \frac{(\cancel{x} + h - \cancel{1}) - (\cancel{x} - \cancel{1})}{h}$$

$$\Rightarrow \lim_{x \to 1^+} = \frac{h}{h} = 1$$

Left hand derivative:

$$\lim_{x \to 1^-} = \frac{f(x-h) - f(x)}{h} \qquad [f(x-h) = -(x-h)+1]$$

$$\Rightarrow \lim_{x \to 1^-} = \frac{(-(x-h)+1) - (-x+1)}{-h}$$

$$\Rightarrow \lim_{x \to 1^-} = \frac{-\cancel{x} + h + \cancel{1} + \cancel{x} - \cancel{1}}{-h} = -1$$

$\therefore$ Right hand derivatives $\neq$ Left hand derivative

So, $f(x)$ is not differentiat at $x=1$.

(proved)

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20b. Sol;

To prove : $C_0C_n + C_1C_{n-1} + C_2C_{n-2} + \cdots + C_nC_0 = \frac{2n!}{n!\,n!}$

We know,

$(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n$ — ①

21 a.

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21 b.

De-moivre's theorem

For any positive integer n,

$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

The above theorem holds for any real number.

To find: square root of $4+4\sqrt{3}i$

Let $4+4\sqrt{3}i = r(\cos\theta + i\sin\theta)$

here $x=4$ and $y=4\sqrt{3}$

then

$r = \sqrt{x^2+y^2}$

$= \sqrt{4^2+(4\sqrt{3})^2}$

$= \sqrt{16+48}$

$= 8$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{4\sqrt{3}}{4} = \sqrt{3}$ [∵ Since x and y both are +ve so, θ lies in 1st quadrant]

$\tan\theta = \tan 60°$

$\therefore \theta = 60°$

$\therefore 4+4\sqrt{3}i = r(\cos\theta + i\sin\theta) = 8(\cos 60° + i\sin 60°)$

Let z be the square root of $4+4\sqrt{3}i$

$z = \sqrt{4+4\sqrt{3}i}$

$z^2 = 4+4\sqrt{3}i = 8(\cos 60° + i\sin 60°)^{1/2}$

$z = [8(\cos 60° + i\sin 60°)]^{1/2}$

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$$= 8^{\frac{1}{2}} \left(\cos \frac{360^\circ n + 60}{2} + i \sin \frac{360^\circ n + 60}{2}\right)$$

where, $n = 0, 1$.

Taking $n = 0$

$\therefore z = 8^{1/2} (\cos 180 \times 0 + 30^\circ) + i \sin (180 \times 0 + 30^\circ)$

$\Rightarrow z = 8^{1/2} \times \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)$

$= 8^{1/2} \times \left(\frac{\sqrt{3} + i}{2}\right)$

$\Rightarrow z = 2\sqrt{2} \left(\frac{\sqrt{3} + i}{\cancel{2}}\right)$

$\therefore z = \sqrt{6} + \sqrt{2} i$

Taking $n = 1$

$\therefore z = 8^{1/2} (\cos (180^\circ + 30^\circ) + i \sin (180^\circ + 30^\circ))$

$= 2\sqrt{2} \left(\frac{-\sqrt{3}}{2} - \frac{1 i}{2}\right)$

$= 2\sqrt{2} \frac{(-\sqrt{3} - i)}{\cancel{2}}$

$= -\sqrt{6} - \sqrt{2} i$

$\therefore$ The square roots are $\pm(\sqrt{6} \pm \sqrt{2} i)$

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22a. Given: $x, y \geq 0$

$x + y \leq 4$

$x - y \leq 1$

$x + 3y = P$

Introducing slack variable,

$x + y + s = 4$

$x - y + t = 1$

$-x - 3y + P = 0$

Writing above eq^n in canonical form

	x	y	s	t	P	RHS	Ratio
s	1	1	1	0	0	4	4 ←
t	1	-1	0	1	0	1	-1
	-1	-3	0	0	1	0	
		↑					

Performing $R_3 \to R_3 *$ [illegible] $3R_1$

	x	y	s	t	P	RHS
s	1	1	1	0	0	4
t	1	-1	0	1	0	1
	1/3	1	0	0	1/-3	0

Performing $R_3 \to R_3 + 3R_1$ and $R_2 \to R_2 + R_1$.

	x	y	s	t	P	RHS
y	1	1	1	0	0	4
t	2	0	1	1	0	5
	2	0	3	0	1	12

Since, the bottom row does not have negative entries. So, maximum value is 12 at (0,4)