Assignment (Simplex Method)

1. Using simplex method, maximize, $Z = 20x + 30y$ subject to $2x + 5y \le 20$, $2x + y \le 12$, $x, y \ge 0$.

→ Soln.

Given,

$$Z = 20x + 30y$$
$$2x + 5y \le 20$$
$$2x + y \le 12$$
$$x, y \ge 0$$

Introducing non-negative slack variables s, t we get.

$$2x + 5y + s = 20$$
$$2x + y + t = 12$$
$$-20x - 30y + z = 0$$

Canonical form of above system is given by,

$$2x + 5y + s + 0.t + 0.z = 20$$
$$2x + y + 0.s + t + 0.z = 12$$
$$-20x - 30y + 0.s + 0t + z = 0$$

This canonical form yields simplex tableau

Initial tableau

BV	x	y	s	t	z	RHS	
s	2	5	1	0	0	20	→
t	2	1	0	1	0	12	
	-20	-30	0	0	1	0	
		↑					

Here, $-30 < -20$, so, y-column is pivot column, also comparing ratios; $\frac{20}{5} < \frac{12}{1}$ i.e. $4 < 12$

So, first row is pivot row & 5 is pivot element.

19/11/2022 18:07

Performing $R_1 \to \frac{1}{5} R_1$

BV	x	y	s	t	z	RHS
y	2/5	1	1/5	0	0	4
t	2	1	0	1	0	12
	-20	-30	0	0	1	0

Performing $R_2 \to R_2 - R_1$ & $R_3 \to R_3 + 30R_1$

BV	x	y	s	t	z	R.H.S.
y	2/5	1	1/5	0	0	4
t	8/5	0	-1/5	1	0	8 →
	-8 ↑	0	6	0	1	120

Again in bottom row, negative entry still present. so. x-column is the pivot column. also comparing ratios:
$\frac{4}{2/5} > \frac{8}{8/5}$ i.e. $10 > 5$ so, $8/5$ is pivot element & second row is pivot row.

Performing $R_2 \to \frac{5}{8} \times R_2$

BV	x	y	s	t	z	R.HS
y	2/5	1	1/5	0	0	4
x	1	0	-1/8	5/8	0	5
	-8	0	6	0	1	120

Performing $R_1 \to R_1 - \frac{2}{5} R_2$ & $R_3 \to R_3 + 8R_2$

BV	x	y	s	t	z	RHS
y	0	1	1/4	-1/4	0	2
x	1	0	-1/8	5/8	0	5
	0	0	5	5	1	160

Since all the entries in the bottom row are non-negative, so, solution given by above tableau is optimum.

$\therefore$ maximum value of $z = 160$ at $(5, 2)$ ans

2. Using simplex method, maximize, $U = 5x + 7y$ subject to $2x + 3y \le 13$, $3x + 2y \le 12$, $x, y \ge 0$

→ Soln.

Introducing slack variables s, t, we get

$$2x + 3y + s = 13$$
$$3x + 2y + t = 12$$
$$-5x - 7y + U = 0$$

Canonical form of above system is,

$$2x + 3y + s + 0.t + 0.U = 13$$
$$3x + 2y + 0.s + t + 0.U = 12$$
$$-5x - 7y + 0.s + 0.t + t = 0$$

This canonical form yields simplex tableau.

BV	x	y	s	t	U	RHS
s	2	3	1	0	0	13 →
t	3	2	0	1	0	12
	-5	-7 ↑	0	0	1	0

Here, $-7 < -5$, so, y-column is pivot column, also comparing ratios, $\frac{13}{3} < \frac{12}{2}$, So, first row is pivot row & 3 is pivot element.

Performing $R_1 \to \frac{1}{3} R_1$

BV	x	y	s	t	U	R.H.S.
~~s~~	$2/3$	1	$1/3$	0	0	$13/3$
t	3	2	0	1	0	12
	-5	-7	0	0	1	0

19/11/2022 18:07

Performing $R_2 \to R_2 - 2R_1$ & $R_3 \to R_3 + 7R_1$

BV	x	y	s	t	u	RHS
y	$2/3$	1	$1/3$	0	0	$13/3$
t	$5/3$	0	$-2/3$	1	0	$10/3$
	$-1/3$	0	$7/3$	0	1	$91/3$

Again in bottom row negative entry still present. So x column is pivot colum, also comparing ratios, $\frac{10/3}{5/3}$ $\frac{13/3}{2/3}$

i.e $2 < \frac{13}{2}$ So, second row is pivot row & $5/3$ is pivot element.

Performing $R_2 \to \frac{3}{5} R_2$

BV	x	y	s	t	u	RHS
y	$2/3$	1	$1/3$	0	0	$13/3$
x	1	0	$-2/5$	$3/5$	0	2
	$-1/3$	0	$7/3$	0	1	$91/3$

Performing $R_1 \to R_1 - \frac{2}{3} R_2$ & $R_3 \to R_3 + \frac{1}{3} R_2$

BV	x	y	s	t	u	RHS
y	0	1	$3/5$	$-2/5$	0	3
x	1	0	$-2/5$	$3/5$	0	2
	0	0	$11/5$	$1/5$	1	31

Since all the entries in the bottom row are non-negative So solution given by above tableau is optimum.

$\therefore$ maximum value of U ~~of~~ $= 31$ at $(2,3)$ ans

3. Using simplex method, maximize, $z = 50x_1 + 80x_2$ subject to $x_1 + 2x_2 \le 32$, $3x_1 + 4x_2 \le 84$, $x_1, x_2 \ge 0$

→ Soln.

Introducing slack variable s, t we get

$$x_1 + 2x_2 + s = 32$$
$$3x_1 + 4x_2 + t = 84$$
$$-50x_1 - 80x_2 + z = 0$$

Canonical form of above system is,

$$x_1 + 2x_2 + s + 0.t + 0.z = 32$$
$$3x_1 + 4x_2 + 0.s + t + 0.z = 84$$
$$-50x_1 - 80x_2 + 0.s + 0.t + z = 0$$

This canonical form yields simplex tableau

BV	x_1	x_2	s	t	z	RHS
s	1	2	1	0	0	32 →
t	3	4	0	1	0	84
	-50	-80 ↑	0	0	1	0

Here, $-80 < -50$ so, x_2 column is pivot column, also comparing ratios $\frac{32}{2} < \frac{84}{4}$ i.e. $16 < 21$. So, first row is pivot row & 2 is pivot element.

Performing $R_1 \to \frac{1}{2} R_1$

BV	x_1	x_2	s	t	z	RHS
x_2	1/2	1	1/2	0	0	16
t	3	4	0	1	0	84
	-50	-80	0	0	1	0

Performing $R_2 \to R_2 - 4R_1$ & $R_3 \to R_3 + 80R_1$

BV	x_1	x_2	s	t	z	RHS
x_2	$1/2$	1	$1/2$	0	0	16
t	1	0	-2	1	0	20
	-10	0	40	0	1	1280

Again, in bottom row negative entry still present so x_1 column is pivot column, also comparing $\frac{16}{1/2} > \frac{20}{1}$

i.e. $32 > 20$. So, second row is pivot row & 1 is pivot element.

Performing $R_1 \to R_1 - \frac{1}{2}R_2$ & $R_3 \to R_3 + 10R_2$

BV	x_1	x_2	s	t	z	RHS
x_2	0	1	$3/2$	$-1/2$	0	6
x_1	1	0	-2	1	0	20
	0	0	20	10	1	1480

Since all the entries in the bottom row are non-negative

So, solution given by above tableau is optimum.

$\therefore$ maximum point of $z = 1480$ at $(20, 6)$ ans

4. Using simplex method, maximize $U = 7x + 5y$ subject to $x + 2y \le 6$, $4x + 3y \le 12$, $x, y \ge 0$.

→ Soln.

Introducing slack variables, s, t we get

$$x + 2y + s = 6$$
$$4x + 3y + t = 12$$
$$-7x - 5y + U = 0$$

Canonical form of above system is,

$$x + 2y + s + 0t + 0U = 6$$
$$4x + 3y + 0.s + t + 0.U = 12$$
$$-7x - 5y + 0.s + 0.t + U = 0$$

This canonical form yields simplex tableau

BV	x	y	s	t	U	R.H.S
s	1	2	1	0	0	6
t	4	3	0	1	0	12 →
	-7	-5	0	0	1	0
	↑					

Here, $-7 < -5$, so, x column is pivot column, also comparing ratios $\frac{6}{1} > \frac{12}{4}$ i.e. $6 > 3$

So, second row is pivot row & 4 is pivot element.

Performing $R_2 \rightarrow \frac{1}{4} R_2$

BV	x	y	s	t	U	RHS
s	1	2	1	0	0	6
x	1	3/4	0	1/4	0	3
	-7	-5	0	0	1	0

19/11/2022 18:07

Date
Page

Performing $R_1 \to R_1 - R_2$ & $R_3 \to R_3 + 7R_2$

BV	x	y	s	t	U	RHS
s	0	5/4	1	-1/4	0	3
x	1	3/4	0	1/4	0	3
	0	1/4	0	7/4	1	21

Since all the entries in the bottom row are non-negative So, solution given by above tableau is optimum.

$\therefore$ maximum point of $U = 21$ at $(3,0)$ ans

5. Using simplex method, maximize $U = 5x + 3y$ subject to $2x + y \le 40$, $x + 2y \le 50$. $x, y \ge 0$.

→ Soln;
Introducing slack variables s, t we get

$$2x + y + s = 40$$
$$x + 2y + t = 50$$
$$-5x - 3y + U = 0$$

Canonical form of above system is.

$$2x + y + s + 0.t + 0.U = 40$$
$$x + 2y + 0.s + t + 0.U = 50$$
$$-5x - 3y + 0.s + 0.t + U = 0$$

This canonical form yields simplex tableau

BV	x	y	s	t	U	RHS
s	2	1	1	0	0	40 →
t	1	2	0	1	0	50
	-5	-3	0	0	1	0
	↑					

Here, $-5 < -3$. So, x-column is pivot column, also comparing ratios, $\frac{40}{2} < \frac{50}{1}$ i.e. $20 < 50$. So, second row is pivot ro[w] P.T.O

19/11/2022 18:07

2 is pivot element.

Performing $R_1 \to \frac{1}{2} R_1$

BV	x	y	s	t	u	RHS
x	1	1/2	1/2	0	0	20
t	1	2	0	1	0	50
	-5	-3	0	0	1	0

Performing $R_2 \to R_2 - R_1$ & $R_3 \to R_3 + 5R_1$

BV	x	y	s	t	u	RHS
x	1	1/2	1/2	0	0	20
t	0	3/2	-1/2	1	0	30
	0	-1/2	5/2	0	1	100

Here, in bottom row negative entry still present. So y column is pivot column, also comparing $\frac{20}{1/2} > \frac{30}{3/2}$

i.e. 40>20. So second row is pivot row & 3/2 is pivot element.

Performing $R_2 \to \frac{2}{3} R_2$

BV	x	y	s	t	u	RHS
x	1	1/2	1/2	0	0	20
y	0	1	-1/3	2/3	0	20
	0	-1/2	5/2	0	1	100

Performing $R_1 \to R_1 - \frac{1}{2} R_2$ & $R_3 \to R_3 + \frac{1}{2} R_2$

BV	x	y	s	t	u	RHS
x	1	0	2/3	-1/3	0	10
y	0	1	-1/3	2/3	0	20
	0	0	7/3	1/3	1	110

19/11/2022 18:07

P.T.O

Since all the entries in the bottom row are non-negative
So, solution given by above tableau is optimum.

$\therefore$ maximum point of $U = 110$ at $(10, 20)$ Ans

6. Maximize $P = x + y$ sub to constraints $x + 2y \le 6$, $3x + 2y \le 12$ $x, y \ge 0$.

→ Soln,

Introducing slack variables s, t we get

$x + 2y + s = 6$

$3x + 2y + t = 12$

$-x - y + P = 0$

Canonical form of above system is,

$x + 2y + s + 0t + 0P = 6$

$3x + 2y + 0.s + t + 0.P = 12$

$-x - y + 0.s + 0.t + P = 0$

This canonical form yields simplex tableau.

BV	x	y	s	t	P	RHS
s	1	2	1	0	0	6
t	3	2	0	1	0	12 →
	-1	-1	0	0	1	0

Performing $R_2 \to \frac{1}{3} R_2$

BV	x	y	s	t	P	RHS
s	1	2	1	0	0	6
x	1	2/3	0	1/3	0	4
	-1	-1	0	0	1	0

19/11/2022 18:07

Performing $R_1 \to R_1 - R_2$ & $R_3 \to R_3 + R_2$

BV	x	y	s	t	P	RHS
s	0	$4/3$	1	$-1/3$	0	2
x	1	$2/3$	0	$1/3$	0	4
	0	$-1/3$		$1/3$	1	4

Here, in bottom row negative entry still present. So y column is pivot column, also comparing $\frac{2}{4/3} < \frac{4}{2/3}$ i.e. $3/2 < 6$. So ~~second~~ first row is pivot row & $4/3$ is pivot element

Performing $R_1 \to \frac{3}{4} R_1$

BV	x	y	s	t	P	RHS
y	0	1	$3/4$	$-1/4$	0	$3/2$
x	1	$2/3$	0	$1/3$	0	4
	0	$-1/3$	0	$1/3$	1	4

Performing $R_2 \to R_2 - \frac{2}{3} R_1$ & $R_3 \to R_3 + \frac{1}{3} R_1$

BV	x	y	s	t	P	RHS
y	0	1	$3/4$	$-1/4$	0	$3/2$
x	1	0	$-1/2$	$1/2$	0	3
	0	0	$1/4$	$1/4$	1	$9/2$

Since, all the entries in the bottom row are non-negative so solution given by above tableau is optimum.

$\therefore$ Maximum point of $P = \frac{9}{2}$ at $(3, \frac{3}{2})$ ans