Chapter - 19

System of Linear Equation

Gauss Elimination Method.

→ Let system of equations be,

$a_{11}x + a_{12}y + a_{13}z = b_1$ —— (1)

$a_{21}x + a_{22}y + a_{23}z = b_2$ —— (2)

$a_{31}x + a_{32}y + a_{33}z = b_3$ —— (3)

Forward Elimination

Eliminating x between eqⁿ (1) & (2) we get, say

$p_1 y + q_1 z = r_1$ —— (4)

Also,

Eliminating x between eqⁿ (2) & (3)

Say, $p_2 y + q_2 z = r_2$ —— (5)

Again, eliminating 'y' between (4) & (5) we get

Say, $\alpha z = \beta$ —— (6)

Now,

We have sub-system

$a_{11}x + a_{12}y + a_{13}z = b_1$ —— (1)

$p_1 y + q_1 z = r_1$ —— (4)

$\alpha z = \beta$ —— (6)

By Backward Substitution

we can find required solution. #

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Note:

Case-I:

If $\alpha \neq 0, \beta \neq 0$ (or, $\beta = 0, \alpha \neq 0$) then given system has unique solution & hence consistent.

Case-II:

If $\alpha = \beta = 0$, Then given system has infinitely many solution & hence consistent.

Case-III:

If $\alpha = 0, \beta \neq 0$ then given system has no solution & hence inconsistent.

Exercise:- 19.1

1 Solve the following system of linear equations by Gauss Elimination method:

a. $4x + 5y = 12$
$3x + 2y = 9$

→ Soln,

Given system,

$4x + 5y = 12$ --- (1)

$3x + 2y = 9$ --- (2)

Forward Elimination

To Eliminating x performing $3[eq^n (1)] - 4[eq^n (2)]$

$12x + 15y = 36$

$\underset{(-)}{12x} \underset{(-)}{+ 8y} \underset{(-)}{= 36}$

$7y = 0$ — (3)

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So,
We have following sub-system

$$3x+2y=9 \quad \text{—— (2)}$$
$$7y=0 \quad \text{—— (3)}$$

Backward Substitution

From (3) $y=0$

When $y=0$, from (2),

$$3x=9$$
$$x=3$$

Req solution is $x=3, y=0$ ans

6. $5x+2y=4$
$7x+3y=5$

→ Soln,

Given system

$$5x+2y=4 \quad \text{—— (1)}$$
$$7x+3y=5 \quad \text{—— (2)}$$

Forward Elimination

To Eliminating x performing $7[eq^n 1]-5[eq^n 2]$

$$35x+14y=28$$
$$\underline{(-)\,35x\overset{(-)}{+}15y=\overset{(-)}{25}}$$
$$-y=3 \quad \text{—— (3)}$$

So,
We have following sub-system,

$$5x+2y=4 \text{ —— (1)}$$
$$-y=3 \text{ —— (3)}$$

Backward Substitution

From (3), $y=-3$

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When $y = -3$, from (1)

$5x + 2\times -3 = 4$

or, $5x - 6 = 4$

or, $x = \frac{10}{5}$

$\therefore x = 2$

$\therefore$ Req solution is $x = 2, y = -3$ <u>an</u>

c. $5x - 3y = 8$

$2x + 5y = 59$

→ Soln,

Given system is.

$5x - 3y = 8$ —— (1)

$2x + 5y = 59$ —— (2)

<u>Forward Elimination</u>

To Eliminating x, performing $2[eq^n (1)] - 5[eq^n (2)]$

$10x - 6y = 16$

$10x + 25y = 295$

$(-) \quad (-) \quad (-)$

$-31y = -279$ —— (3)

So,

We have following sub-system.

$5x - 3y = 8$ —— (1)

$-31y = -279$ —— (3)

<u>Backward Substitution</u>

From (3), $y = 9$

When $y = 9$, from (1)

$5x - 3\times 9 = 8$

or, $5x = 35$

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$\therefore x = 7$

$\therefore$ Required solution is $x = 7, y = 9$ <u>an</u>

d. $2x - 3y = 7$
$3x + y = 5$

$\rightarrow$ Soln,

Given system is

$2x - 3y = 7$ — (1)
$3x + y = 5$ — (2)

<u>Forward Elimination</u>

To Eliminating x, performing $3[eq^n (1)] - 2[eq^n (2)]$

$$\begin{array}{r} 6x - 9y = 21 \\ 6x + 2y = 10 \\ (-) \quad (-) \quad (-) \\ \hline -11y = 11 \end{array}$$ — (3)

So,
we have following substitution

$2x - 3y = 7$ — (1)
$-11y = 11$ — (3)

<u>Backward Substitution</u>

From (3), $y = -1$
When, $y = -1$, from (1)

$2x - 3\times -1 = 7$
or, $2x = 4$
$\therefore x = 2$

$\therefore$ Required solution is $x = 2, y = -1$ <u>an</u>

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2. Solve the following system of linear equations by Gauss elimination method:

a. $5x - y + 4z = 5$

$2x + 3y + 5z = 2$

$5x - 2y + 6z = -1$

→ Soln,

Given system,

$5x - y + 4z = 5$ —— (1)

$2x + 3y + 5z = 2$ —— (2)

$5x - 2y + 6z = -1$ —— (3)

Foward Elimination

To Eliminating x between eq^n (1) & (2) by performing $2[eq^n (1)] - 5[eq^n (2)]$

we get,

$$\cancel{10x} - 2y + 8z = 10$$
$$\underline{\cancel{10x} + 15y + 25z = 10}$$
$$(-) \quad (-) \quad (-) \quad (-)$$
$$-17y - 17z = 0 \text{ —— (4)}$$

Again,

Eliminating x between (1) & (3) by performing eq^n (1) - eq^n (3)

$$\cancel{5x} - y + 4z = 5$$
$$\underline{\cancel{5x} - 2y + 6z = -1}$$
$$(-) \quad (+) \quad (-) \quad (+)$$
$$y - 2z = 6 \text{ —— (5)}$$

Also, eliminating 'y' between (4) & (5) by performing eq^n (4) + 17 eq^n (5)

we get,

$$-\cancel{17y} - 17z = 0$$
$$\underline{\cancel{17y} - 34z = 102}$$
$$-51z = 102 \text{ —— (6)}$$

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Now,
we have sub-system

$$5x - y + 4z = 5 \quad \text{---} \; (1)$$
$$y - 2z = 6 \quad \text{---} \; (5)$$
$$-51z = 102 \quad \text{---} \; (6)$$

Backward Substitution

From (6), $z = -2$

When, $z = -2$, from (5)

$$y = 6 + 2\times(-2) = 2$$

When, $y = 2$, $z = -2$, from (1)

$$5x = 5 + 2 - 4(-2)$$
$$\text{or, } 5x = 15$$
$$\therefore x = 3$$

$\therefore$ Req. solution is $x = 3, y = 2, z = -2$ ans

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6. $x - y + 2z = 7$
$3x + 4y - 5z = -5$
$2x - y + 3z = 12$

→ Soln,
Given system,

$$x - y + 2z = 7 \quad \text{---} \; (1)$$
$$3x + 4y - 5z = -5 \quad \text{---} \; (2)$$
$$2x - y + 3z = 12 \quad \text{---} \; (3)$$

Forward Elimination

To Eliminating x between eq (1) & (2) by performing $3[eq^n(1)] - eq^n(2)$
we get, $\cancel{3x} - 3y + 6z = 21$

$$\underset{(-)}{\cancel{3x}} \underset{(-)}{+} 4y \underset{(+)}{-} 5z = \underset{(+)}{-}5$$

$$-7y + 11z = 26 \quad \text{---} \; (4)$$

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Again,
Eliminating x between (1) & (3) by performing 2[eqⁿ(1)] − eqⁿ(3)

$$2x - 2y + 4z = 14$$
$$2x - y + 3z = 12$$
$$(-)\quad (+)\quad (-)\quad (-)$$
$$-y + z = 2 \quad \text{——} (5)$$

Also eliminating y between (4) & (5) by performing eqⁿ(4) − 7[eqⁿ(5)]

$$-7y + 11z = 26$$
$$-7y + 7z = 14$$
$$(+)\quad (-)\quad (-)$$
$$4z = 12 \quad \text{——} (6)$$

Now,
we have sub-system

$$x - y + 2z = 7 \quad \text{——} (1)$$
$$-y + z = 2 \quad \text{——} (5)$$
$$4z = 12 \quad \text{——} (6)$$

Backward Substitution

from (6), $z = 3$

when $z = 3$ from (5)

$$-y + 3 = 2$$
$$\text{or, } -y = -1$$
$$\therefore y = 1$$

When $y = 1$, $z = 3$ from (1)

$$x - 1 + 2 \times 3 = 7$$
$$\text{or, } x = 7 - 5$$
$$\therefore x = 2$$

$\therefore$ Required solution is, $x = 2, y = 1, z = 3$ ans

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c. $2x+3y+3z=5$

$x-2y+z=-4$

$3x-y-2z=3$

→ Soln,

Given system is,

$2x+3y+3z=5$ ——— (1)

$x-2y+z=-4$ ——— (2)

$3x-y-2z=3$ ——— (3)

Forward Elimination

To Eliminating x between (1) & (2) by performing eqⁿ(1) - 2[eqⁿ(2)]

we get,

$$\cancel{2x}+3y+3z=5$$
$$\cancel{2x}-4y+2z=-8$$
$$(-)\quad(+)\quad(-)\quad(+)$$
$$7y+z=13 \text{ ——— (4)}$$

Again,

Eliminating x between (2) & (3) by performing 3[eqⁿ(2)] - eqⁿ(3)

$$\cancel{3x}-6y+3z=-12$$
$$\cancel{3x}-y-2z=3$$
$$(-)\quad(+)\quad(+)\quad(-)$$
$$-5y+5z=-15 \text{ ——— (5)}$$

Also, eliminating y between (4) & (5) by performing 5(eqⁿ(4))+7(eqⁿ(5))

$$\cancel{35y}+5z=65$$
$$-\cancel{35y}+35z=-105$$
$$40z=-40 \text{ ——— (6)}$$

Now,

We have sub-system,

$2x+3y+3z=5$ ——— (1)

$-5y+5z=-15$ ——— (5)

$40z=-40$ ——— (6)

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Back-ward Substitution

from (6), $z = -1$

When $z = -1$ from (5)

$-5y + 5 \times -1 = -15$

or, $-5y = -10$

$\therefore y = 2$

When, $y = 2$, $z = -1$ from (1)

$2x + 3 \times 2 + 3 \times -1 = 5$

or, $2x = 5 - 6 + 3$

or, $x = \frac{2}{2}$

$\therefore x = 1$

$\therefore$ Required solution is $x = 1, y = 2, z = -1$ Ans

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d. $x + 2y + 3z = 14$

$3x + 4y + 2z = 17$

$2x + 3y + z = 11$

→ Soln.

Given system is,

$x + 2y + 3z = 14$ —— (1)

$3x + 4y + 2z = 17$ —— (2)

$2x + 3y + z = 11$ —— (3)

Forward Elimination

To eliminating x ~~performing~~ between (1) & (2) by performing $3[eq^n (1)] - eq^n (2)$, we get,

$3x + 6y + 9z = 42$

$\underline{\overset{(-)}{3x} + \overset{(-)}{4y} + \overset{(-)}{2z} = \overset{(-)}{17}}$

$2y + 7z = 25$ —— (4)

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Again,

Eliminating x between (1) & (3) by performing $2[eq^n (1)] - eq^n (3)$

$$\begin{array}{r} 2x + 4y + 6z = 28 \\ 2x + 3y + z = 11 \\ (-) \quad (-) \quad (-) \quad (-) \\ \hline y + 5z = 17 \end{array} \quad \text{—— (5)}$$

Also, eliminating y between (4) & (5) by performing $eq^n (4) - 2[eq^n (5)]$

$$\begin{array}{r} 2y + 7z = 25 \\ 2y + 10z = 34 \\ (-) \quad (-) \quad (-) \\ \hline -3z = -9 \end{array} \quad \text{—— (6)}$$

Now,

We have sub-system

$x + 2y + 3z = 14$ —— (1)

$y + 5z = 17$ —— (5)

$-3z = -9$ —— (6)

Backward Substitution

from (6), $z = 3$

When $z = 3$, from (5)

$y + 5 \times 3 = 17$

or, $y = 17 - 15$

$\therefore y = 2$

When $y = 2$, $z = 3$, from (1)

$x + 2 \times 2 + 3 \times 3 = 14$

or, $x = 14 - 4 - 9$

$\therefore x = 1$

$\therefore$ Required solution is $x = 1$, $y = 2$, $z = 3$ Ans

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3. Test the consistency of the following system of equations by Gauss Elimination method.

a. $x + 3y = 5$

$3x + y = 4$

→ Soln,

Given system is,

$x + 3y = 5$ ——— (1)

$3x + y = 4$ ——— (2)

Forward elimination

To eliminating x by performing $3[\text{eq}^n (1)] - \text{eq}^n (2)$, we get

$$\begin{aligned} 3x + 9y &= 15 \\ \underset{(-)}{3x} + \underset{(-)}{y} &= \underset{(-)}{4} \\ \hline 8y &= 11 \end{aligned}$$ ——— (3)

So, we have following sub-system

$x + 3y = 5$ ——— (1)

$8y = 11$ ——— (3)

Backward Substitution

from (3), $y = 11/8$

when $y = \frac{11}{8}$, from (1)

$x + 3 \times \frac{11}{8} = 5$

or, $x = 5 - \frac{33}{8}$

$\therefore x = \frac{7}{8}$

Req. solution is, $x = \frac{7}{8}$, $y = \frac{11}{8}$

Hence, given system has unique solution & hence consistent. ans

C. $-2x+5y=3$
$6x-15y=-9$

→ Soln.

Given system is.

$$-2x+5y=3 \quad \text{—— (1)}$$
$$6x-15y=-9 \quad \text{—— (2)}$$

Forward Elimination

To eliminating x by performing $3[eq^n(1)] + eq^n(2)$, we get,

$$-6x+15y=9$$
$$6x-15y=-9$$
$$\overline{0.y=0} \quad \text{—— (3)}$$

So, we have following sub-system.

$$-2x+5y=3 \text{ —— (1)}$$
$$0.y=0 \quad \text{—— (3)}$$

Here, in eq^n (3), any real value of z satisfies the equation,

So, we can take $y=k$, $k \in R$

When, $y=k$, from (1)

$$-2x+5k=3$$
$$\text{or, } -2x=3-5k$$
$$\therefore x = \frac{3-5k}{-2}$$

$\therefore$ ~~Req~~ q solution is, $x = -\dfrac{3-5k}{2}$, $y=k, k \in R$

Hence, given system has infinitely many solutions and hence consistent. ans

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c. $2x - y + 4z = 4$
$x + 2y - 3z = 1$
$3x + 3z = 6$

→ Soln
Given system is,
$2x - y + 4z = 4$ —— (1)
$x + 2y - 3z = 1$ —— (2)
$3x + 3z = 6$ —— (3)

Forward Elimination
To Eliminating x between (1) & (2) by performing eqⁿ(1) - 2[eqⁿ(2)]
we get,

$$\cancel{2x} - y + 4z = 4$$
$$\underset{(-)}{\cancel{2x}} \underset{(-)}{+ 4y} \underset{(+)}{- 6z} = \underset{(-)}{2}$$
$$-5y + 10z = 2 \text{ —— (4)}$$

Again,
Eliminating x between (2) & (3) by performing 3[eqⁿ(2)] - eqⁿ(3)

$$\cancel{3x} + 6y - 9z = 3$$
$$\underset{(-)}{\cancel{3x}} \underset{(-)}{+ 0} \underset{(-)}{+ 3z} = \underset{(-)}{6}$$
$$6y - 12z = -3 \text{ —— (5)}$$

Also, eliminating 'y' between (4) & (5) by performing 6[eqⁿ(4)] + 5[eqⁿ(5)]
we get,

$$-\cancel{30y} + 60z = 12$$
$$\cancel{30y} - 60z = -15$$
$$0.z = -3 \text{ —— (6)}$$

Now,
We have sub-system,

$2x - y + 4z = 4$ —— (1)
$6y - 12z = -3$ —— (5)
$0.z = -3$ —— (6)

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From (6)

Here, there is no real value of z satisfies equation so, system has no solution & hence inconsistent. ans

f. $x+3y+4z=8$
$2x+y+2z=5$
$5x+2z=7$

→ Soln,

Given system is,

$x+3y+4z=8$ —— (1)

$2x+y+2z=5$ —— (2)

$5x+2z=7$ —— (3)

Forward Elimination

To eliminating x between (1) & (2) by performing 2[eqⁿ (1)] - eqⁿ (2) we get,

$$\begin{array}{r} \cancel{2x}+6y+8z=16 \\ \underset{(-)}{\cancel{2x}} \underset{(-)}{+y} \underset{(-)}{+2z} = \underset{(-)}{5} \\ \hline 5y+6z=11 \end{array}$$ —— (4)

Again,

Eliminating x between (2) & (3) by performing 5[eqⁿ (2)] - 2[eqⁿ (3)] we get,

$$\begin{array}{r} \cancel{10x}+5y+10z=25 \\ \underset{(-)}{\cancel{10x}} \underset{(-)}{+4z} = \underset{(-)}{14} \\ \hline 5y+6z=11 \end{array}$$ —— (5)

Also, eliminating 'y' between (4) & (5) by performing eqⁿ (4) - eqⁿ (5)

$$\begin{array}{r} \cancel{5y}+6z=11 \\ \underset{(-)}{\cancel{5y}} \underset{(-)}{+6z} = \underset{(-)}{11} \\ \hline 0.z=0 \end{array}$$ —— (6)

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Now,

We have sub-system

$$x + 3y + 4z = 8 \text{ --- (1)}$$
$$5y + 6z = 11 \text{ --- (5)}$$
$$0.z = 0 \text{ --- (6)}$$

Here in eqn (6), any real value of z satisfies the equation.
So, we can take $z = k$, $k \in R$

When, $z = k$ from (5)

$$5y = 11 - 6k$$
$$\therefore y = \frac{11 - 6k}{5}$$

Also,
When $y = \frac{11-6k}{5}$, $z = k$ then from (1)

$$x = 8 - 3\left(\frac{11-6k}{5}\right) - 4k$$
$$= \frac{40 - 33 + 18k - 20k}{5}$$
$$= \frac{7 - 2k}{5}$$

$\therefore$ Req. solution is, $x = \frac{7-2k}{5}$, $y = \frac{11-6k}{5}$, $z = k$, $k \in R$

Hence, given system has infinitely many solutions and hence consistent.