Chapter 13
Correlation and Regression

Exercise 13.1

1. Solution:

a. Here, cov (x, y) = –16.5

var (x) = 2.89

var (y) = 100

Coefficient off correlation (r) = $\frac{\text{cov}(x, y)}{\sqrt{\text{val}(x)} \cdot \sqrt{\text{var}(y)}}$

$= \frac{-16.5}{\sqrt{2.89} \cdot \sqrt{100}}$

$= \frac{-16.5}{1.7 \times 10}$

$= \frac{-16.5}{17}$

$= -0.97$

b. Here,

Given, N = 13

$\Sigma x = 117$

$\Sigma x^3 = 1,313$

$\Sigma y = 260$

$\Sigma y^2 = 6580$

$\Sigma xy = 2827$

Coefficient of correlation (r) = $\frac{n\Sigma xy - \Sigma x . \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2} . \sqrt{n\Sigma y - (\Sigma y)^2}}$

$= \frac{3 \times 2827 - 117 \times 260}{\sqrt{13 \times 1313} - \sqrt{13 \times 6580 - ....}}$

$= \frac{36751 - 30420}{\sqrt{3380} - \sqrt{17940}}$

$= \frac{6331}{58.13 \times 133.94}$

$= 0.81$

c. Solution:

Here, n = 15

$\sigma x = 3.2$

$\sigma y = 3.y$

$\Sigma(x - \overline{x})(y - \overline{y}) = 122$

Coefficient of correlation (r) = $\frac{\Sigma(x - \overline{x})(y - \overline{y})}{n\sigma x\sigma y}$

$= \frac{122}{15 \times 3.2 \times 3.y}$

$= \frac{122}{163.2}$

$= 0.75$

2. a. Solution:

Karl Pearson's coefficient of correlation between 'x' and 'y' (i) = 0.28

Cor (x, y) = 0.76

Val (x) = 9

σy = ?

Now, We have,

Coefficient of correlation (r) = $\frac{\text{cov (x, y)}}{\sqrt{\text{var(x)}}.\sqrt{\text{var(y)}}}$

or, $0.26 = \frac{0.76}{\sqrt{9}\,.\,\sqrt{\text{var(y)}}}$

or, $0.36 = \frac{1}{3.\sqrt{\text{var(y)}}}$

or, $1.105\,.\,\sqrt{\text{var(y)}} = 1$

or, $\sqrt{\text{var (y)}} = 0.904$

$\therefore$ $\sigma_y = 0.904$

b. Solution:

Correlation coefficient between x and y (r) = 0.85

Cov (x,y) = 6.5

Var (x) = 6.1

Standard derivation of y (σ_y) = ?

Now, we have,

Coefficient of correlation (r) = $\frac{\text{cov(x,y)}}{\sqrt{\text{var(x)}}\,.\,\sqrt{\text{var(y)}}}$

or, $0.85 = \frac{6.5}{\sqrt{6.1}\,.\,\sqrt{\text{var(y)}}}$

or, $0.13076 \times 24698\,.\,\sqrt{\text{var(y)}} = 1$

or, $\sqrt{\text{var(y)}} = \frac{1}{0.32295}$

or, $\sqrt{\text{var(y)}} = 3.096$

or, $\sigma_y = 3.1$

Hence, the required σ_y is 3.1

3. Solution:

a. Here,

Maths(x)	Biology (y)	$x = (x-\bar{x})$	$y=(y-\bar{y})$	xy	x^2	y^2
48	45	14	10	140	196	100
35	20	1	–15	–15	1	225
17	40	–17	5	–85	289	25
23	25	–11	–10	110	121	100
47	45	13	10	130	169	100
$\Sigma x = 170$	$\Sigma y = 175$			$\Sigma xy=280$	$\Sigma x^2=776$	$\Sigma y^2=550$

$\bar{x} = \frac{\Sigma x}{n} = \frac{170}{5} = 34$, $\bar{y} = \frac{\Sigma y}{N} = \frac{175}{5} = 35$

Karl Pearn's coefficient of correlation (r) = $\frac{\Sigma xy}{\sqrt{\Sigma x^2}\,.\,\sqrt{\Sigma y^2}}$

$= \frac{280}{\sqrt{776}\,.\,\sqrt{550}}$

$= \frac{280}{\sqrt{776}\,.\,\sqrt{550}}$

$$= \frac{280}{27.85 \times 23.45}$$

$$= \frac{280}{653.0325}$$

$$= 0.42$$

b. Calculation of correlation coefficient

Data of price (Rs.) (x)	Demand (Rs.) (y)	u = x – 19	v = v₁ – 20	u^2	v^2	uv
14	24	–5	–4	25	16	20
16	22	–3	–2	9	4	6
19	20	0	0	0	0	0
22	24	3	4	9	16	12
24	23	5	3	25	9	15
30	28	11	8	121	64	88
		11	9	189	109	141

∴ Required correlation coefficient is given by

$$r = \frac{n\Sigma uv - \Sigma u \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\sqrt{n\Sigma v^2 - (\Sigma v)^2}}$$

$$= \frac{6 \times 141 - 11 \times 9}{\sqrt{6 \times 189 - (11)^2}\sqrt{6 \times 109 - (9)^2}}$$

$$= \frac{846 - 99}{\sqrt{1013}\sqrt{573}} = \frac{747}{31.83 \times 23.94} = \frac{747}{762.01} = 0.98$$

c. Calculation of co-varience and correlation coefficient

x	y	$x-\bar{x}$	$(x-\bar{x})^2$	$y-\bar{y}$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
20	7	5	25	–5	25	–25
10	15	–5	25	3	9	–15
20	12	5	25	0	0	0
10	16	–5	25	4	16	–20
17	17	2	4	5	25	10
12	10	–3	9	–2	4	6
15	11	0	0	–1	1	0
16	8	1	1	–4	16	–4
Σx = 120	Σy = 96		114		94	$\Sigma(x-\bar{x})(y-\bar{y})=-48$

Here, $\bar{x} = \frac{\Sigma x}{n}$ and $\bar{y} = \frac{\Sigma y}{n}$

⇒ $\bar{x} = \frac{120}{8}$ and $\bar{y} = \frac{96}{8}$

∴ $\bar{x} = 15$ and $\bar{y} = 12$

∴ Co–varience of (x, y) = $\frac{\Sigma(x-\bar{x})(y-\bar{y})}{n} = \frac{-48}{8} = -6$

And, the correlation coefficient is

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{(y-\bar{y})^2}} = \frac{-48}{\sqrt{114}\sqrt{94}} = \frac{-48}{10.67 \times 9.70} = \frac{-48}{103.50} = -0.463$$

4. Solution:

a. Here,

x	y	$x=(x-\bar{x})$	$y=(y-\bar{y})$	xy	x^2	y^2
6	9	–0	1	0	0	1

2	11	–4	3	–12	16	9
10	5	4	–3	–12	16	9
4	8	–2	0	0	4	0
8	7	2	–1	–2	4	1

$\Sigma xyh = -26, \Sigma x^2 = 40, \Sigma y^2 = 20$

Hence,

$\overline{y} = \frac{\Sigma y}{n}, \overline{x} = 6$

or, $8 = \frac{35 + a}{5}$

or, $40 - 35 + a$

$\therefore$ $a = 5$

Coefficient of correlation (r) $= \frac{\Sigma xy}{\sqrt{\Sigma x^2}.\sqrt{\Sigma y^2}} = \frac{-26}{\sqrt{40}.\sqrt{20}} = \frac{-26}{\sqrt{800}} = \frac{-26}{28.28} = -0.92$

b. Solution:

x	y	$x=(x-\overline{x})$	$y=(y-\overline{y})$	x^2	y^2	xy
10	9	–3	–5	9	25	15
12	12	–1	–2	1	4	2
20	15	7	1	49	1	7
$x_1(6)$	18	–7	4	49	16	–28
16	14	3	0	9	0	0
14	16	1	2	1	4	2
$\Sigma x = 72 + x$	$\Sigma y = 84$	0	0	118	50	–2

It is given that,

$\overline{x} = 13$ and $\overline{y} = \frac{\Sigma y}{n}$

$\Rightarrow \frac{\Sigma x}{n} = 13$ and $\overline{y} = \frac{84}{6}$

$\Rightarrow$ $\frac{72 + x_1}{6} = 13$ and $\overline{y} = 14$

$\therefore$ $x_1 = 6$

$\therefore$ The correlation coefficient is given by

$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2}\sqrt{\Sigma y^2}} = \frac{-2}{\sqrt{118}\sqrt{50}}$

$= \frac{-2}{10.86 \times 7.07} = \frac{-2}{76.78}$

$= -0.026$

5. Solution:

x	y	x^2	y^2	xy
41	22	1618	484	902
44	24	1936	576	1051
45	25	2025	625	1125
48	27	2304	729	1296
40	21	1600	441	840
42	22	1764	484	924
44	23	1936	529	1012
Σx=304	Σy=164	Σx^2 = 13266	Σy^2=3868	Σxy=7155

No. of items (n) = 7

Coefficient of collection (r) $= \frac{n\Sigma xy - \Sigma x.\Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}.\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$

$= \frac{7\times7155 - 304\times164}{\sqrt{7\times13246 - (304)^2}.\sqrt{7\times3868.(64)^2}}$

$= \frac{223}{\sqrt{306.\sqrt{180}}}$

$= 0.976$

b. Solution

Here,

x	y	xy	x^2	y^2
10	9	96	100	81
12	12	144	144	144
20	16	300	400	225
6	18	108	36	324
16	14	224	256	190
14	16	224	196	256
$\Sigma x=78$	$\Sigma y=8y$	$\Sigma xy=1090$	$\Sigma x^2=1132$	$\Sigma y^2=1226$

Here,

$\overline{x} = 13$

We have, $\overline{x} = \frac{\Sigma x}{n}$

or, $13 = \frac{72 + 9}{6}$

or, $78 = 72+9$

$\therefore$ $a = 6$

Now, Coefficient of correlation (r) $= \frac{n\Sigma xy - \Sigma x.\ \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\ .\ \sqrt{n\Sigma y^2 - (\Sigma y)^2}}$

$= \frac{6\times1090 - 78\times84}{\sqrt{6\times1132 - (78)^2}\sqrt{6\times1226 - (34)^2}}$

$= \frac{-12}{\sqrt{708}\sqrt{300}}$

$= \frac{-12}{26.60\times1732} = -0.01$

6. Solution:

Math (x)	Eng(y)	xy	x^2	y^2
45	35	1575	2025	1225
70	90	6300	4900	8100
65	70	4550	4225	4900
30	40	1200	900	1600
90	95	8550	8100	9025
40	40	1600	1600	1600
50	60	3000	2500	3600
75	80	6000	5625	6400
85	80	6800	7225	6400
60	50	3000	3600	2500
$\Sigma x=610$	$\Sigma y=640$	$\Sigma xy=42575$	$\Sigma x^2=40700$	$\Sigma y^2=45350$

Coefficient of correlation (r) $= \frac{n\Sigma xy - \Sigma x.\ \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\ .\ \sqrt{n\Sigma y^2 - (\Sigma y)^2}}$

$= \frac{10\times42575 - 610\times640}{\sqrt{10\times40700 - (610)^2}\ .\ \sqrt{10\times45360 - ...}}$

$$= \frac{425750 - 390400}{\sqrt{34900}\,.\,\sqrt{43900}}$$
$$= \frac{35350}{186.81 \times 209.52}$$
$$= \frac{35350}{39140.4312}$$
$$= 0.9033$$

7. Solution

Here, No. of observations (n) = 9

$\Sigma xy = 731$

$\Sigma x^2 = 285$

$\Sigma xy^2 = 2085$

$\overline{x} = 5$

$\overline{y} = 15$

Coefficient of co-relation (r) $= \frac{\Sigma xy - n\overline{x}\,\overline{y}}{\sqrt{\Sigma x^2 - n\overline{x}^2}\,.\,\sqrt{\Sigma y^2 - n\overline{y}^2}}$

$$= \frac{731 - 9 \times 5 \times 15}{\sqrt{285 - 9 \times 25}\,.\,\sqrt{2085 - 9 \times 225}}$$
$$= \frac{56}{7.74 \times 7.74}$$
$$= \frac{56}{7.74 \times 7.74}$$
$$= 0.934$$

Yes, it is positive.

8. Solution:

Here,

Collected $\Sigma x = 120 - 8 - 12 + 8 + 10 = 118$

Collected $\Sigma y = 90 - 10 - 7 + 12 + 8 = 93$

Collected $\Sigma x^2 = 600 - 8^2 - 12^2 + 8^2 + 10^2 = 556$

Collected $\Sigma y^2 = 250 - 10^2 - 7^2 + 12^2 + 8^2 = 309$

Collected $\Sigma xy = 356 - 8 \times 10 - 12 \times 7 + 8 \times 12 + 10 \times 8 = 368$

Now,

Corrected value of f = 8

$$r_e = \frac{n\Sigma xy - \Sigma x.\,\Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\,.\,\sqrt{n\Sigma y^2 - (\Sigma y^2)}}$$
$$= \frac{30 \times 368 - 118 \times 93}{\sqrt{30 \times 556 - (118)^2}\,\sqrt{30 \times 309 - (93)^2}}$$
$$= \frac{11040 - 10974}{\sqrt{16680 - 13924}.\,\sqrt{9270 - 8649}}$$
$$= \frac{66}{52.49 \times 24.91}$$
$$= \frac{66}{1307.5259}$$
$$= 0.05$$

9. Here,

x	Rank (R_x)	y	Rank (R_y)	$d = R_x - R_y$	d^2
39	8	47	10	–2	4
65	6	53	8	–2	4

62	7	58	7	0	0
90	2	86	2	0	0
82	3	62	5	–2	4
75	5	68	4	1	1
25	10	60	6	4	16
98	1	91	1	0	0
36	9	51	9	0	0
78	4	84	3	1	1
					$\Sigma d^2 = 30$

$$\text{Rank } (\rho) = 1 - \frac{6 - \sum d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6 \times 30}{10 \times 99}$$

$$= 0.818$$

10. Here,

Officer (x)	Officer (y)	xy	x^2	y^2
1	1	1	1	1
7	6	42	49	36
4	5	20	16	25
2	2	4	4	4
3	3	9	9	9
6	4	24	36	16
5	7	35	25	49
9	11	99	81	121
10	8	80	100	64
8	10	80	64	100
11	9	99	121	81
$\Sigma x = 66$	$\Sigma y = 66$	$\Sigma xy = 493$	$\Sigma x^2 = 506$	$\Sigma y^2 = 506$

$$\text{Correlation coefficient (r)} = \frac{n\Sigma xy - \Sigma x.\ \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\ .\ \sqrt{n\Sigma y^2 - (\Sigma y^2)}}$$

$$= \frac{11 \times 493 - 66 \times 66}{\sqrt{11 \times 506 - (66)^2}\ \sqrt{11 \times 506 - (66)^2}}$$

$$= \frac{5423 - 4356}{\sqrt{1210}\ .\ \sqrt{1210}}$$

$$= \frac{1067}{1210}$$

$$= 0.882$$

11. Calculation for spearman's rank correlation

Teaching method	Rank of st A (R_A)	Rank of std. B (R_B)	$d = R_A - R_B$	d^2
I	2	1	1	1
II	1	3	–2	4
III	3	2	1	1
IV	5	4	1	1
V	4	7	–3	9
VI	6	5	1	1
VII	7	6	1	1
				$\Sigma d^2 = 18$

$$\text{Rank } (\rho) = 1 - \frac{6\sum d^2}{n(n^2-1)}$$
$$= 1 - \frac{6 \times 18}{7 \times 48} = 1 - \frac{108}{336} = \frac{228}{336} = 0.68$$

Exercise 13.2

1. **Solution:**

a. Here, $\sigma_x = 20$, $\sigma_y = 15$, $r = 0.48$

The regression coefficients are $b_{yx} = r\,\frac{\sigma_y}{\sigma_x} = 0.48 \times \frac{15}{20} = 0.36$

and $b_{xy} = r\,\frac{\sigma_x}{\sigma_y} = 0.48 \times \frac{20}{15} = 0.64$

b. $\sigma_x = 8$, $\sigma_y = 10$, $r = -0.6$

The regression coefficients are

$$b_{yx} = r\,\frac{\sigma_y}{\sigma_x} = -0.6 \times \frac{10}{8} = -0.75$$

and $b_{xy} = r\,\frac{\sigma_x}{\sigma_y} = -0.6 \times \frac{8}{10} = -0.48$

2. **Solution:**

a. Here, $b_{yx} = 0.35$, $b_{xy} = 1.8$

Now, correlation coefficient $(r) = \sqrt{b_{yx} \, . \, b_{xy}} = \sqrt{0.35 \times 1.8} = \sqrt{0.63} = 0.7937$

b. We have, $\Sigma x = 60$, $\Sigma y = 40$, $\Sigma xy = 1150$

$\Sigma x^2 = 4160$, $\Sigma y^2 = 1720$, $N = 10$

Here, $b_{yx} = \frac{N\Sigma xy - \Sigma x\Sigma y}{N\Sigma x^2 - (\Sigma x)^2} = \frac{10 \times 1150 - 60 \times 40}{10 \times 4160 - (60)^2}$

$$= \frac{11500 - 2400}{4160 - 3600} = \frac{9100}{38000} = 0.2394$$

$b_{xy} = \frac{N\Sigma xy - \Sigma x\Sigma y}{N\Sigma y^2 - (\Sigma y)^2} = \frac{10 \times 1150 - 60 \times 40}{10 \times 1720 - (40)^2} = \frac{9100}{15600} = 0.5833$

And, $\overline{x} = \frac{\Sigma x}{N} = \frac{60}{10} = 6$, $\overline{y} = \frac{\Sigma y}{N} = \frac{40}{10} = 4$

Now, Regression equation of y on x is,

$y - \overline{y} = b_{yx}(x - \overline{x})$

$\Rightarrow$ $y - 4 = 0.2394\,(x - 6)$

$\Rightarrow$ $y - 4 = 0.2394x - 1.4364$

$\Rightarrow$ $yh = 2.5636 + 0.2394x$

Regression equation of x on y is,

$x - \overline{x} = b_{xy}(y - \overline{y})$

$\Rightarrow$ $x - 6 = 0.5833\,(Y - 4)$

$\Rightarrow$ $x - 6 = 0.5833y - 2.3332$

$\Rightarrow$ $x = 3.6668 + 0.5833y$

And, the correlation coefficient is

$r = \sqrt{b_{yx} \times b_{xy}} = \sqrt{0.2394 \times 0.5833} = \sqrt{0.1396} = 0.3736$

c. We have,

$b_{yx} = 2.002$, $b_{xy} = -0.461$. $\overline{x} = 87.2$,

$\overline{y} = 127.2$

$\therefore$ Correlation coefficient is,

$$r = \sqrt{-2.002 \times -0.461} = 0.9606 > 0$$

Since $b_{yx} < 0$, $b_{xy} < 0$ and $r > 0$

So, it is not possible.

3. Solution:

a. Here, s$\sum xy = 750$, $\sum x^2 = 2085$, $\sum y^2 = 285$, $\sum x = 135$, $\sum y = 45$, N = 9

Now, The regression coefficient of x only is

$$b_{xy} = \frac{n\sum xy - \sum x.\sum y}{n\sum y^2 - (\sum y)^2}$$

$$= \frac{9 \times 750 - 135 \times 45}{9 \times 285 - (45)^2} = \frac{6750 - 6075}{2565 - 2025} = \frac{675}{540} = 1.25$$

b. Here, $\sum x = 60$, $\sum y = 40$, $\sum xy = 1150$, $\sum x^2 = 4160$, $\sum y^2 = 1720$, n = 10

Now, the point through which the regression lines intersect to each other is

$$(\overline{x}, \overline{y}) = \left(\frac{\sum x}{n}, \frac{\sum y}{n}\right) = \left(\frac{60}{10}, \frac{40}{10}\right) = (6, 4)$$

Since the equation of regression line of y on x is

$y - \overline{y} = b_{yx}(x - \overline{x})$ ... ... (i) and x on y is

$x - \overline{x} = b_{xy}(y - \overline{y})$ ... ... (ii) where

$$b_{yx} = \frac{n\sum xy - \sum x.\sum y}{n\sum x^2 - (\sum x^2)} = \frac{10 \times 1150 - 60 \times 40}{10 \times 4160 - 60^2}$$

$$= \frac{11500 - 2400}{41600 - 3600}$$

$$= \frac{9100}{38000} = 0.239$$

and $b_{xy} = \frac{n\sum xy - \sum x.\sum y}{n\sum y^2 - (\sum y)^2} = \frac{10 \times 1150 - 60 \times 40}{10 \times 1720 - 40^2}$

$$= \frac{9100}{15600} = 0.583$$

Hence, from (i) and (ii), the required equations are

$y - 4 = 0.239(x - 6)$

or, $y = 2.566 + 0.239x$

and $x - 6 = 0.583(y - 4)$

or, $x = 3.668 + 0.583y$

Also, correlation coefficient (r) $= \sqrt{b_{yx}.b_{xy}}$

$= \sqrt{0.239 \times 0.583}$

$= \sqrt{0.139337} = 0.373$

c. Here, $b_{yx} = -2.002$ and $b_{xy} = -0.461$

Now, $r = \sqrt{b_{yx}.b_{xy}} = \sqrt{-2.002 \times -0.461} = \sqrt{0.922922} = -0.961$

Also, **Error! Bookmark not defined.** $\overline{x} = 87.2$, $\overline{y} = 127.2$, y = 133, x = ?

Using, $x - \overline{x} = b_{xy}(y - \overline{y})$,

or, $x - 87.2 = -0.461(y - 127.2)$

or, $x - 87.2 = -461(133 - 127.2)$

or, $x - 87.2 = -2.6738$

or, $x = 84.5262$

d. Let average price in Birgunj $(\overline{x})$ = Rs. 65

Average price in Kathmandu $(\overline{y})$ = Rs. 67

$\sigma_x = 2.5$, $\sigma_y = 3.5$, r = 0.08

Now, $b_{yx} = r.\frac{\sigma_y}{\sigma_x} = 0.08 \times \frac{3.5}{2.5} = 0.112$

The equation of regression line of y on x is

$y - \overline{y} = b_{yx}(x - \overline{x})$

or, $y - 67 = 0.112(x - 65)$

or, $y - 67 = 0.112 \times -7.28$

or, $y = 59.72 + 0.112x$

If x = Rs. 70, then y = 59.72 + 0.112 × 70 = Rs. 67.56

4. Solution:

a. Here, the regression equations ae $3x + 2y - 26 = 0$; $6x + y - 31 = 0$

or, $2y = -3x + 26$ and $6x = -y + 31$

or, $y = \frac{-3}{2}x + 13$, $x = -\frac{1}{6}y + \frac{31}{6}$

Implies the regression coefficient as $b_{yx} = -\frac{3}{2}$ and $b_{xy} = -\frac{1}{6}$

Now, correlation coefficient (r) $= \sqrt{byx \cdot bxy}$

$$= \sqrt{\frac{-3}{2} \times -\frac{1}{6}} = \sqrt{\frac{1}{4}} = -0.5$$

After solving the given equations, we get the intersection point (x, y) = (4, 7)

i.e, means of $x = \overline{x} = 4$ and means of $y = \overline{y} = 7$

b. We have, the given two regression equations are

$3x + 4y = 65$ and $3x + y = 31$

Since $(\overline{x}, \overline{y})$ lies on the given regression lines.

$3\overline{x} + 4\overline{y} = 65$ ... ... (i)

$3\overline{x} + \overline{y} = 32$ ... ... (ii)

Subtracting (ii) from (i), we get

$3\overline{y} = 33$

$\therefore \quad \overline{y} = 11$

from (i), $3\overline{x} + 4\times11 = 65 \Rightarrow 3\overline{x} = 65 - 44 = 21$

$\therefore \quad \overline{x} = 7\ 33$

$\therefore \quad \overline{y} = 11$

For the regression coefficients, we have

$3x + 4y = 65$

$\Rightarrow \quad 4y = 65 - 3x$

$\therefore \quad y = \frac{65}{4} - \frac{3}{4}x$ which is in the form of $y = a + bx$; where

$b = -\frac{3}{4} \therefore b = b_{yx} = -\frac{3}{4}$

Again, we have,

$3x + y = 32$

$\Rightarrow 3x = 32 - y$

$\therefore \quad x = \frac{32}{3} - \frac{1}{3}y$ which is in the form of $x = a + by$; where $b = -\frac{1}{3}$

$\therefore \quad b = b_{xy} = -\frac{1}{3}$

$\therefore \quad$ Correlation coefficient (r) $= \sqrt{b_{xy} + b_{xy}} = \pm\sqrt{-\frac{3}{4} \times -\frac{1}{3}}$

$\therefore \quad r = -\frac{1}{2}$

5. Solution,

Here, $b_{xy} = 1.5$, $b_{yx} = 0.65$

$\overline{x} = 36$ and $\overline{y} = 52$

Regression equation of y on x is

$y - \overline{y} = b_{yx}(x - \overline{x})$

$\Rightarrow \quad y - 52 = 0.65 (x - 36)$

$\Rightarrow \quad y = 0.65x - 23.4 + 52$

$\therefore \quad y = 0.65x + 28.6$ ... ... (i)

And, the regression equation of x on y is

$x - \overline{x} = b_{xy}(y - \overline{y})$

$\Rightarrow \quad x - 36 = 1.5 (y - 52)$

$\Rightarrow \quad x = 1.5y - 78 + 36$

$\therefore \quad x = 1.5y - 42$ ... ... (ii)

When x = 60 then from (i),

$y = 0.65 \times 60 + 28.6 = 67.6$

6. Solution:

a. Given, $\overline{x} = 20$, $\overline{y} = 120$, $cov_x = 25$, $cov_y = 28.83$, $r = 0.8$, $x = ?$ if $y = 150$

$cov_x = 25$, then $25 = \frac{\sigma_x}{\overline{x}} \times 100$

or, $\frac{25 \times 20}{100} = \sigma_x$

or, $\sigma_x = 5$

$Cov_y = 28.83 \Rightarrow 28.83 = \frac{\sigma_y}{\overline{y}} \times 100$

or, $\frac{28.83 \times 120}{100} = \sigma_y$

or, $\sigma_y = 34.596$

$\therefore \quad b_{xy} = r\frac{\sigma_x}{\sigma_y} = 0.8 \times \frac{5}{34.596} = 0.1156$

Now, the equation of regression line of x on y is

$x - \overline{x} = b_{xy}(y - \overline{y})$

or, $x - 20 = 0.1156 (y - 120)$

or, $x = 0.1156y - 13.872 + 20$

$\therefore \quad x = 0.1156y + 6.128$

when $y = 150$, $x = 0.1156 \times 150 + 6.128 = 23.5$

b. Given,

$n = 50$

$\frac{\sigma_y}{\sigma_x} = \frac{5}{2}$

$b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{5r}{2}$ ... ... (i)

and $b_{xy} = \frac{2r}{5}$

Also, $4y - 5x - 8 = 0$

$\Rightarrow \quad y = \frac{5x}{4} + 2$

i.e. $b_{yx} = \frac{5}{4}$

from (i), $\frac{5r}{2} = \frac{5}{4}$

or, $r = \frac{1}{2}$

Now, $\overline{x} = 40$. Let the average marks for mathematics by $\overline{y}$.

So, $4\overline{y} - 5\overline{x} = 8$

$\Rightarrow \quad 4\overline{y} = 8 + 200$

$\Rightarrow \quad \overline{y} = \frac{208}{4} = 52$

7. Here, average of rainfall $(\overline{x})$ = 26.7cm

   Average of yield $(\overline{y})$ = 508.4kg
   $\sigma_x = 4.6$, $\sigma_y = 36.8$, r = 0.52

   For x on y:
   $b_{xy} = r\frac{\sigma_x}{\sigma_y} = 0.52 \times \frac{4.6}{36.8} = 0.065$

   So, the equation is $x - \overline{x} = b_{xy}(y - \overline{y})$
   or, $x - 26.7 = 0.065(y - 508.4)$
   or, $x - 26.7 = 0.065y - 33.046$
   or, $x = 0.065y - 6.346$
   When yield (y) = 600kg, $x = 0.065 \times 600 - 6.346$
   $= 32.654$cm
   For y on x
   $b_{yx} = r\frac{\sigma_y}{\sigma_x} = 0.52 \times \frac{36.8}{4.6} = 4.16$

   So, the equation is $y - \overline{y} = b_{yx}(x - \overline{x})$
   or, $y - 5084 = 4.16(x - 26.7)$
   or, $y - 5084 = 4.16x - 111.072$
   or, $y = 4.16x + 397.328$
   When rainfall (x) = 29cm
   $y = 4.16 \times 29 + 397.328 = 517.968$kg

8. Given, Mean no. of workers on strike $(\overline{x})$ = 800

   Mean loss of daily production $(\overline{y})$ = 35000
   Standard deviation of no. of workers on strike (σ_x) = 100
   Standard deviation of daily production (σ_y) = 2,000
   Coefficient of correlation between x and y (r) = 0.8
   When x = 1800, y = ?
   Now, $b_{yx} = r\frac{\sigma_y}{\sigma_x} = 0.8 \times \frac{2000}{100} = 16$

   The equation of regression of y on x is $y - \overline{y} = b_{yx}(x - \overline{x})$
   or, $y - 35000 = 16(x - 800)$
   or, $y - 35000 = 16x - 12800$
   or, $y = 16x - 22200$
   When x = 1800, $y = 16 \times 1800 - 22200 = 6600$

9. **Solution:**

x	y	xy	x^2
1	50	50	1
5	60	300	25
6	80	480	36
8	100	800	64
10	110	1100	100
$\sum x = 30$	$\sum y = 400$	$\sum xy = 2730$	$\sum x = 226$

Here, $\overline{x} = \frac{\sum x}{n} = \frac{30}{5} = 6$

$\overline{y} = \frac{\sum y}{n} = \frac{400}{5} = 80$

$b_{yx} = \dfrac{n\Sigma xy - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2} = \dfrac{5 \times 273 - 30 \times 400}{5 \times 226 - (30)^2} = \dfrac{13650 - 12000}{1130 - 900} = \dfrac{1650}{230} = 7.174$

The regression equation of y on x is

$y - \overline{y} = b_{yx}(x - \overline{x})$

or, $y - 80 = 7.174(x - 6)$

or, $y - 80 = 7.174x - 43.04$

or, $y = 7.174x + 36.96$

When $x = 15000$,

$y = 7.174 \times 15000 + 36.96$

$= \text{Rs. } 107645.66$

10. Calculation regression equation of x on y

x	y	x^2	y^2	xy
2	18	4	324	36
4	12	16	144	48
5	10	25	100	50
6	8	36	64	48
8	7	64	49	56
11	5	121	25	55
36	**60**	**266**	**706**	**293**

Here, $\overline{x} = \dfrac{\Sigma x}{n}$ and $\overline{y} = \dfrac{\Sigma y}{n}$

$\Rightarrow \overline{x} = \dfrac{36}{6}$ and $\overline{y} = \dfrac{60}{6}$

$\therefore \overline{x} = 6$ $\quad \therefore \overline{y} = 10$

Again, $b_{xy} = \dfrac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma y^2 - (\Sigma y)^2} = \dfrac{6 \times 293 - 36 \times 60}{6 \times 706 - (60)^2}$

$= \dfrac{1758 - 2160}{4236 - 3600} = \dfrac{-402}{636} = -0.6320$

$\therefore$ Regression equation of x on y is,

$x - \overline{x} = b_{xy}(y - \overline{y})$

$\Rightarrow x - 6 = -0.6320(y - 10)$

$\Rightarrow x - 6 = -0.6320y + 6.32$

$\therefore x = 12.32 - 0.6320y$

When $y = 12$ then $x = 12.32 - 0.6320 \times 12$

$\therefore x = 4.736$

11. Here, $n = 25$, $\Sigma x = 125$, $\Sigma y = 100$, $\Sigma x^2 = 650$, $\Sigma y^2 = 460$, $\Sigma xy = 508$

But (8, 12) and (6, 8) were copied wrong as (6, 14) and (8, 6) respectively.

So, correct values are

$n = 25$, $\Sigma x = 125 + 8 - 6 + 6 - 8 = 125$

$\Sigma y = 100 + 12 - 14 + 8 - 6 = 100$

$\Sigma x^2 = 650 + 8^2 - 6^2 + 8^2 = 650$

$\Sigma y^2 = 460 + 12^2 - 14^2 + 8^2 - 6^2 = 436$

$\Sigma xy = 508 + 8 \times 12 + 6 \times 8 - 6 \times 14 - 8 \times 6 = 520$

Now, $b_{xy} = \dfrac{n\Sigma xy - \Sigma x.\Sigma y}{n\Sigma y^2 - (\Sigma y)^2} = \dfrac{25 \times 520 - 125 \times 100}{25 \times 436 - (100)^2} = \dfrac{13000 - 12500}{10900 - 100000} = \dfrac{5}{9}$

and $b_{yx} = \dfrac{n\Sigma xy - \Sigma x.\Sigma y}{n\Sigma y^2 - (\Sigma y)^2} = \dfrac{25 \times 520 - 12500}{25 \times 650 - (125)^2} = \dfrac{500}{625} = \dfrac{4}{5}$

Now, coefficient of correlation (r) $= \sqrt{b_{yx} \times b_{xy}} = \sqrt{\dfrac{5}{9} \times \dfrac{4}{5}} = \dfrac{2}{3}$

Now, $\overline{x} = \frac{\sum x}{n} = \frac{125}{25} = 5$

$\overline{y} = \frac{\sum y}{n} = \frac{100}{25} = 4$

The equation of regression line of x on y is

$x - \overline{x} = b_{xy} (y - \overline{y})$

or, $x - 5 = \frac{5}{9} (y - 4)$

or, $9x - 45 = 5y - 20$

or, $9x - 5y = 25$

and the equation of regression line of y on x is

$y - \overline{y} = b_{yx} (x - \overline{x})$

or, $y - 4 = \frac{4}{5} (x - 5)$

or, $5y - 20 = 4x - 20$

or, $4x - 5y = 0$