Chapter-7

Matrix Based system of linear Equations.

1. Row equivalent matrix method
2. Inverse matrix method.
3. Cramer's Rule.

* System of linear Equation

Two or more equations constitute a system of equations. For eg: $a_{11}x + a_{12}y = c_1$

$$a_{21}x + a_{22}y = c_2$$

is a system of equation of two variables.

And.

$$a_{11}x + a_{12}y + a_{13}z = d_1$$
$$a_{21}x + a_{22}y + a_{23}z = d_2$$
$$a_{31}x + a_{32}y + a_{33}z = d_3$$

is a system of equation of three variables.

- Consistent and Inconsistent system

A system of linear equation is said to be consistent if it has atleast one solution otherwise it is inconsistent.

- Dependent and Independent system

A system of equation is said to be independent if it has almost one solution,

[At most one solution $\longrightarrow$ no solution, unique solution]

otherwise it is dependent.

[Dependent $\longrightarrow$ Infinitely many solution]

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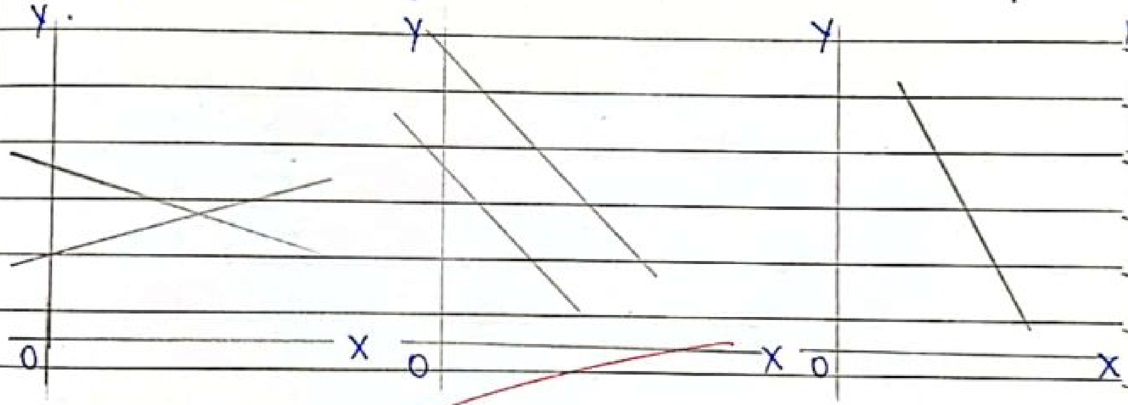
System of Equation

(solution exists) — Consistent
- Independent → unique $soln$
- Dependent → indefinitely manly $soln$

Solution doesn't exist — Inconsistent

- Having unique solution → consistent and independent
- Having no solution → Inconsistent and independent
- Having infinitely many solution → consistent and dependent



unique solution (consistent and independent)

no solution [inconsistent and independent]

infinitely many solution [consistent and dependent]

<u>Note:</u>

Consider a system of equations

$$a_{11}x + a_{12}y = c_1 \quad \text{—— (i)}$$
$$a_{21}x + a_{22}y = c_2 \quad \text{—— (ii)}$$

(i) If $\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = \frac{c_1}{c_2}$, then lines are parallel and coincidence. In this case, system has infinitely many $soln$.

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(ii) If $\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} \neq \frac{c_1}{c_2}$, then the line are parallel and distinct. In this case, system has no solution.

(iii) If $\frac{a_{11}}{a_{21}} \neq \frac{a_{12}}{a_{22}}$, then the lines are intersected at a point. In this case, system has unique solution.

Exercise 7.1

1. By drawing graph or otherwise, classify each of the following system of equations.

a. $4x-3y=-6$
$-4x+2y=16$

Soln;

Given system is, $4x-3y=-6$
$-4x+2y=16$

Here,

$a_{11}=4$ $\quad a_{12}=-3$ $\quad c_1=-6$

$a_{21}=-4$ $\quad a_{22}=2$ $\quad c_2=16$

Now,

a. $\Rightarrow \dfrac{a_{11}}{a_{21}} = \dfrac{4}{-4} = -1$

b. $\Rightarrow \dfrac{a_{12}}{a_{22}} = \dfrac{-3}{2}$

Here,

$$\frac{a_{11}}{a_{21}} \neq \frac{a_{12}}{a_{22}}$$

So, the given system has unique solution. Hence, the system is consistent and independent.

b. $2x-y=3$
$-4x+2y=6$

Soln,

Given system is, $2x-y=3$
$-4x+2y=6$

Here, $a_{11}=2$ $\quad a_{12}=-1$ $\quad c_1=3$

$a_{21}=-4$ $\quad a_{22}=2$ $\quad c_2=6$

Now,

$$\frac{a_{11}}{a_{21}} = \frac{2}{-4} = \frac{-1}{2}$$

and $\frac{a_{12}}{a_{22}} = \frac{-1}{2}$ $\qquad$ $\frac{c_1}{c_2} = \frac{3}{6} = \frac{1}{2}$

Here, $\frac{a_{11}}{a_{21}} = \frac{a_{22}}{a_{22}} \neq \frac{c_1}{c_2}$

So, the given system has no solution. Hence, the system is inconsistent and independent.

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c. $-6x + 4y = 10$
$3x - 2y = -5$

Soln;

Given system is,

$$-6x + 4y = 10$$
$$3x - 2y = -5$$

Here, $a_{11} = -6$ $\quad a_{12} = 4$ $\quad c_1 = 10$
$a_{21} = 3$ $\quad a_{22} = -2$ $\quad c_2 = -5$

Now,

$$\frac{a_{11}}{a_{21}} = \frac{-6}{3} = -2 \qquad \frac{a_{12}}{a_{22}} = \frac{4}{-2} = -2$$

and $\frac{c_1}{c_2} = \frac{10}{-5} = -2$

Here,

$$\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = \frac{c_1}{c_2}$$

So, the given system has infinitely many solutions. Hence, system is consistent and dependent.

d. $7x+2y=15$

$x+y=5$

soln,

Given solution is, $7x+2y=15$

$x+y=5$

Here, $a_{11}=7$ $a_{12}=2$ $c_1=15$

$a_{21}=1$ $a_{22}=1$ $c_2=5$

Now,

$\frac{a_{11}}{a_{21}}=\frac{7}{1}=7$ $\frac{a_{12}}{a_{22}}=\frac{2}{1}=2$

Here,

$\frac{a_{11}}{a_{21}} \neq \frac{a_{12}}{a_{22}}$

So, the given system has unique solution. Hence, system is consistent and independent.

- Row-equivalent method.

Consider a system of equation,

$$a_{11}x + a_{12}y + a_{13}z = c_1$$
$$a_{21}x + a_{22}y + a_{23}z = c_2$$
$$a_{31}x + a_{32}y + a_{33}z = c_3$$

The corresponding argumented matrix is

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & c_1 \\ a_{21} & a_{22} & a_{23} & c_2 \\ a_{31} & a_{32} & a_{33} & c_3 \end{array}\right]$$

We transform above matrix in the following form by using elementary row operations.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & d_1 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & d_3 \end{array}\right]$$

Then,

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$d_1 = x$
$d_2 = y$
$d_3 = z$

- Elementary row operation

(i) Interchanging only two rows
i.e $R_i \longleftrightarrow R_j$ Eg: $R_1 \longleftrightarrow R_2$, $R_2 \longleftrightarrow R_3$

(ii) Multiplying all elements of any row by non zero constant
i.e $R_i \to mR_i$ Eg: $R_1 \to 2R_1$, $R_2 \to 3R_2$

(iii) $R_i \to R_i \pm mR_j$ Eg: $R_1 \to R_1 - R_2$
$R_1 \to R_1 - 2R$
$R_2 \to R_2 - 2R_3$

2. Solve the following systems by using row equivalent matrix method.

a. $x + y = 5$
$2x + 3y = 12$
Soln;
Given system of equation is,
$$x + y = 5$$
$$2x + 3y = 12$$
The corresponding augmented matrix is,
$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 2 & 3 & 12 \end{array}\right]$$
$R_2 \to R_2 - 2R_1$
$$\sim \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 2 \end{array}\right]$$

$R_1 \to R_1 - R_2$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right]$$

$\therefore$ The required solution is, $x = 3$

$y = 2$

b. $2x + 12y = 16$

$3x + 10y = 8$

Soln;

Given system of equation is,

$$2x + 12y = 16$$
$$3x + 10y = 8$$

The corresponding augmented matrix is,

$$\left[\begin{array}{cc|c} 2 & 12 & 16 \\ 3 & 10 & 8 \end{array}\right]$$

$R_1 \to \frac{1}{2} R_1$

$$\sim \left[\begin{array}{cc|c} 1 & 6 & 8 \\ 3 & 10 & 8 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$

$$\sim \left[\begin{array}{cc|c} 1 & 6 & 8 \\ 0 & -8 & -16 \end{array}\right]$$

$R_2 \to \frac{1}{-8} R_2$

$$\sim \left[\begin{array}{cc|c} 1 & 6 & 8 \\ 0 & 1 & 2 \end{array}\right]$$

$R_1 \to R_1 - 6R_2$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & 2 \end{array}\right]$$

$\therefore$

The required solution is,

$x = -4$

$y = 2$

c. $x-3y=-1$
$4x-y=7$
Soln;
Given system of equation is,
$$x-3y=-1$$
$$4x-y=7$$
The corresponding augmented matrix is,

$$\left[\begin{array}{cc|c} 1 & -3 & -1 \\ 4 & -1 & 7 \end{array}\right]$$

$R_2 \to R_2 - 4R_1$

$$\sim \left[\begin{array}{cc|c} 1 & -3 & -1 \\ 0 & 11 & 11 \end{array}\right]$$

$R_2 \to \frac{1}{11} R_2$

$$\sim \left[\begin{array}{cc|c} 1 & -3 & -1 \\ 0 & 1 & 1 \end{array}\right]$$

$R_1 \to R_1 + 3R_2$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

∴ The required ~~eqn~~ soln is,
$$x=2$$
$$y=1$$

d. $8x-3y=-31$
$2x+6y=26$
Soln;
Given system of eqn is,
$$8x-3y=-31$$
$$2x+6y=26$$
The corresponding augmented matrix is,

$$\left[\begin{array}{cc|c} 8 & -3 & -31 \\ 2 & 6 & 26 \end{array}\right]$$

$R_1 \to \frac{1}{8} R_1$

$$\sim \left[\begin{array}{cc|c} 1 & -3/8 & -31/8 \\ 2 & 6 & 26 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$$\sim \left[\begin{array}{cc|c} 1 & -3/8 & -31/8 \\ 0 & 27/4 & 135/4 \end{array}\right]$$

$R_2 \to \frac{4}{27} R_2$

$$\sim \left[\begin{array}{cc|c} 1 & -3/8 & -31/8 \\ 0 & 1 & 5 \end{array}\right]$$

$R_1 \to R_1 + \frac{3}{8} R_2$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 5 \end{array}\right]$$

∴ The required solution is,

$x = -2$

$y = 5$

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e. $5x - 3y = -2$

$4x + 2y = 5$

soln;

Given system of equation is,

$5x - 3y = -2$

$4x + 2y = 5$

The corresponding augment matrix is,

$$\left[\begin{array}{cc|c} 5 & -3 & -2 \\ 4 & 2 & 5 \end{array}\right]$$

$R_1 \to \frac{1}{5} R_1$

$$\sim \left[\begin{array}{cc|c} 1 & -3/5 & -2/5 \\ 4 & 2 & 5 \end{array}\right]$$

$R_2 \to R_2 - 4R_1$

$$\sim \left[\begin{array}{cc|c} 1 & -3/5 & -2/5 \\ 0 & 22/5 & 33/5 \end{array}\right]$$

$R_2 \to \frac{5}{22} R_2$

$$\sim \left[\begin{array}{cc|c} 1 & -3/5 & -2/5 \\ 0 & 1 & 3/2 \end{array}\right]$$

$R_1 \to R_1 + \frac{3}{5} R_2$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & 1/2 \\ 0 & 1 & 3/2 \end{array}\right]$$

∴ The required solution is,

$x = 1/2$

$y = 3/2$

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f. $2/x + 3/y = 2$

$4/x - 5/y = 7$

soln;

Given system of eqn is,

$2/x + 3/y = 2$

$4/x - 5/y = 7$

The corresponding augmented matrix is,

$$\left[\begin{array}{cc|c} 2 & 3 & 2 \\ 4 & -5 & 7 \end{array}\right]$$

$R_1 \to \frac{1}{2} R_1$

$$\sim \left[\begin{array}{cc|c} 1 & 3/2 & 1 \\ 4 & -5 & 7 \end{array}\right]$$

$R_2 \to R_2 - 4R_1$

$$\sim \left[\begin{array}{cc|c} 1 & 3/2 & 1 \\ 0 & -11 & 3 \end{array}\right]$$

$R_2 \to \frac{-1}{11} R_2$

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$$\sim \left[\begin{array}{cc|c} 1 & 3/2 & 1 \\ 0 & 1 & -3/11 \end{array}\right]$$

$R_1 \longrightarrow R_1 - \frac{3}{2}R_2$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & 31/22 \\ 0 & 1 & -3/11 \end{array}\right]$$

∴ The required solution is,

$$\frac{1}{x} = \frac{31}{22} \quad \text{and} \quad \frac{1}{y} = \frac{-3}{11}$$

$$\therefore x = \frac{22}{31} \quad \text{and} \quad \therefore y = -\frac{11}{3}$$

3. use row-equivalent matrix method to solve the system of equations:

a. $x+y+z=1$
$x+2y+3z=-1$
$2x-y+2z=-4$

Soln;

Given system of eqn is,

$$x+y+z=1$$
$$x+2y+3z=-1$$
$$2x-y+2z=-4$$

The corresponding augmented matrix is,

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & -1 \\ 2 & -1 & 2 & -4 \end{array}\right]$$

$R_2 \longrightarrow R_2 - R_1$, $R_3 \longrightarrow R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -2 \\ 0 & -3 & 0 & -6 \end{array}\right]$$

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$R_1 \to R_1 - R_2$, $R_3 \to R_3 + 3R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 6 & -12 \end{array}\right]$$

$R_3 \to \frac{1}{6} R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$R_1 \to R_1 + R_3$, $R_2 \to R_2 - 2R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$\therefore$ The required sol^n is, $x = 1$
$y = 2$
$z = -2$

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6. $x + 4y + z = 18$
$3x + 3y - 2z = 2$
$0.x - 4y + z = -7$

Sol^n;

Given system of equation is,

$$x + 4y + z = 18$$
$$3x + 3y - 2z = 2$$
$$0 \cdot x - 4y + z = -7$$

The corresponding augmented matrix is,

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 18 \\ 3 & 3 & -2 & 2 \\ 0 & -9 & 1 & -7 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 18 \\ 0 & -9 & -5 & -52 \\ 0 & -4 & 1 & -7 \end{array}\right]$$

$R_2 \to -\frac{1}{9} R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 4 & 1 & 18 \\ 0 & 1 & 5/9 & 52/9 \\ 0 & -4 & 1 & -7 \end{array}\right]$$

$R_1 \to R_1 - 4R_2$, $R_3 \to R_3 + 4R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -11/9 & -46/9 \\ 0 & 1 & 5/9 & 52/9 \\ 0 & 0 & 29/9 & 145/9 \end{array}\right]$$

$R_3 \to \frac{9}{29} R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -11/9 & -46/9 \\ 0 & 1 & 5/9 & 52/9 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$R_1 \to R_1 + \frac{11}{9} R_3$, $R_2 \to R_2 - \frac{5}{9} R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$\therefore$ The required eqn is,

$x = 1$

$y = 3$

$z = 5$

c. $9y - 5x = 3$

$x + z = 1$

$z + 2y = 2$

Soln;

Given system of eqn is, $-5x + 9y + 0.z = 3$

$x + 0y + z = 1$

$0.x + 2y + z = 2$

The corresponding augmented matrix.

$$\left[\begin{array}{ccc|c} -5 & 9 & 0 & 3 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 2 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ -5 & 9 & 0 & 3 \\ 0 & 2 & 1 & 2 \end{array}\right]$$

$R_2 \rightarrow R_2 + 5R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 9 & 5 & 8 \\ 0 & 2 & 1 & 2 \end{array}\right]$$

$R_2 \rightarrow 1/9 R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 5/9 & 8/9 \\ 0 & 2 & 1 & 2 \end{array}\right]$$

$R_3 \rightarrow R_3 - 2R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 5/9 & 8/9 \\ 0 & 0 & -1/9 & 2/9 \end{array}\right]$$

$R_3 \rightarrow -9/1 \, R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 5/9 & 8/9 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$R_1 \rightarrow R_1 - R_3$, $R_2 \rightarrow R_2 - \frac{5}{9} R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

∴ The required solution is,

$x = 3$

$y = 2$

$z = -2$

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d. $x - y + 2z = 0$

$x - 2y + 3z = -1$

$2x - 2y + z = -3$

soln;

Given system of eqn is,

$x - y + 2z = 0$

$x - 2y + 3z = -1$

$2x - 2y + z = -3$

The corresponding augmented matrix is,

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 1 & -2 & 3 & -1 \\ 2 & -2 & 1 & -3 \end{array}\right]$$

$R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & -3 & -3 \end{array}\right]$$

$R_2 \rightarrow -1 R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -3 & -3 \end{array}\right]$$

$R_1 \to R_1 + R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -3 & -3 \end{array}\right]$$

$R_3 \to -\frac{1}{3} R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$R_1 \to R_1 - R_3$, $R_2 \to R_2 + R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$\therefore$ The required solution is,

$x = 0$

$y = 2$

$z = 1$

---

e. $2x - y + 4z = -3$

$x + 0.y - 4z = 5$

$6x - y + 2z = 10$

soln;

Given system of eqn is,

$2x - y + 4z = -3$

$x + 0.y - 4z = 5$

$6x - y + 2z = 10$

The corresponding augmented matrix is,

$$\left[\begin{array}{ccc|c} 2 & -1 & 4 & -3 \\ 1 & 0 & -4 & 5 \\ 6 & -1 & 2 & 10 \end{array}\right]$$

$R_1 \longleftrightarrow R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -4 & 5 \\ 2 & -1 & 4 & -3 \\ 6 & -1 & 2 & 10 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 6R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -4 & 5 \\ 0 & -1 & 12 & -13 \\ 0 & -1 & 26 & -20 \end{array}\right]$$

$R_2 \to -1R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -4 & 5 \\ 0 & 1 & -12 & 13 \\ 0 & -1 & 26 & -20 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -4 & 5 \\ 0 & 1 & -12 & 13 \\ 0 & 0 & 14 & -7 \end{array}\right]$$

$R_3 \to 1/14\, R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -4 & 5 \\ 0 & 1 & -12 & 13 \\ 0 & 0 & 1 & -1/2 \end{array}\right]$$

$R_1 \to R_1 + 4R_3$, $R_2 \to R_2 + 12R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -1/2 \end{array}\right] \quad \therefore \quad \begin{array}{l} x = 3 \\ y = 7 \\ z = -1/2 \end{array}$$

f. $x + 2y - 3z = 9$
$2x - y + 2z = -8$
$3x - y - 4z = 3$
Soln;
Given system of eqn is,

$$x + 2y - 3z = 9$$
$$2x - y + 2z = -8$$
$$3x - y - 4z = 3$$

The corresponding augmented matrix is,

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 2 & -1 & 2 & -8 \\ 3 & -1 & -4 & 3 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 8 & -26 \\ 0 & -7 & 5 & -24 \end{array}\right]$$

$R_2 \to -1/5 R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & 1 & -8/5 & 26/5 \\ 0 & -7 & 5 & -24 \end{array}\right]$$

$R_1 \to R_1 - 2R_2$, $R_3 \to R_3 + 7R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1/5 & -7/5 \\ 0 & 1 & -8/5 & 26/5 \\ 0 & 0 & -31/5 & 62/5 \end{array}\right]$$

$R_3 \to -5/31 R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1/5 & -7/5 \\ 0 & 1 & -8/5 & 26/5 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$R_1 \to R_1 - \frac{1}{5}R_3$, $R_2 \to R_2 + \frac{8}{5}R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$\therefore$ The required solution is,

$x = -1$

$y = 2$

$z = -2$

4. To control a certain crop disease, it is necessary to use 7 units of chemical A, 10 units of chemical B and 6 units of chemical C. One barrel of spray P contains 1,4,2 units of chemical, one barrel of spray Q contains 3,2,2 units of chemical and one barrel of spray R contains 4,3,2 units of chemical respectively.

a. Formulate the simultateneous linear system.

chemicals → spray ↓	A (7)	B (10)	C (6)
P	1	4	2
Q	3	2	2
R	4	3	2

Let x, y, z be the quantities of types of spray P, Q, R respectively. Then, the correspondting system of eqn describing the given problem is,

$$\left.\begin{array}{l} x + 3y + 4z = 7 \\ 4x + 2y + 3z = 10 \\ 2x + 2y + 2z = 6 \end{array}\right\} \text{—} (A)$$

b. Write the linear system in matrix form as:

The system (A) can be written as,

$$\begin{bmatrix} 1 & 3 & 4 \\ 4 & 2 & 3 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 10 \\ 6 \end{bmatrix}$$

i. coefficient matrix

$$= \begin{bmatrix} 1 & 3 & 4 \\ 4 & 2 & 3 \\ 2 & 2 & 2 \end{bmatrix}$$

ii. variable matrix

$$= \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

iii. constant matrix

$$= \begin{bmatrix} 7 \\ 10 \\ 6 \end{bmatrix}$$

c. Express the matrix in augmented matrix form and solve it to find the quantity of each spray.

The matrix (A) can be expressed as:

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 4 & 2 & 3 & 10 \\ 2 & 2 & 2 & 6 \end{array}\right]$$

$R_2 \rightarrow R_2 - 4R_1$, $R_3 \rightarrow R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & -10 & -13 & -18 \\ 0 & -4 & 6 & -8 \end{array}\right]$$

$R_2 \rightarrow \frac{1}{-10} R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 1 & 13/10 & 9/5 \\ 0 & -4 & -6 & -8 \end{array}\right]$$

$R_1 \rightarrow R_1 - 3R_2$, $R_3 \rightarrow R_3 + 4R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1/10 & 8/5 \\ 0 & 1 & 13/10 & 9/5 \\ 0 & 0 & -4/5 & -4/5 \end{array}\right]$$

$R_3 \rightarrow -5/4\, R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1/10 & 8/5 \\ 0 & 1 & 13/10 & 9/5 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$R_1 \rightarrow R_1 - 1/10\, R_3$, $R_2 \rightarrow R_3 - 13/10\, R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$\therefore$ The required solution is,

$x = 3/2$

$y = 1/2$

$z = 1$.

- Cramers Rule (Determinant Method)

Consider a system of eqns.

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

coefficient of x	coefficient of y	constant terms
a_1	b_1	c_1
a_2	b_2	c_2

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$Dx \text{ or } D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

$$Dy \text{ or } D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

Then the solution is,

$$x = \frac{Dx}{D} \qquad y = \frac{Dy}{D}$$

For three variables,

consider a system,

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

coefficient of x	coeff. of y	coeff. of z	constant terms
a_1	b_1	c_1	d_1
a_2	b_2	c_2	d_2
a_3	b_3	c_3	d_3

Then,

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad D_x \text{ or } D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$D_y \text{ or } D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \qquad D_z \text{ or } D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Then the solution is,

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \quad \text{and} \quad z = \frac{D_z}{D}$$

Note:

(i) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

(ii) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(iii) If $D_x = D_y = D_z = 0$ and $D = 0$ then the system has infinitely many solutions.

(iv) If $D_x \neq 0$, $D_y \neq 0$, $D_z \neq 0$ but $D = 0$ then the system has no solution.

(v) If $D_x = 0$, $D_y = 0$, $D_z = 0$ but $D \neq 0$ then the system has

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(0,0,0) solution (i.e trivial solution), In this case, $d_1 = d_2 = d_3 = 0$.

vi. If $D \neq 0$, then the system has unique solution.

- Inverse Matrix Method

consider a system,

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

This can be written as,

$$\begin{bmatrix} a_{11}x + a_{12}y + a_{13}z \\ a_{21}x + a_{22}y + a_{23}z \\ a_{31}x + a_{32}y + a_{33}z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

i.e $AX = B$ ——— (1)

$$|A| = \begin{vmatrix} a_{12} & a_{22} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$= \text{---}$

→ $|A| \neq 0$ then, A^{-1} exists.

Now,

cofactor of $a_{11} = C_{11} = + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

cofactor of $a_{12} = C_{12} = - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

Matrix of cofactors $= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

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Adjoint of A = Adj(A)

= Transpose of matrix of co factor

$$= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

Now,

$$A^{-1} = \frac{1}{|A|} \times Adj.$$

From (1)

$$AX = B$$

$$\Rightarrow X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$$

$\therefore x = m_1 \quad y = m_2 \quad z = m_3$

Note:

(i) Two matrices A and B are inverse of each other if,

$$AB = BA = I$$

i.e $A^{-1} = B$ or, $B^{-1} = A$

* $A \cdot A^{-1} = A^{-1} A = I$

(ii) $AX = B$

$\Rightarrow A^{-1}(AX) = A^{-1}B$ (If A is non singular ($|A| \neq 0$)

$\Rightarrow (A^{-1}A)X = A^{-1}B$ then A^{-1} exists)

$\Rightarrow IX = A^{-1}B$

$\Rightarrow X = A^{-1}B$

$\therefore AX = B \Leftrightarrow X = A^{-1}B$

Exercise 7.2

1. Solve the equations by cramer's rule and by inverse matrix method.

a. $x+y=4$

$+3x-2y=17$

soln,

By Cramer's Rule,

coefficient of x	coefficient of y	constant terms
1	1	4
3	-2	17

$D = \begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix}$ $= -2-3$ $= -5$

$Dx = \begin{vmatrix} 4 & 1 \\ 17 & -2 \end{vmatrix}$ $= -8-17$ $= -25$

$Dy = \begin{vmatrix} 1 & 4 \\ 3 & 17 \end{vmatrix}$ $= 17-12$ $= 5$

Then, the solution is,

$x = \frac{Dx}{D} = \frac{-25}{-5}$ and $y = \frac{Dy}{D} = \frac{5}{-5}$

$\therefore x = 5$ $\quad$ $\therefore y = -1$

By Inverse matrix method,

$$\begin{bmatrix} 1 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 17 \end{bmatrix}$$

i.e $AX = B$ —— (1)

$|A| = \begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix}$

$= -5$ $\quad |A| \neq 0$ $\quad \therefore A^{-1}$ exists.

Now,

$$Adj = \begin{bmatrix} -2 & -1 \\ -3 & 1 \end{bmatrix}$$

So, $A^{-1} = \frac{1}{|A|} . Adj$

$$= \frac{1}{-5} \begin{bmatrix} -2 & -1 \\ -3 & 1 \end{bmatrix}$$

From (1)

$$A^{-1} B = X$$

$$\Rightarrow \frac{1}{-5} \begin{bmatrix} -2 & -1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 17 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow \frac{1}{-5} \begin{bmatrix} -8-17 \\ -12+17 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow \frac{1}{-5} \begin{bmatrix} -25 \\ 5 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$\therefore x = 5$ and $y = -1$.

b. $2x - y = 5$

$x - 2y = 1$

Soln;

By Cramer's Rule.

coeff. of x	coeff. of y	constant
2	-1	5
1	-2	1

$D = \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} = -3$ $\quad Dx = \begin{vmatrix} 5 & -1 \\ 1 & -2 \end{vmatrix} = -9$ $\quad Dy = \begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = -3$

Then, the solution is,

$x = \frac{D_x}{D} = \frac{-9}{-3}$ and $y = \frac{D_y}{D} = \frac{-3}{-3}$

$\therefore x = 3$ $\therefore y = 1$

By matrix inverse method.

$$\begin{bmatrix} 2 & -1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

i.e $AX = B$ —— (1)

$$|A| = \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix}$$

$= -3$

Then,

$$Adj(A) = \begin{bmatrix} -2 & 1 \\ -1 & 2 \end{bmatrix}$$

And,

$$A^{-1} = \frac{1}{|A|} Adj$$

$$= \frac{1}{-3}\begin{bmatrix} -2 & 1 \\ -1 & 2 \end{bmatrix}$$

From (1)

$AX = B$

or, $X = A^{-1}B$

or, $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -2 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 1 \end{bmatrix}$

or, $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -10+1 \\ -5+2 \end{bmatrix}$

or, $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -9 \\ -3 \end{bmatrix}$

or, $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\therefore x = 3$ and $y = 1$.

c. $3x+4$…

$15x+2$…

Soln;

By cra…

coeff. …

3

15

$D = \begin{vmatrix} 3 \\ 15 \end{vmatrix}$…

$= 60-$…

$= 0$

$D = 0$

By Inver…

$\begin{bmatrix} 3 \\ 15 \end{bmatrix}$…

$|A| = \begin{vmatrix} 3 \\ 1… \end{vmatrix}$…

$= 0$

Since, |A…

A…

Hence, …

d. $2/3x + y =$…

$x + \frac{y}{4} = 14$…

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c. $3x+4y=-2$
$15x+20y=24$
Soln;
By cramer's rule,

coeff. of x	coeff. of y	constant term
3	4	-2
15	20	24

$$D=\begin{vmatrix} 3 & 4 \\ 15 & 20 \end{vmatrix}$$
$= 60-60$
$= 0$
$D=0$ $\therefore$ Solution doesn't exist.

By Inverse matrix method.

$$\begin{bmatrix} 3 & 4 \\ 15 & 20 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -2 \\ 24 \end{bmatrix}$$
i.e $AX=B$ — (1)

$$|A|=\begin{vmatrix} 3 & 4 \\ 15 & 20 \end{vmatrix}$$
$= 0$
Since, $|A|=0$
A^{-1} doesn't exist
Hence, the solution doesn't exist.

d. $2/3x+y=16$
$x+\frac{y}{4}=14$

Soln;

The given system of eqn are,

$$2x+3y=48$$
$$4x+y=56$$

By cramer's rule,

coeff. of x	coeff. of y	constant term
2	3	48
4	1	56

$D=\begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix}$
$= 2-12$
$=-10$

$Dx=\begin{vmatrix} 48 & 3 \\ 56 & 1 \end{vmatrix}$
$=48-168$
$=-120$

$Dy=\begin{vmatrix} 2 & 48 \\ 4 & 56 \end{vmatrix}$
$=112-192$
$=-80$

Then, the solution is,

$x=\frac{Dx}{D}=\frac{-120}{-10}$ and $y=\frac{Dy}{D}=\frac{-80}{-10}$

$\therefore x=12$ and $y=8$

By Inverse Matrix method

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 48 \\ 56 \end{bmatrix}$$

i.e $AX=B$ —— ①

$|A|=\begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix}$

$=-10\neq 0$

$\therefore A^{-1}$ exists

$Adj(A)=\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|} . \text{Adj}$$

$$= \frac{1}{-10}\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}$$

From ①

$AX = B$

or, $X = A^{-1}B$

or, $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 48 \\ 56 \end{bmatrix}$

or, $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} 48-168 \\ 112-192 \end{bmatrix}$

or, $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} -120 \\ -80 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \end{bmatrix}$

$\therefore x = 12$ and $y = 8$.

$$3x + \frac{4}{y} = 10$$

$$-2x + \frac{3}{y} = -1$$

Soln;

By Cramer's rule

coeff of x	coeff of $1/y$	constant term
3	4	10
-2	3	-1

$D = \begin{vmatrix} 3 & 4 \\ -2 & 3 \end{vmatrix} = 17$ $D_x = \begin{vmatrix} 10 & 4 \\ -1 & 3 \end{vmatrix} = 34$ $D_y = \begin{vmatrix} 3 & 10 \\ -2 & -1 \end{vmatrix} = 17$

Then, the solution is,

$$x = \frac{D_x}{D} \qquad \frac{1}{y} = \frac{D_y}{D}$$

$$= \frac{34}{17} \qquad = \frac{17}{17}$$

$$= 2 \qquad \frac{1}{y} = 1$$

$$\therefore x = 2 \text{ and } y = 1$$

By Inverse Matrix method.

$$\begin{bmatrix} 3 & 4 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ 1/y \end{bmatrix} = \begin{bmatrix} 10 \\ -1 \end{bmatrix}$$

i.e $AX = B$ ——— (1)

$$|A| = \begin{vmatrix} 3 & 4 \\ -2 & 3 \end{vmatrix}$$

$$= 17$$

$$\text{Adj}(A) = \begin{bmatrix} 3 & -4 \\ 2 & 3 \end{bmatrix}$$

So,

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj}$$

$$= \frac{1}{17} \begin{bmatrix} 3 & -4 \\ 2 & 3 \end{bmatrix}$$

From (1)

$$AX = B$$

or, $X = A^{-1}B$

or, $$\begin{bmatrix} x \\ 1/y \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 2 & 3 \end{bmatrix} \cdot \frac{1}{17} \cdot \begin{bmatrix} 10 \\ -1 \end{bmatrix}$$

or, $$\begin{bmatrix} x \\ 1/y \end{bmatrix} = \frac{1}{17} \begin{bmatrix} 34 \\ 17 \end{bmatrix}$$

or, $\begin{bmatrix} x \\ 1/y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\therefore x = 2$ and $\frac{1}{y} = 1$

$\therefore y = 1$

f. $3x = 4y - 11$

$5y = -2x + 31$

soln;

The given eqns are:

$$3x - 4y = -11$$
$$2x + 5y = 31$$

By Cramer's rule

coeff. of x	coeff. of y	constant term.
3	−4	−11
2	5	31

$D_y = \begin{vmatrix} 3 & -11 \\ 2 & 31 \end{vmatrix} = 93 - (-22) = 115$

$D = \begin{vmatrix} 3 & -4 \\ 2 & 5 \end{vmatrix} = 15 - (-8) = 23$

$D_x = \begin{vmatrix} -11 & -4 \\ 31 & 5 \end{vmatrix} = -55 - (-124) = 69$

Then, the solution is,

$x = \frac{D_x}{D} = \frac{69}{23} = 3$ and $y = \frac{D_y}{D} = \frac{115}{23} = 5$

$\therefore x = 3$ and $y = 5$

By Inverse Matrix method.

$$\begin{bmatrix} 3 & -4 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -11 \\ 31 \end{bmatrix}$$

i.e $AX = B$ ——— (1)

$$|A| = \begin{vmatrix} 3 & -4 \\ 2 & 5 \end{vmatrix}$$

$= 23$

$$Adj(A) = \begin{vmatrix} 5 & 4 \\ -2 & 3 \end{vmatrix}$$

So,

$$A^{-1} = \frac{1}{23} \begin{vmatrix} 5 & 4 \\ -2 & 3 \end{vmatrix}$$

From (1)

$AX = B$

or, $X = A^{-1}B$

or, $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ -2 & 3 \end{bmatrix} \frac{1}{23} \cdot \begin{bmatrix} -11 \\ 31 \end{bmatrix}$

or, $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{23} \begin{bmatrix} -55-(-124) \\ 93-(-22) \end{bmatrix}$

or, $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{23} \begin{bmatrix} 69 \\ 115 \end{bmatrix}$

or, $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$\therefore x = 3$ and $y = 5$.

R.K.
05-19

Good

Exercise 7.2 (continued)

2. Solve the following system of simultaneous linear equations by matrix inversion method.

a. $x-y+z=4$
$x+y+z=2$
$2x+y-3z=0$

Soln;

The given system of eqn is,

$$x-y+z=4$$
$$x+y+z=2$$
$$2x+y-3z=0$$

which can be written as;

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$$

i.e $AX = B$ ——— (1)

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & -3 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}$$

$= 1(-3-1) + 1(-3-2) + 1(1-2)$

$= 1(-4) + 1(-5) + 1(-1)$

$= -4-5-1$

$= -10 \neq 0$

$\therefore A^{-1}$ exists

Now,

cofactor of $a_{12} = C_{12} = -\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -(-3-2) = 5$

cofactor of $a_{13} = C_{13} = +\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1-2 = -1$

cofactor of $a_{21} = C_{21} = -\begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = -(3-1) = -2$

cofactor of $a_{22} = C_{22} = +\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -3-2 = -5$

cofactor of $a_{23} = C_{23} = -\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = -(1+2) = -3$

cofactor of $a_{31} = C_{31} = +\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -1-1 = -2$

cofactor of $a_{32} = C_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$

cofactor of $a_{33} = C_{33} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1+1 = 2$

$$\text{Adj}(A) = \begin{bmatrix} -4 & 5 & -1 \\ -2 & -5 & -3 \\ -2 & 0 & 2 \end{bmatrix}^T \qquad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$= \begin{bmatrix} -4 & -2 & -2 \\ 5 & -5 & 0 \\ -1 & -3 & 2 \end{bmatrix}$$

Then,

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj}$$

$$= \frac{1}{-10} \begin{bmatrix} -4 & -2 & -2 \\ 5 & -5 & 0 \\ -1 & -3 & 2 \end{bmatrix}$$

Now,

From ①

$$AX = B$$

or, $X = A^{-1}B$

or, $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} 4 & -2 & -2 \\ 5 & -5 & 0 \\ -1 & -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$$

or, $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} -16-4 \\ 20-10 \\ -4-6 \end{bmatrix}$$

or, $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} -20 \\ 10 \\ -10 \end{bmatrix}$$

or, $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\therefore x = 2$, $y = -1$ and $z = 1$.

$2x - 3y - z = 4$
$x - 2y - z = 1$
$x - y + 2z = 9$

Soln;

The given system of eqn is,

$$2x - 3y - z = 4$$
$$x - 2y - z = 1$$
$$x - y + 2z = 9$$

This can be written as,

$$\begin{bmatrix} 2 & -3 & -1 \\ 1 & -2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 9 \end{bmatrix}$$

i.e $AX = B$ ——— ①

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$$|A| = \begin{vmatrix} 2 & -3 & -1 \\ 1 & -2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} -2 & -1 \\ -1 & 2 \end{vmatrix} + 3\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix}$$

$= 2(-4-1) + 3(2+1) - 1(-1+2)$

$= 2(-5) + 3(3) - 1(1)$

$= -10 + 9 - 1$

$= -2 \neq 0$

$\therefore A^{-1}$ exists.

Now,

cofactor of $a_{11} = C_{11} = +\begin{vmatrix} -2 & -1 \\ -1 & 2 \end{vmatrix} = -4-1 = -5$

cofactor of $a_{12} = C_{12} = -\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} = -(2+1) = -3$

cofactor of $a_{13} = C_{13} = +\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} = -1+2 = 1$

cofactor of $a_{21} = C_{21} = -\begin{vmatrix} -3 & -1 \\ -1 & 2 \end{vmatrix} = -(-6-1) = 7$

cofactor of $a_{22} = C_{22} = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 4+1 = 5$

cofactor of $a_{23} = C_{23} = -\begin{vmatrix} 2 & -3 \\ 1 & -1 \end{vmatrix} = -(-2+3) = -1$

cofactor of $a_{31} = C_{31} = \begin{vmatrix} -3 & -1 \\ -2 & -1 \end{vmatrix} = 3-2 = 1$

cofactor of $a_{32} = C_{32} = -\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = -(-2+1) = 1$

cofactor of $a_{33} = C_{33} = \begin{vmatrix} 2 & -3 \\ 1 & -2 \end{vmatrix} = -4+3 = -1$

$$\mathrm{Adj}(A) = \begin{bmatrix} -5 & -3 & 1 \\ 7 & 5 & -1 \\ 1 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} -5 & 7 & 1 \\ -3 & 5 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

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So, $A^{-1} = \frac{1}{|A|} . Adj$

$$= \frac{1}{-2}\begin{bmatrix} -5 & 7 & 1 \\ -3 & 5 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

From (1)

$AX = B$

or, $X = A^{-1}B$

or, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} -5 & 7 & 1 \\ -3 & 5 & 1 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 9 \end{bmatrix}$

$$= \frac{1}{-2}\begin{bmatrix} -20+7+9 \\ -12+5+9 \\ 4-1-9 \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} -4 \\ 2 \\ -6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$\therefore$ The values are : $x = 2$, $y = -1$ and $z = 3$

c. $3x + 5y = 2$

$2x - 3z = -7$

$4y + 2z = 2$

Soln;

The given system of eqn is,

$3x + 5y + 0.z = 2$

$2x + 0.y - 3z = -7$

$0.x + 4y + 2z = 2$

This can be written as,

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$$\begin{bmatrix} 3 & 5 & 0 \\ 2 & 0 & -3 \\ 0 & 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 2 \end{bmatrix}$$

i.e $AX = B$ ——— (1)

$$|A| = \begin{vmatrix} 3 & 5 & 0 \\ 2 & 0 & -3 \\ 0 & 4 & 2 \end{vmatrix}$$

$$= 3\begin{vmatrix} 0 & -3 \\ 4 & 2 \end{vmatrix} - 5\begin{vmatrix} 2 & -3 \\ 0 & 2 \end{vmatrix}$$

$= 3(12) - 5(4)$

$= 36 - 20$

$= 16 \neq 0$

$\therefore A^{-1}$ exists

Now,

cofactor of $a_{11} = C_{11} = +\begin{vmatrix} 0 & -3 \\ 4 & 2 \end{vmatrix} = 12$

cofactor of $a_{12} = C_{12} = -\begin{vmatrix} 2 & -3 \\ 0 & 2 \end{vmatrix} = -4$

cofactor of $a_{13} = C_{13} = +\begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix} = 8$

cofactor of $a_{21} = C_{21} = -\begin{vmatrix} 5 & 0 \\ 4 & 2 \end{vmatrix} = -10$

cofactor of $a_{22} = C_{22} = +\begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} = 6$

cofactor of $a_{23} = C_{23} = -\begin{vmatrix} 3 & 5 \\ 0 & 4 \end{vmatrix} = -12$

cofactor of $a_{31} = C_{31} = +\begin{vmatrix} 5 & 0 \\ 0 & -3 \end{vmatrix} = -15$

cofactor of $a_{32} = C_{32} = -\begin{vmatrix} 3 & 0 \\ 2 & -3 \end{vmatrix} = 9$

cofactor of $a_{33} = C_{33} = +\begin{vmatrix} 3 & 5 \\ 2 & 0 \end{vmatrix} = -10$

$$Adj(A) = \begin{bmatrix} 12 & -4 & 8 \\ -20 & 6 & -12 \\ -15 & 9 & -10 \end{bmatrix}^T$$

$$= \begin{bmatrix} +12 & -10 & -15 \\ -4 & 6 & 9 \\ 8 & -12 & -10 \end{bmatrix}$$

Then,

$$A^{-1} = \frac{1}{|A|} \cdot Adj$$

$$= \frac{1}{16} \begin{bmatrix} 12 & -10 & -15 \\ -4 & 6 & 9 \\ 8 & -12 & -10 \end{bmatrix}$$

Now, From (1)

$$AX = B$$

or, $X = A^{-1}B$

or,
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{16} \begin{bmatrix} 12 & -10 & -15 \\ -4 & 6 & 9 \\ 8 & -12 & -10 \end{bmatrix} \begin{bmatrix} 2 \\ -7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{16} \begin{bmatrix} 24 + 70 - 30 \\ -8 - 42 + 18 \\ 16 + 84 - 20 \end{bmatrix}$$

$$= \frac{1}{16} \begin{bmatrix} 64 \\ -32 \\ 80 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -2 \\ 5 \end{bmatrix}$$

$\therefore$ The required solution is,

$x = 4$

$y = -2$

$z = 5$ //

d. $x-3y-7z=6$
$2x+3y+z=9$
$4x+y=7$
Soln;
The given system of eqn is,

$$x-3y-7z=6$$
$$2x+3y+z=9$$
$$4x+y+0.z=7$$

which can be written as,

$$\begin{bmatrix} 1 & -3 & -7 \\ 2 & 3 & 1 \\ 4 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ 7 \end{bmatrix}$$

i.e $AX=B$ —— (1)

$$|A| = \begin{vmatrix} 1 & -3 & -7 \\ 2 & 3 & 1 \\ 4 & 1 & 0 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 4 & 0 \end{vmatrix} - 7\begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix}$$

$$= 1(-1) + 3(-4) - 7(2-12)$$
$$= -1 - 12 + 70$$
$$= 57 \neq 0$$

i.e A^{-1} exists

Now,

cofactor of $a_{11} = C_{11} = \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} = -1$

cofactor of $a_{12} = C_{12} = -\begin{vmatrix} 2 & 1 \\ 4 & 0 \end{vmatrix} = 4$

cofactor of $a_{13} = C_{13} = +\begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix} = 2-12 = -10$

cofactor of $a_{21} = C_{21} = -\begin{vmatrix} -3 & -7 \\ 1 & 0 \end{vmatrix} = -7$

cofactor of a_{22} = $C_{22} = + \begin{vmatrix} 1 & -7 \\ 4 & 0 \end{vmatrix} = 28$

cofactor of a_{23} = $C_{23} = - \begin{vmatrix} 1 & -3 \\ 4 & 1 \end{vmatrix} = -(1+12) = -13$

cofactor of a_{31} = $C_{31} = + \begin{vmatrix} -3 & -7 \\ 3 & 1 \end{vmatrix} = -3+21 = 18$

cofactor of a_{32} = $C_{32} = - \begin{vmatrix} 1 & -7 \\ 2 & 1 \end{vmatrix} = -(1+14) = -15$

cofactor of a_{33} = $C_{33} = + \begin{vmatrix} 1 & -3 \\ 2 & 3 \end{vmatrix} = 3+6 = 9$

Now,

$$Adj(A) = \begin{bmatrix} -1 & 4 & -10 \\ -7 & 28 & -13 \\ 18 & -15 & 9 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & -7 & 18 \\ 4 & 28 & -15 \\ -10 & -13 & 9 \end{bmatrix}$$

Then,

$$A^{-1} = \frac{1}{|A|} . Adj$$

$$= \frac{1}{57} \begin{bmatrix} -1 & -7 & 18 \\ 4 & 28 & -15 \\ -10 & -13 & 9 \end{bmatrix}$$

From ①

$$AX = B$$

or, $X = A^{-1}B$

or, $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{57} \begin{bmatrix} -1 & -7 & 18 \\ 4 & 28 & -15 \\ -10 & -13 & 9 \end{bmatrix} \begin{bmatrix} 6 \\ 9 \\ 7 \end{bmatrix}$$

$$= \frac{1}{57} \begin{bmatrix} -6-63+126 \\ 24+252-105 \\ -60-117+63 \end{bmatrix} = \frac{1}{57} \begin{bmatrix} 57 \\ 171 \\ -114 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$$

$\therefore x = 1$, $y = 3$ and $z = -2$.

$x+2y+z=7$

$2x-y+z=3$

$3x+y+2z=8$

Soln;

The given system of eqn is,

$$x+2y+z=7$$
$$2x-y+z=3$$
$$3x+y+2z=8$$

which can be written as,

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 8 \end{bmatrix}$$

i.e $AX=B$

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix}$$

$$= 1(-2-1) - 2(4-3) + 1(2+3)$$

$$= 1(-3) - 2(1) + 1(5)$$

$$= -3-2+5$$

$$= 0$$

$\therefore A^{-1}$ doesn't exist

Hence, the system has no solution.

3. Solve the following system of eqn using Cramer's rule.

a. $2x - 3y - z = 4$
$x - 2y - z = 1$
$x - y + 2z = 9$

Soln;

The given system of eqn is,

$$2x - 3y - z = 4$$
$$x - 2y - z = 1$$
$$x - y + 2z = 9$$

coeff. of x	coeff of y	coeff of z	constant term
2	-3	-1	4
1	-2	-1	1
1	-1	2	9

$$D = \begin{vmatrix} 2 & -3 & -1 \\ 1 & -2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} -2 & -1 \\ -1 & 2 \end{vmatrix} + 3\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix}$$

$= 2(-4-1) + 3(2+1) - 1(-1+2)$

$= 2(-5) + 3(3) - 1(1)$

$= -10 + 9 - 1$

$= -2 \neq 0$

$\therefore$ System has solution.

$$Dx = \begin{vmatrix} 4 & -3 & -1 \\ 1 & -2 & -1 \\ 9 & -1 & 2 \end{vmatrix}$$

$$= 4\begin{vmatrix} -2 & -1 \\ -1 & 2 \end{vmatrix} + 3\begin{vmatrix} 1 & -1 \\ 9 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & -2 \\ 9 & -1 \end{vmatrix}$$

$= 4(-4-1) + 3(2+9) - 1(-1+18) = 4(-5) + 3(11) - 1(17)$

$= -20 + 33 - 17$

$= -4$

$$D_y = \begin{vmatrix} 2 & 4 & -1 \\ 1 & 1 & -1 \\ 1 & 9 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & -1 \\ 9 & 2 \end{vmatrix} - 4\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix}$$

$= 2(2+9) - 4(2+1) - 1(9-1)$

$= 2(11) - 4(3) - 1(8)$

$= 22 - 12 - 8$

$= 2$

$$D_z = \begin{vmatrix} 2 & -3 & 4 \\ 1 & -2 & 1 \\ 1 & -1 & 9 \end{vmatrix}$$

$$= 2\begin{vmatrix} -2 & 1 \\ -1 & 9 \end{vmatrix} + 3\begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix}$$

$= 2(-18+1) + 3(9-1) + 4(-1+2)$

$= 2(-17) + 3(8) + 4(1)$

$= -34 + 24 + 4$

$= -6$

Then, the solution is,

$x = \frac{D_x}{D} = \frac{-4}{-2}$, $y = \frac{D_y}{D} = \frac{2}{-2}$ and $z = \frac{D_z}{D} = \frac{-6}{-2}$

$\therefore x = 2$, $y = -1$ and $z = 3$

b. $x+y+z=-1$

$3x+y+z=1$

$4x-2y+2z=0$

Soln,

The given system of eqn is,

$x+y+z=-1$

$3x+y+z=1$

$4x-2y+2z=0$

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coeff. of x	coeff of y	coeff. of z	constant
1	1	1	-1
3	1	1	1
4	-2	2	0

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \\ 4 & -2 & 2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 1 \\ -2 & 2 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 4 & -2 \end{vmatrix}$$

$= 1(2+2) - 1(6-4) + 1(-6-4)$

$= 1(4) - 1(2) + 1(-10)$

$= 4 - 2 - 10$

$= -8$

$$Dx = \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & -2 & 2 \end{vmatrix}$$

$$= -1\begin{vmatrix} 1 & 1 \\ -2 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix}$$

$= -1(2+2) - 1(2) + 1(-2)$

$= -1(4) - 2 - 2$

$= -4 - 2 - 2$

$= -8$

$$Dy = \begin{vmatrix} 1 & -1 & 1 \\ 3 & 1 & 1 \\ 4 & 0 & 2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 4 & 0 \end{vmatrix}$$

$= 1(2) + 1(6-4) + 1(-4)$

$= 2 + 1(2) - 4$

$= 2 + 2 - 4$

$= 0$

$$D_z = \begin{vmatrix} 1 & 1 & -1 \\ 3 & 1 & 1 \\ 4 & -2 & 0 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 1 \\ -2 & 0 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ 4 & 0 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ 4 & -2 \end{vmatrix}$$

$= 1(2) - 1(-4) - 1(-6-4)$

$= 2 + 4 + 10$

$= 16$

Then, the solution is,

$x = \frac{D_x}{D} = \frac{-8}{-8}$, $y = \frac{D_y}{D} = \frac{0}{-8}$ and $z = \frac{D_z}{D} = \frac{16}{-8}$

$\therefore x = 1$, $y = 0$ and $z = -2$

c. $6y + 6z = -1$

$8x + 6z = -1$

$4x + 9y = 8$

Soln;

The given eqn is

$0.x + 6y + 6z = -1$

$8x + 0y + 6z = -1$

$4x + 9y + 0z = 8$

coeff. of x	coeff. of y	coeff. of z	constant term
0	6	6	-1
8	0	6	-1
4	9	0	8

$$D = \begin{vmatrix} 0 & 6 & 6 \\ 8 & 0 & 6 \\ 4 & 9 & 0 \end{vmatrix}$$

$$= -6\begin{vmatrix} 8 & 6 \\ 4 & 0 \end{vmatrix} + 6\begin{vmatrix} 8 & 0 \\ 4 & 9 \end{vmatrix}$$
$$= -6(-24) + 6(72)$$
$$= 144 + 432$$
$$= 576$$

$$D_x = \begin{vmatrix} -1 & 6 & 6 \\ -1 & 0 & 6 \\ 8 & 9 & 0 \end{vmatrix}$$
$$= -1\begin{vmatrix} 0 & 6 \\ 9 & 0 \end{vmatrix} - 6\begin{vmatrix} -1 & 6 \\ 8 & 0 \end{vmatrix} + 6\begin{vmatrix} -1 & 0 \\ 8 & 9 \end{vmatrix}$$
$$= -1(-54) - 6(-48) + 6(-9)$$
$$= +54 + 288 - 54$$
$$= 288$$

$$D_y = \begin{vmatrix} 0 & -1 & 6 \\ 8 & -1 & 6 \\ 4 & 8 & 0 \end{vmatrix}$$
$$= 1\begin{vmatrix} 8 & 6 \\ 4 & 0 \end{vmatrix} + 6\begin{vmatrix} 8 & -1 \\ 4 & 8 \end{vmatrix}$$
$$= 1(-24) + 6(64+4)$$
$$= -24 + 6(68)$$
$$= 384$$

$$D_z = \begin{vmatrix} 0 & 6 & -1 \\ 8 & 0 & -1 \\ 4 & 9 & 8 \end{vmatrix}$$
$$= -6\begin{vmatrix} 8 & -1 \\ 4 & 8 \end{vmatrix} - 1\begin{vmatrix} 8 & 0 \\ 4 & 9 \end{vmatrix}$$
$$= -6(68) - 1(72)$$
$$= -480$$

Then the solution is,

$x = \frac{D_x}{D} = \frac{288}{576} = \frac{1}{2}$, $y = \frac{D_y}{D} = \frac{384}{576} = \frac{2}{3}$ and $z = \frac{D_z}{D} = \frac{-480}{576} = \frac{-5}{6}$

d. $x+4y+z=18$
$3x+3y-2z=2$
$0\cdot x-4y+z=-7$

Soln;
The given system of eqn is,

$$x+4y+z=18$$
$$3x+3y-2z=2$$
$$0\cdot x-4y+z=-7$$

coeff. of x	coeff. of y	coeff. of z	constant term
1	4	1	18
3	3	-2	2
0	-4	1	-7

$$D=\begin{vmatrix} 1 & 4 & 1 \\ 3 & 3 & -2 \\ 0 & -4 & 1 \end{vmatrix}$$

$$=1\begin{vmatrix} 3 & -2 \\ -4 & 1 \end{vmatrix}-4\begin{vmatrix} 3 & -2 \\ 0 & 1 \end{vmatrix}+1\begin{vmatrix} 3 & 3 \\ 0 & -4 \end{vmatrix}$$

$$=1(3-8)-4(3)+1(-12)$$
$$=-5-12-12$$
$$=-29$$

$$Dx=\begin{vmatrix} 18 & 4 & 1 \\ 2 & 3 & -2 \\ -7 & -4 & 1 \end{vmatrix}$$

$$=18\begin{vmatrix} 3 & -2 \\ -4 & 1 \end{vmatrix}-4\begin{vmatrix} 2 & -2 \\ -7 & 1 \end{vmatrix}+1\begin{vmatrix} 2 & 3 \\ -7 & -4 \end{vmatrix}$$

$$=18(3-8)-4(2-14)+1(-8+21)$$
$$=18(-5)-4(-12)+1(13)$$
$$=-90+48+13$$
$$=-29$$

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$$D_y = \begin{vmatrix} 1 & 18 & 1 \\ 3 & 2 & -2 \\ 0 & -7 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & -2 \\ -7 & 1 \end{vmatrix} - 18\begin{vmatrix} 3 & -2 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 2 \\ 0 & -7 \end{vmatrix}$$

$$= 1(2-14) - 18(3) + 1(-21)$$

$$= -12 - 54 - 21$$

$$= -87$$

$$D_z = \begin{vmatrix} 1 & 4 & 18 \\ 3 & 3 & 2 \\ 0 & -4 & -7 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & 2 \\ -4 & -7 \end{vmatrix} - 4\begin{vmatrix} 3 & 2 \\ 0 & -7 \end{vmatrix} + 18\begin{vmatrix} 3 & 3 \\ 0 & -4 \end{vmatrix}$$

$$= 1(-21+8) - 4(-21) + 18(-12)$$

$$= 1(-13) + 84 - 216$$

$$= -145$$

Then the solution is,

$$x = \frac{D_x}{D} = \frac{-29}{-29}, \quad y = \frac{D_y}{D} = \frac{-87}{-29} \text{ and } z = \frac{D_z}{D} = \frac{-145}{-29}$$

$$\therefore x = 1, \quad y = 3 \text{ and } z = 5$$

4. Rinav sells 7 shares of A and buys 9 shares of B, thus increasing his cash by 70Rs, Arnav sells 9 shares of A and buys 14 shares of B thus decreasing cash by Rs 80

a. Formulate the simultaneous linear system
b. Express the linear system in determinant form.
c. Using cramer's rule, find the price per share of A and B.

Soln;

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	A	B	
Rinav	7	9	70
Arnav	9	14	-80

a. Let x and y be per unit price of shares of A and B respectively.
The system of equation describing given problem is,

$$7x - 9y = 70$$
$$9x - 14y = -80$$

b. $D = \begin{vmatrix} 7 & -9 \\ 9 & -14 \end{vmatrix}$

$= -98 + 81$

$= -17$

c. $Dx = \begin{vmatrix} 70 & -9 \\ -80 & -14 \end{vmatrix}$

$= -980 - 720$

$= -1700$

$Dy = \begin{vmatrix} 7 & 70 \\ 9 & -80 \end{vmatrix}$

$= -560 - 630$

$= -1190$

Then the solution is,

$x = \frac{Dx}{D} = \frac{-1700}{-17} = 100$ and

$y = \frac{Dy}{D} = \frac{-1190}{17} = 70$

e. coeff of x coeff. of y. constant term.

coeff of x	coeff. of y	constant term
7	-9	70
9	-14	-80

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5\. A transport company has three types of trucks: A, B and C which are designed to carry three different sizes of boxes, P, Q and R per load as shown below

Types of trucks:

Boxes ↓	A	B	C
P	2	5	2
Q	3	2	5
R	1	9	0

Each type of boxes should be used to carry exactly 18 boxes of size P, 18 boxes of size Q and 21 boxes of size R.

a. Formulate the simultaneous linear system

b. Express the linear system in determinant form.

c. Using Cramer's rule, solve the linear system and find the number of truck.

Soln;

a. Let x, y and z be the ~~types~~ numbers of trucks respectively, The system of eqn describing given problem is,

$$2x + 5y + 2z = 18$$
$$3x + 2y + 5z = 18$$
$$x + 9y + 0.z = 21$$

b.

coeff. of x	coeff. of y	coeff. of z	constant term
2	5	2	18
3	2	5	18
1	9	0	21

b. $D = \begin{vmatrix} 2 & 5 & 2 \\ 3 & 2 & 5 \\ 1 & 9 & 0 \end{vmatrix}$

$= 2\begin{vmatrix} 2 & 5 \\ 9 & 0 \end{vmatrix} - 5\begin{vmatrix} 3 & 5 \\ 1 & 0 \end{vmatrix} + 2\begin{vmatrix} 3 & 2 \\ 1 & 9 \end{vmatrix}$

$= 2(-45) - 5(5) + 2(27-2)$

$= -15$

c. $D_x = \begin{vmatrix} 18 & 5 & 2 \\ 18 & 2 & 5 \\ 21 & 9 & 0 \end{vmatrix}$

$= 18\begin{vmatrix} 2 & 5 \\ 9 & 0 \end{vmatrix} - 5\begin{vmatrix} 18 & 5 \\ 21 & 0 \end{vmatrix} + 2\begin{vmatrix} 18 & 2 \\ 21 & 9 \end{vmatrix}$

$= 18(-45) - 5(-105) + 2(162-42)$

$= -810 + 525 + 240$

$= -45$

$D_y = \begin{vmatrix} 2 & 18 & 2 \\ 3 & 18 & 5 \\ 1 & 21 & 0 \end{vmatrix}$

$= 2\begin{vmatrix} 18 & 5 \\ 21 & 0 \end{vmatrix} - 18\begin{vmatrix} 3 & 5 \\ 1 & 0 \end{vmatrix} + 2\begin{vmatrix} 3 & 18 \\ 1 & 21 \end{vmatrix}$

$= 2(-105) - 18(-5) + 2(63-18)$

$= -210 + 90 + 90$

$= -30$

$D_z = \begin{vmatrix} 2 & 5 & 18 \\ 3 & 2 & 18 \\ 1 & 9 & 21 \end{vmatrix}$

$= 2\begin{vmatrix} 2 & 18 \\ 9 & 21 \end{vmatrix} - 5\begin{vmatrix} 3 & 18 \\ 1 & 21 \end{vmatrix} + 18\begin{vmatrix} 3 & 2 \\ 1 & 9 \end{vmatrix}$

$= 2(42-162) - 5(63-18) + 18(27-2)$

$= -15$

Then, the solution is,

$x = \frac{D_x}{D} = \frac{-45}{-15}$, $y = \frac{D_y}{D} = \frac{-30}{-15}$ and $z = \frac{D_z}{D} = \frac{-15}{-15}$

$\therefore x = 3$, $y = 2$ and $z = 1$

$\therefore$ The number of trucks at A, B, C are 3, 2, 1 respectively.

6. The price of commodities X, Y and Z are respectively x, y and z rupees per unit. Mr. A purchases 4 units and sells 3 units of X and 5 units of Y. Mr B purchases 3 units of Y and sells 2 units of X and 1 unit of Z. Mr C purchases 1 unit of X and sells 4 units of Y and 6 units of Z. In this process, A, B, C earn zero profit, Rs 5000 and Rs 13000 profits respectively.

a. Formulate the simultaneous linear system.
b. Express the linear system in determinant form.
c. Using cramer's rule, find the prices per unit of three commodities.

Soln;

	X	Y	Z	
A	3	5	-4	0
B	2	-3	1	5000
C	-1	4	6	13000

coeff of x	coeff. of y	coeff. of z	constant term
3	5	-4	0
2	-3	1	5000
-1	4	6	13000

a. If x, y, z be the per unit price of commodities X, Y, Z respectively. Then the system of eq^n describing given word problem is

$$3x + 5y - 4z = 0$$
$$2x - 3y + z = 5000$$
$$-x + 4y + 6z = 13000$$

b. $D = \begin{vmatrix} 3 & 5 & -4 \\ +2 & -3 & 1 \\ -1 & 4 & 6 \end{vmatrix}$

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$$= 3\begin{vmatrix} -3 & 1 \\ 4 & 6 \end{vmatrix} - 5\begin{vmatrix} 2 & 1 \\ -1 & 6 \end{vmatrix} - 4\begin{vmatrix} 2 & -3 \\ -1 & 4 \end{vmatrix}$$

$= 3(-18-4) - 5(12+2) - 4(8-3)$

$= -151$

$$Dx = \begin{vmatrix} 0 & 5 & -4 \\ 5000 & -3 & 1 \\ 13000 & 4 & 6 \end{vmatrix}$$

$$= -5\begin{vmatrix} 5000 & 1 \\ 13000 & 6 \end{vmatrix} - 4\begin{vmatrix} 5000 & -3 \\ 13000 & 4 \end{vmatrix}$$

$= -5(17000) - 4(20000+39000)$

$= -321000$

$$Dy = \begin{vmatrix} 3 & 0 & -4 \\ 2 & 5000 & 1 \\ -1 & 13000 & 6 \end{vmatrix}$$

$$= 3\begin{vmatrix} 5000 & 1 \\ 13000 & 6 \end{vmatrix} - 4\begin{vmatrix} 2 & 5000 \\ -1 & 13000 \end{vmatrix}$$

$= 3(30000-13000) - 4(26000+5000)$

$= -73000$

$$Dz = \begin{vmatrix} 3 & 5 & 0 \\ 2 & -3 & 5000 \\ 1 & 4 & 13000 \end{vmatrix}$$

$$= 3\begin{vmatrix} -3 & 5000 \\ 4 & 13000 \end{vmatrix} - 5\begin{vmatrix} 2 & 5000 \\ 1 & 13000 \end{vmatrix}$$

$= 3(-39000-20000) - 5(26000+5000)$

$= -332000$

Then the solution is,

$x = \frac{Dx}{D} = \frac{-321000}{-151}$, $y = \frac{Dy}{D} = \frac{-73000}{-151}$ & $z = \frac{Dz}{D} = \frac{-332000}{-151}$

$\therefore x = 2125.82$, $y = 483.44$ & $z = 2198.67$

R.K
05/19
Good