∴ Exercise: 16.2

Evaluate.

1. $\int \sqrt{x^2-36}\, dx$

$= \int \sqrt{x^2-6^2}\, dx$

$= \frac{x}{2}\sqrt{x^2-36} + \frac{36}{2}\ln|x+\sqrt{x^2-36}| + C.$

$= \frac{x}{2}\sqrt{x^2-36} - 18\ln(x+\sqrt{x^2-36}) + C.$

2. $\int \sqrt{1-4x^2}\, dx$

$= \frac{1}{2}\int \sqrt{\frac{1}{4}-x^2}\, dx$

$= \frac{1}{2}\int \sqrt{\left(\frac{1}{2}\right)^2-(x^2)}\, dx$

$= \frac{1}{2}\cdot\frac{x}{2}\cdot\sqrt{\frac{1}{4}-x^2} + \frac{1}{4\cdot 2}\sin^{-1}x\cdot 2 + C.$

$= \frac{x}{8}\sqrt{1-4x^2} - \frac{1}{8}\sin^{-1}4x + C$

3. $\int \sqrt{3x^2+5}\, dx$

$= \sqrt{3}\int \sqrt{x^2+\left(\frac{\sqrt{5}}{\sqrt{3}}\right)^2}\, dx$

$= \sqrt{3}\left(\frac{x}{2}\sqrt{x^2+\frac{5}{3}} + \frac{\frac{5}{3}}{2}\ln\left|x+\sqrt{x^2+\frac{5}{3}}\right|\right) + C$

$= \frac{x}{2}\sqrt{3x^2+5} + \frac{5}{2\sqrt{3}}\ln|\sqrt{3}x+\sqrt{3x^2+5}| + C$ //

4. $\int \sqrt{3-2x-x^2}\,dx$

$= \int \sqrt{-(x^2+2x-3)}\,dx$

$= \int \sqrt{-\{(x)^2+2\cdot x\cdot 1+(1)^2-1-3\}}\,dx$

$= \int \sqrt{2^2-(x+1)^2}\,dx$

$= \frac{x+1}{2}\sqrt{4-(x+1)^2}+2\sin^{-1}\left(\frac{x+1}{2}\right)+C$ //

5. $\int \sqrt{5-2x+x^2}\,dx$

$= \int \sqrt{(x)^2-2\cdot x\cdot 1+(1)^2-1+5}\,dx$

$= \int \sqrt{(x-1)^2+2^2}\,dx$

$= \left(\frac{x-1}{2}\right)\sqrt{(x-1)^2+4}+\frac{2^2}{2}\ln\left|(x-1)+\sqrt{(x-1)^2+4}\right|+C$

$= \frac{x-1}{2}\sqrt{x^2-2x+5}+2\ln\left|(x-1)+\sqrt{x^2-2x+5}\right|+C.$

6. $\int \sqrt{18x-x^2-65}\,dx$

$= \int \sqrt{-(x^2-18x+65)}\,dx$

$= \int \sqrt{-(x^2-2x\cdot 9+9^2-81+65)}\,dx$

$= \int \sqrt{4^2-(x-9)^2}\,dx$

$= \frac{x-9}{2}\sqrt{16-x^2+18x-81}+8\sin^{-1}\left(\frac{x-9}{4}\right)+C$

$= \frac{x-9}{2}\sqrt{18x-x^2+65}+8\sin^{-1}\left(\frac{x-9}{4}\right)+C$ //

7. $\int \sqrt{5x^2+8x+4}\ dx$

Soln

$$I = \sqrt{5}\int \sqrt{(x)^2 + 2.x.\frac{4}{5} + \left(\frac{4}{5}\right)^2 - \frac{16}{25} + \frac{4}{5}}\ dx$$

$$= \sqrt{5}\int \sqrt{\left(x+\frac{4}{5}\right)^2 + \frac{4}{25}}\ dx$$

$$= \sqrt{5}\left[\frac{x+\frac{4}{5}}{2}\sqrt{\left(x+\frac{4}{5}\right)^2+\left(\frac{2}{5}\right)^2} + \frac{4}{25\times 2}\ln\left|\left(x+\frac{4}{5}\right) + \sqrt{\left(x+\frac{4}{5}\right)^2+\left(\frac{2}{5}\right)^2}\right| + C\right]$$

$$= \frac{\sqrt{5}}{\sqrt{5}}\left(\frac{5x+4}{10}\sqrt{5x^2+8x+4} + \frac{2}{5\sqrt{5}}\ln\left|\frac{5x+4}{5} + \sqrt{x^2+\frac{8x}{5}+\frac{4}{5}}\right|\right) + C.$$

8. $\int \sqrt{(x-\alpha)(\beta-x)}\ dx$

$$= \int \sqrt{\beta x - x^2 - \alpha\beta + \alpha x}\ dx$$

$$= \sqrt{-x^2 + (\alpha+\beta)x - \alpha\beta}\ dx$$

$$= \sqrt{-\left(x^2 - 2x\cdot\left(\frac{\alpha+\beta}{2}\right) + \left(\frac{\alpha+\beta}{2}\right)^2\right) + \left(\frac{\alpha+\beta}{2}\right)^2 - 2\alpha\beta}\ dx$$

$$= \int \sqrt{\left(\frac{\alpha-\beta}{2}\right)^2 - \left(x-\frac{\alpha+\beta}{2}\right)^2}\ dx \quad \left[\because (a+b)^2 - 4ab = (a-b)^2\right]$$

$$= \frac{1}{4}(2x-\alpha-\beta)\sqrt{(x-\alpha)(\beta-x)} + \left(\frac{\alpha-\beta}{8}\right)^2 \sin^{-1}\left(\frac{2x-\alpha-\beta}{\alpha-\beta}\right) + c.$$

9. $\int \sqrt{2ax-x^2}\, dx$

$$= \int \sqrt{a^2+2ax-x^2+a^2}\, dx$$

$$= \int \sqrt{-(x^2-2ax+a^2-a^2)}\, dx$$

$$= \int \sqrt{a^2-(x-a)^2}\, dx$$

$$= \frac{(x-a)}{2}\sqrt{a^2-x^2+2ax-a^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right)+c$$

$$= \frac{(x-a)}{2}\sqrt{2ax-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right)+c$$

10. $\int (2x-3)\sqrt{x^2-4x+5}\, dx$

$$= \int (2x-4)\sqrt{x^2-4x+5}\, dx - \int \sqrt{x^2-4x+5}\, dx$$

$$= \frac{2}{3}(x^2-4x+5)^{3/2} - \int \sqrt{x^2-4x+4+1}\, dx$$

$$= \frac{2}{3}(x^2-4x+5)^{3/2} - \int \sqrt{(x-2)^2+(1)^2}\, dx$$

$$= \frac{2}{3}(x^2-4x+5)^{3/2} - \left\{\left(\frac{x-2}{2}\right)\sqrt{x^2-4x+5} + \frac{1}{2}\right.$$

$$\left. \ln\left(x-2+\sqrt{x^2-4x+5}\right)\right\} + c.$$

11 $\int (2-x)\sqrt{16-6x-x^2}\,dx$

$\frac{1}{2}\int (4-2x)\sqrt{16-6x-x^2}\,dx$

$= \frac{10}{2}\int \sqrt{16-6x-x^2}\,dx + \frac{1}{2}\int (-6-2x)\sqrt{16-6x-x^2}\,dx$

$= 5\int \sqrt{25-9-6x-x^2}\,dx + \frac{1}{2}\cdot\frac{2}{3}(16-6x-x^2)^{3/2}$

$= 5\int \sqrt{(5)^2-(x+3)^2}\,dx + \frac{1}{3}(16-6x-x^2)^{3/2}$

$= \frac{5(x+3)}{2}\sqrt{16-6x-x^2} + \frac{125}{2}\sin^{-1}\left(\frac{x+3}{5}\right) + \frac{1}{3}(16-6x-x^2)^{3/2}$

$+C.$

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13. $\int (2x+3)\sqrt{x^2-2x-3}\,dx$

$\int (2x-2)\sqrt{x^2-2x-3}\,dx + 5\int \sqrt{x^2-2x+1-4}\,dx$

$= \frac{2}{3}(x^2-2x-3)^{3/2} + 5\int \sqrt{(x-1)^2-(2)^2}\,dx$

$= \frac{2}{3}(x^2-2x-3)^{3/2} + \frac{5(x-1)}{2}\sqrt{(x-1)^2-(2)^2} + \ln\frac{4(x-1+\sqrt{(x-1)^2-4})}{2} + c$

$= \frac{2}{3}(x^2-2x-3)^{3/2} + \frac{5x-5}{2}\sqrt{(x-1)^2-4} + 10\ln(x-1+\sqrt{(x-1)^2-4}) + C.$

14. $\int e^{3x} \sin 5x \, dx$

$$\frac{e^{3x}}{3^2+5^2}\left[3\cos 3\sin 5x - 5\cos 5x\right) + c.$$

$$= \frac{e^{3x}}{34}\left(3\sin 5x - 5\cos 5x\right] + c.$$

15. $\int e^x \cos 3x \, dx$

$$= \frac{e^x}{1^2+3^2}\left(3\sin 3x + \cos 3x\right) + c$$

$$= \frac{e^x}{10}\left(\cos 3x + 3\sin 3x\right) + c.$$

16. $\int e^{2x} \sin(x+1) \, dx$

$$= \frac{e^{2x}}{2^2+(x+1)^2}\left(2\sin(x+1) - (x+1)\cos(x+1)\right) + c$$

$$= \frac{e^{2x}}{x^2+2x+5}\left(2\sin(x+1) - \cos(x+1) - x\cos(x+1)\right) + c.$$

16 $\int e^{2x} \sin(x+1) \, dx$

$$= \frac{e^{2x}}{2^2+1^2}\left(2\sin(x+1) - \cos(x+1)\right) + c$$

$$= \frac{e^{2x}}{5}\left(2\sin(x+1) - \cos(x+1)\right) + c_{//}$$