Differential Approximation

Let $y = f(x)$ be a function.
Approximate change in y:
$dy = f'(x)dx$, where $dx = \Delta x$

Actual change in y.
$$\Delta y = f(x+\Delta x) - f(x)$$

Percentage Error change in y: $\frac{\Delta y - dy}{y} \times 100\%$.

Error in approximation $= \Delta y - dy$

Exercise 15.7

1 Find Δy and dy

a) $y = x^3 + 3$ for $x = 2$ and $\Delta x = 0.1$

Soln

Here,

$y = x^3 + 3$

$dy = f'(x)dx$

$\therefore \frac{dy}{dx} = 3x^2$

At $x = 2$,

$\frac{dy}{dx} = 3 \times 2^2 = 12$.

$\therefore f'(x) = 12$

When $x = 2$,

~~Also,~~

$dx = \Delta x = 0.1$, then,

$dy = f'(x) \cdot dx$

$= 12 \times 0.1$

$= 1.2$.

For Δy,

$\Delta y = f(x+\Delta x) - f(x)$

$= f(2+0.1) - f(2)$

$= (2.1)^3 + 3 - 2^3 - 3$

$= 1.261$.

$\therefore \Delta y = 1.261$, $dy = 1.2$.

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b) $y = \sqrt{x}$ for $x = 4$ and $\Delta x = 0.41$

Sol^n

$y = \sqrt{x}$

Diff both sides w.r.t x.

$\frac{dy}{dx} = \frac{d\sqrt{x}}{dx}$

$= \frac{1}{2\sqrt{x}}$

At $x = 2$,

$\frac{dy}{dx} = \frac{1}{2\sqrt{2}}$.

~~Also,~~

$dy = f'(x)dx$.

At $x = 4$, $\Delta x = 0.41 = dx$

or $dy = \frac{1}{2\sqrt{4}} \times 0.41 = 0.1025$.

$\Delta y = f(x+\Delta x) - f(x)$

$= f(4+0.41) - f(4)$

$= 0.1$

$\therefore \Delta y = 0.1, \quad dy = 0.144 \; 0.1025$

c) Soln

$y = x^2 + 3x$

$f(x) = x^2 + 3x$

$x = 2$

$\Delta x = 0.2$

Now,

$\Delta y = f(x + \Delta x) - f(x)$

$= f(2 + 0.2) - f(2)$

$= 2.2^2 + 3 \times 2.2 - 2^2 + 3 \times 2$

$= 1.44.$

Now,

$dy = f'(x)dx$

At $x = 2, \quad \Delta x = dx = 0.2.$

Now,

$dy = (2x + 3) \times 0.2$

$= (2 \times 2 + 3) \times 0.2$

$= 1.4$

$\therefore dy = 1.4, \quad \Delta y = 1.44.$

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2 Soln

Let 'v' denote volume of cube having side 'x' meter.

Then,

$V = x^3 \quad -(1)$

Diff (1) w.r.t x,

$\frac{dv}{dx} = 3x^2$

or, $dv = 3x^2 \cdot dx$ [$dx = \Delta x = 0.02x$]

$= 3x^2 \times 0.02x$

$= 0.06x^3 m^3$ //

3. Soln

Here

$x + \Delta x = 1.99$

$x = 2$

$dx = \Delta x = -0.01$

Now,

$y = f(x) = x^4 - 10$

approximate

For ~~exact~~ change,

$dy = f'(x)dx$

$= 4x^3 \cdot (-0.01)$

$= -0.04x^3$

At $x = 2$

$dy = -0.32$

For ~~approximate~~ exact change,

$\Delta y = f(x + \Delta x) - f(x)$

$= f(1.99) - f(2)$

$= (1.99)^4 - 10 - (2^4 - 10)$

$= -0.317$.

$\therefore \Delta y = -0.317$ and $dy = -0.32$.

Q.4. Soln

Here

$x = 3cm$

$x + \Delta x = 3.01 cm$

$\therefore \Delta x = 0.01$.

Now,
let V be the volume of Sphere,
Then,
Approximate change in its volume is:

$$\frac{dy}{dx} = \frac{d\frac{4}{3}\pi x^3}{dx}$$

$$\therefore dy = \frac{4}{3}\pi 3x^2 \cdot dx$$

$$\text{or } dy = 4\pi x^2 \cdot dx$$

$$= 4\pi \cdot 3^2 \times 0.01$$

$$= 0.36\pi \text{ cm}^3$$

5) Soln

Here
Let l be the edge of cube, then.

$\Delta l = 0.01$ cm

Now,
A be the ~~cur~~ surface area of cube
Then,
approximate change in its area is,

$$\frac{dA}{dl} = \frac{d6l^2}{dl}$$

$$\text{or } dA = 12l \cdot dl$$

$$\text{or, } dA = 12 \times 10 \times 0.01$$

$$= 1.2 \text{ cm}^2.$$

Now,
% error in surface area $= \frac{\Delta y - dy}{y} \times 100\%$

$$= \frac{1.2006 - 1.2}{600} \times 100$$

$= 0.0001$ cm.

where,

$\Delta y = f(10.01) - f(10)$

$= 6(10.01)^2 - 600$

$= 1.2006$ ll.

$\therefore$ % error in surface area $= 0.0001$ cm.

6) Soln

Here,

Let A denote the area of circular copper plate with radius (x) cms.

Then,

$A = \pi r^2$

Also,

When radius increases from 5cm to 5.06 cm,

$x = 5, dx = \Delta x = 0.06$ cm

Now,

$\Delta A = f(5.06) - f(5)$

$= (5.06)^2\pi - \pi \cdot 5^2$

$= 0.6036\pi$.

For approximate change, dA.

$\frac{dA}{dx} = \frac{d\pi r^2}{dx}$

$= \pi 2x$

or, $dA = \pi \cdot 2x \cdot dx$.

$= \pi \times 2 \cdot 5 \times 0.06$

$= 0.6\pi$.

7) The radius of a sphere is found by measurement to be 20cm with a possible error of 0·02 for a centimeter. Find the consequent error in surface.

Solution,

Here,

Let the radius of a sphere be r,

Given,

$r = 20cm$

Also,

$dr = \Delta r = 0.02 cm.$

$r + \Delta r = 20.02 cm.$

We know,

Surface area of sphere $(A) = 4\pi r^2$.

Now,

Diff. both sides w.r.t.r,

$$\frac{dA}{dr} = \frac{4\pi dr^2}{dr}$$

$$\therefore dA = 8\pi r dr.$$

$$= 8\pi r \times 0.02$$

$$= 0.16\pi r$$

When $r = 20cm$,

$$dA = 0.16\pi \times 20$$

$$= 10.05 cm^2.$$

$\therefore$ Consequent error in surface

$= 10.05 cm^2.$