Exercise: 15.6.

1 By using L'Hospital's rule, evaluate:

1(a) $\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9} \left(\frac{0}{0}\right)$

Using L'Hospital's rule,

$\lim_{x \to 3} \frac{3x^2}{2x}$

$= \frac{\cancel{3} \cdot 3^2}{2 \cdot \cancel{3}}$

$= \frac{9}{2}$.

b) $\lim_{x \to a} \frac{x^n - a^n}{x - a} \left(\frac{0}{0}\right)$

$= \lim_{x \to a} \frac{nx^{n-1} \cancel{- na^{n-1}} - 0}{1}$

$= na^{n-1}$.

c) $\lim_{x \to 0} \frac{3x - \sin x}{x} \left(\frac{0}{0}\right)$

Using L'Hospital's rule,

$= \lim_{x \to 0} 3 - \cos x$

$= 3 - \cos 0$

$= 3 - 1$

$= 2$.

1d) $\lim_{x \to \infty} \frac{5x^2 + 4x - 3}{2x^2 - 3x + 5} \left(\frac{\infty}{\infty}\right)$

$= \lim_{x \to \infty} \frac{5 \times 2x + 4}{4x - 3} \left(\frac{\infty}{\infty}\right)$

a) $\lim_{x \to \infty} \frac{5x^2+4x-3}{2x^2-3x+5} \left(\frac{\infty}{\infty}\right)$

Using L'Hopital Rule

$\lim_{x \to \infty} \frac{10x+4}{4x-3} \left(\frac{\infty}{\infty}\right)$

Again,

Using L'Hopital Rule

$\lim_{x \to \infty} \frac{10}{4}$

$= \frac{5}{2}$.

e) $\lim_{x \to 0} \frac{e^x+e^{-x}-2\cos x}{\sin^2 x}, \left(\frac{0}{0}\right)$

$=$ ~~lim~~ Using L'Hopital Rule,

$\lim_{x \to 0} \frac{2(e^x+e^{-x}+2\sin x)}{-\cancel{\sin} \cos 2x}$.

$= \frac{2(e^0+e^{-0}+2\sin 0)}{\cos 2\cdot 0}$

$= \frac{2(1+1+0)}{1}$

$=$

e) $\lim_{x \to 0} \frac{e^x+e^{-x}-2\cos x}{\sin^2 x} \left(\frac{0}{0}\right)$

$= \lim_{x \to 0} \frac{2(e^x+e^{-x}-2\cos x)}{1-\cos 2x} \cdot \left(\frac{0}{0}\right)$.

Using L'Hopital rule,

$= 2 \cdot \lim_{x \to 0} \frac{e^x-e^{-x}+2\sin x}{2\sin 2x} \left(\frac{0}{0}\right)$

$\cancel{= \frac{2}{2} \cdot e^0+e^0+2\sin}$

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Using L'Hopital Rule,

$$\lim_{x\to 0} \frac{e^x + e^{-x} + 2\cos x}{2\cos 2x}$$

$$= \frac{e^0 + e^{-0} + 2\cos 0}{2\cos 2\cdot 0}$$

$$= \frac{1+1+2}{2}$$

$$= \cancel{2}.$$

f. $\lim_{x\to 0} \frac{x-\sin x}{\tan^3 x}, \left(\frac{0}{0}\right)$

Using L'Hopital's rule.

$$\lim_{x\to 0} \frac{1-\cos x}{3\tan^2 x \cdot \sec^2 x} \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{1-\cos x}{3\tan^2 x(1+\tan^2 x)}$$

$$= \lim_{x\to 0} \frac{1-\cos x}{3\tan^2 x + 3\tan^4 x}, \left(\frac{0}{0} \text{ form}\right)$$

Using L'Hopital rule,

$$\lim_{x\to 0} \frac{\sin x}{6\tan x \cdot \sec^2 x + 12\tan^3 x \cdot \sec^2 x}$$

$$= \lim_{x\to 0} \frac{\sin x}{\sec^2 x(6\tan x + 12\tan^3 x)}, \left(\frac{0}{0}\right)$$

Using L'Hopital rule.

$$\lim_{x\to 0} \frac{\cos x}{\sec^2 x(6\sec^2 x + 36\tan^2 x\sec^2 x) + (6\tan x + 12\tan^3 x)\cdot 2\sec^2 x\cdot\tan x}$$

$$= \frac{\cos 0}{\sec^2 0(6\sec^2 0 + 36\tan^2 0\cdot\sec^2 0) + (6\tan 0 + 12\tan^3 0)\cdot 2\sec^2 0\cdot\tan 0}$$

$$= \frac{1}{6+0}$$

$$= \frac{1}{6}$$

g) $\lim_{x \to 0} \frac{x - \tan x}{\tan^3 x} \left(\frac{0}{0}\right)$

Using l' Hopital Rule,

$$\lim_{x \to 0} \frac{1 - \sec^2 x}{3\tan^2 x \sec^2 x}$$

$$= \lim_{x \to 0} \frac{-\tan^2 x}{3\tan^2 x \sec^2 x}$$

$$= \lim_{x \to 0} \frac{-1}{3\sec^2 x}$$

$$= \frac{-1}{3 \cdot \sec^2 0}$$

$$= \frac{-1}{3}$$

h) $\lim_{x \to \frac{\pi}{2}} \frac{\tan 5x}{\tan x}, \left(\frac{\infty}{\infty}\right) = \frac{\cot x}{\cot 5x} \left(\frac{0}{0}\right)$

Taking l'Hopital rule,

$$\lim_{x \to \frac{\pi}{2}} \frac{-\operatorname{cosec}^2 x}{-5 \cdot \operatorname{cosec}^2 5x}$$

$$= \frac{-\operatorname{cosec}^2 \frac{\pi}{2}}{-5 \operatorname{cosec}^2 \frac{5\pi}{2}}$$

$$= \frac{1}{5}$$

1) $\lim_{x \to 0} \frac{(e^x - 1)\tan x}{x^2}, \left(\frac{0}{0}\right)$

Using l'Hopital Rule,

$\lim_{x \to 0} \frac{\tan x e^x + e^x \sec^2 x - \sec^2 x}{2x} \left(\frac{0}{0}\right)$.

Using l'Hopital rule

$\lim_{x \to 0} \frac{e^x \tan x + e^x \sec^2 x + e^x \sec^2 x + 2e^x \sec^2 x \tan x - 2\sec^2 x . \tan x}{2}$

$= \frac{e^0 \tan 0 + e^0 \sec^2 0 + e^0 \sec^2 0 + 2e^0 \sec^2 0 \tan 0 - 2\sec^2 0 \tan 0}{2}$

$= \frac{0 + 1 + 1 + 0 + 0}{2}$

$= 1$. //

1) $\lim_{x \to 0} \frac{\ln \tan x}{\ln x} . \left(\frac{\infty}{\infty}\right)$

Using l'Hopital rule,

$\lim_{x \to 0} . \frac{\frac{1}{\tan x} \sec^2 x}{\frac{1}{x}}$

$= \lim_{x \to 0} 2x \operatorname{cosec} 2x$.

Using l'Hopital rule

$\lim_{x \to 0} 2\operatorname{cosec} 2x +$

j) $\lim_{x \to 0^+} \frac{\ln \tan x}{\ln x} \left(\frac{0}{0}\right)$

Soln

Here

Using L'Hopital rule,

$\lim_{x \to 0^+} \frac{x \sec^2 x}{\tan x} \left(\frac{0}{0}\right)$

Using L'Hopital rule,

$\lim_{x \to 0^+} \frac{\sec^2 x + 2x \sec^2 x \tan x}{\sec^2 x}$

$= \frac{\sec^2 0 + 2 0 \cdot \sec^2 0 \cdot \tan 0}{\sec^2 0}$

$= \frac{1 + 0}{1}$

$= 1$

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20) $\lim_{x \to \infty} \frac{x^4}{e^x}, \left(\frac{\infty}{\infty}\right)$

Using L'Hopital rule

$\lim_{x \to \infty} \frac{4x^3}{e^x} \left(\frac{\infty}{\infty}\right)$

Using L'Hopital rule

$\lim_{x \to \infty} \frac{12x^2}{e^x} \left(\frac{\infty}{\infty}\right)$

Using L'Hopital rule

$\lim_{x \to \infty} \frac{24x}{e^x} \left(\frac{\infty}{\infty}\right)$

Using L'Hopital rule

$\lim_{x \to \infty} \frac{24}{e^x}$

$= \frac{24}{e^\infty} = \frac{24}{\infty} = 0.$

b) $\lim_{x \to +\infty} \frac{\ln(x^2+1)}{\ln(x^3+1)}, \left(\frac{\infty}{\infty}\right)$

Using L Hopital rule

$\lim_{x \to +\infty} \frac{(x^3+1)2\cancel{x}}{(x^2+1)3x^{\cancel{2}}}$

$= \lim_{x \to +\infty} \frac{2x^3+2}{3x^3+3x}$

$= \lim_{x \to +\infty} \frac{\cancel{x^3}\left(2+\frac{2}{x^3}\right)}{\cancel{x^3}\left(3+\frac{3}{x^2}\right)}$

$= \frac{2+\frac{2}{\infty}}{3+\frac{3}{\infty}}$

$= \frac{2}{3}$

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c) $\lim_{x \to 0^+} x^x$

Soln

let $y = x^x$

taking ln on both sides,

$\ln y = x \ln x$

Taking lim $x \to 0^+$ on both sides

$\lim_{x \to 0^+} \ln y = \lim_{x \to 0} x \ln x$

or, $\ln y = \lim_{x \to 0} \frac{\ln x}{\frac{1}{x}} \left(\frac{\infty}{\infty}\right)$

Using L Hopital's rule,

$= \lim_{x \to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$

$$= \lim_{x \to 0^+} -x.$$

$$= 0.$$

$\therefore \ln y = 0$

or $\ln y = \ln 1$

$\therefore y = 1.$

$\lim_{x \to 0} x^x = 1.$

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a) $\lim_{x \to 0^+} \sin x \ln x^2$

$$= \lim_{x \to 0^+} \frac{2 \ln x}{\operatorname{cosec} x} \left(\frac{\infty}{\infty}\right)$$

Using L'Hopital Rule,

$$\lim_{x \to 0^+} \frac{\frac{2}{x}}{-\operatorname{cosec} x \cot x}$$

$$= -2 \lim_{x \to 0^+} \frac{\sin x \tan x}{x}$$

$$= -2 . \lim_{x \to 0^+} \frac{\sin x}{x} \lim_{x \to 0^+} \tan x$$

$$= -2 \times 1 \times 0$$

$$= 0 \,//$$

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2d) $\lim_{x \to 0} \left[\frac{1}{x^2} - \frac{1}{\sin^2 x}\right]$ $(\infty - \infty)$ form

$= \lim_{x \to 0} \frac{\sin^2 x - x^2}{x^2 \sin^2 x} \left(\frac{0}{0}\right)$

Using L'Hopital Rule.

$\lim_{x \to 0} \frac{\sin 2x - 2x}{\sin^2 x \cdot 2x + x^2 \sin x} \left(\frac{0}{0}\right)$

Using L'Hopital rule

$\lim_{x \to 0} \frac{2\cos 2x - 2}{2x\sin 2x + 2\sin^2 x + 2x\sin 2x + x^2 2\cos 2x}$

$\left(\frac{0}{0}\right)$ form

$= \lim_{x \to 0} \frac{2\cos 2x - 2}{4x\sin 2x + 2\sin^2 x + 2x^2\cos 2x} \left(\frac{0}{0}\right)$

Using L'Hopitalrule

$\lim_{x \to 0} \frac{-4\sin 2x}{6\sin 2x + 12x\cos 2x - 4x^2 \sin 2x} \left(\frac{0}{0}\right)$

Using L'Hopitalrule,

$\lim_{x \to 0} \frac{-8\cos 2x}{12\cos 2x + 12(\cos 2x + x(-2\sin 2x)) - 4(\sin 2x \cdot (2x) + x^2 2\cos 2x)}$

$= \frac{-8 \times 1}{12 \times 1 + 12(1 + 0) - 4(0 + 0)}$

$= \frac{-8}{24} = \frac{-1}{3}$ //