Extra Questions

a) $2+5+10+17+26+\cdots$

Solution

Given series is

$2+5+10+17+26+\cdots$

Let t_n be the n^{th} term and S_n be the sum of n terms then.

$$S_n = 2+5+10+17+26+\cdots+t_n$$
$$-S_n = \quad -2-5-10-17-\cdots-t_{n-1}-t_n$$
$$0 = 2+3+5+7+9+\cdots+(t_n-t_{n-1})-t_n$$

or, $t_n = 2+3+5+7+9+\cdots+ \cancel{t}\ n^{th}$ term

$= 2 + \frac{n-1}{2}[2\times3+((n-1)-1).2]$

$= 2+\frac{n-1}{2}[6+(2n-4)]$

$= 2+\frac{n-1}{2}[2n+2]$

$= 2+n^2-1$

$= n^2+1$

$\therefore t_n = n^2+1$

Now, $S_n = \Sigma t_n$

$= \Sigma(n^2+1)$

$= \Sigma n^2 + \Sigma 1$

$= \frac{n(n+1)(2n+1)}{6} + n$

$= \frac{n[(n+1)(2n+1)+6]}{6}$

b) $3+5+8+12+17+ ----$

Solution

Given series is:

$3+5+8+12+17+ ---$

Let t_n be the nth term and S_n be the sum of n terms.

Then,

$S_n = 3+5+8+12+17+ --- + t_n$

$-S_n = \quad -3-5-8-12- ...... -t_{n-1}-t_n$

$0 = 3+2+3+4+5+ ... +(t_n-t_{n-1})-t_n$

or, $t_n = 3+2+3+4+5+ ---- + n^{th}$ term

$= 3+\frac{n-1}{2}[2\times2+(n-1)(n-1-1)1]$

$= 3+\frac{n-1}{2}[4+n-2]$

$= 3+\frac{n-1}{2}(2+n)$

$$= \frac{6+2n+n^2-2-n}{2}$$

$$= 2+\frac{n}{2}+\frac{n^2}{2}$$

Now,

$$S_n = \Sigma t_n$$

$$= \Sigma\left(2+\frac{n}{2}+\frac{n^2}{2}\right)$$

$$= \Sigma 2 + \frac{1}{2}\Sigma n + \frac{1}{2}\Sigma n^2$$

$$= 2n + \frac{1}{2}\frac{n(n+1)}{2} + \frac{1}{2}\frac{n(n+1)(2n+1)}{6}$$

$$= \frac{24n+3n(n+1)+n(n+1)(2n+1)}{12}$$

$$\therefore S_n = \frac{n(24+3(n+1)+(n+1)(2n+1))}{12}$$

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c) $2+4+8+14+22+---$

Solution

Given series is:

$2+4+8+14+22+---$

Let t_n be the n^{th} term and S_n be the sum of n terms then

$$S_n = 2+4+8+14+22+\ldots\ldots+t_n$$

$$-S_n = 2+4+8+14+---+t_{n-1}+t_n$$

$$0 = 2+2+4+6+8+----+(t_n-t_{n-1})-t_n$$

$$\text{or; } t_n = 2+\frac{n-1}{2}(2\times2+(n-1-1)2)$$

$$= 2+\frac{n-1}{2}(4+2n-4)$$

$$\therefore t_n = 2+n^2-2$$

Now,

$$S_n = \Sigma t_n$$

$$= \Sigma(2+n^2-n)$$

$$= 2n + \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$$= \frac{12n + n(n+1)(2n+1) - 3n(n+1)}{6}$$

$$\therefore S_n = \frac{n(12 + (n+1)(2n+1) - 3(n+1))}{6}$$

d) $3+7+13+21+31+ - - - -$

Solution

Given series is:

$3+7+13+21+31+ \cdots$

Let t_n be the nth term and S_n be the sum of nth terms.

Then,

$$S_n = 3+7+13+21+31+ - - - + t_n$$

$$-S_n = \quad 3 + 7 + 13 + 21 + - - - + t_{n-1} + t_n$$

$$0 = 3+4+6+8+10+ - - - + (t_n - t_{n-1}) - t_n$$

or, $t_n = 3+4+6+8+10+ - - - + t_n$

or; $t_n = 3 + \frac{n-1}{2}[2.4 + (n-1-1)2]$

$$= 3 + \frac{n-1}{2}[8+2n-4]$$

$$= 3 + \frac{n-1}{2}[4+2n]$$

$$= 3 + \frac{4n+2n^2-4-2n}{2}$$

$$= 3 + \frac{2n^2+2n-4}{2}$$

$$= 3 + n^2 + n - 2$$

$$\therefore t_n = n^2+n+1$$

Now,

$$S_n = \Sigma t_n$$

$$= \Sigma(n^2+n+1)$$

$$= \Sigma n^2 + \Sigma n + \Sigma 1$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + n$$

$= \frac{n(n+1)(2n+1) + 3n(n+1) + 6n}{6}$

$\therefore S_n = \frac{n(n+1)(2n+1) + 3(n+1) + 6}{6}$

e) $2+5+10+17+26+---$

Solution

Given series is:

$2+5+10+17+26+---$

let t_n be the n^{th} term and S_n be the sum of n terms.

Then,

$S_n = 2+5+10+17+26+---+t_n$

$-S_n = \pm 2 \pm 5 \pm 10 \pm 17 \pm --- \pm t_{n-1} \pm t_n$

$0 = 2+3+5+7+9+---+(t_n - t_{n-1}) - t_n$

or, $t_n = 2+3+5+7+9+---+t_n$

or, $t_n = 2 + \frac{n-1}{2}[2\times 3+(n-1-1)2]$

or; $t_n = 2+\frac{n-1}{2}[6+(n-2)2]$

$= 2+\frac{n-1}{2}[2+2n]$

$= 2+(n-1)(n+1)$

$= 2+n^2-1$

$\therefore t_n = 1+n^2$

Now,

$S_n = \Sigma t_n$

$= \Sigma(1+n^2)$

$= \Sigma 1 + \Sigma n^2$

$= n + \frac{n(n+1)(2n+1)}{6}$

$\therefore S_n = \frac{n(6+(n+1)(2n+1))}{6}$

b) $5+11+19+29+41+\cdots$

Solution:

Given series is:

$5+11+19+29+41+\cdots$

Let t_n be the n^{th} term and S_n be the sum of n terms.

$S_n = 5+11+19+29+41+\cdots+t_n$

$-S_n = \quad +5+11+19+29+\cdots+t_{n-1}+t_n$

$0 = 5+6+8+10+12+\cdots+(t_{n-1}-t_n)-t_n$

or, $t_n = 5+6+8+10+12+\cdots+t_n$

$= 5+\frac{(n-1)}{2}[2\times6+(n-1-1)2]$

$= 5+(n-1)[6+n-2]$

$= 5+(n-1)(4+n)$

$= 5+(4n+n^2-4-n)$

$= 5+(n^2+3n-4)$

$\therefore t_n = n^2+3n+1$

Now,

$S_n = \Sigma t_n$

$= \Sigma(n^2+3n+1)$

$= \Sigma n^2+3\Sigma n+\Sigma 1$

$= \frac{n(n+1)(2n+1)}{6}+\frac{3n(n+1)}{2}+n$

$= \frac{n(n+1)(2n+1)+9n(n+1)+6n}{6}$

$\therefore S_n = \frac{n[(n+1)(2n+1)+9(n+1)+6]}{6}$

9) $5+7+13+31+85+ \dots$

Solution

Given series is:

$5+7+13+31+85+ \dots$

Let t_n be the nth term and S_n be the sum of n terms, then

$$S_n = 5+7+13+31+85+ \dots + t_n$$
$$-S_n = \cancel{\pm 5} \pm 5 \pm 7 \pm 13 \pm 31 \pm \dots + t_{n-1} \pm t_n$$
$$0 = 5+2+6+18+54+ \dots + (t_n - t_{n-1}) - t_n$$

or, $t_n = 5+2+6+18+54+ \dots +$ nth term

$$= 5 + \frac{2[(3^{n-1}) - 1]}{3-1}$$

$$= 5 + \frac{2(3^{n-1} - 1)}{2}$$

$$= 5 + 3^{n-1} - 1$$

$$\therefore t_n = 4 + 3^{n-1}$$

Now,

$$S_n = \cancel{\Sigma t_n}\ \Sigma t_n$$
$$= \Sigma(3^{n-1} + 4)$$
$$= \Sigma 3^{n-1} + \Sigma 4$$
$$= (3^0 + 3^1 + 3^2 + \dots + 3^{n-1}) + 4n$$
$$= (1 + 3 + 3^2 + \dots + 3^{n-1}) + 4n$$
$$= \frac{1(3^n - 1)}{3-1} + 4n$$

$$\therefore S_n = \frac{1}{2}(3^n - 1) + 4n$$

h) $2+5+14+41+\dots$

Solution

Given series is:

$2+5+14+41+\dots$

Let t_n be the nth term and S_n be the sum of n terms.

Then,

$$S_n = 2+5+14+41+\dots+t_n$$
$$-S_n = \quad +2+5+14+\dots+t_{n-1}+t_n$$
$$0 = 2+3+9+27+\dots+(t_{n-1}-t_n)-t_n$$

or; $t_n = 2+3+9+27+\dots+ n$

$$= 2+\frac{3(3^{n-1}-1)}{3-1}$$

$$= \frac{4+3^{n-1+1}-3}{2}$$

$$= \frac{4+3^n-3}{2}$$

$$\therefore t_n = \frac{1+3^n}{2}$$

Now,

$$S_n = \Sigma t_n$$

$$= \Sigma\left(\frac{1}{2}+\frac{3^n}{2}\right)$$

$$= \Sigma\frac{1}{2}+\frac{1}{2}\Sigma 3^n$$

$$= \frac{n}{2}+\frac{1}{2}(3+9+27+\dots+3^n)$$

$$= \frac{n}{2}+\frac{1}{2}\,\frac{3(3^n-1)}{2}$$

$$= \frac{n}{2}+\frac{3(3^n-1)}{4}$$

$$\therefore S_n = \frac{2n+3(3^n-1)}{4}$$