Mathematical Induction

Two ways of proving results:

(i) Inductive method

- Particular to general
- Simple to complex

(ii) Deductive Method

- General to particular
- Complex to simple.

$$(a+b)^2 = (a+b)(a+b)$$
$$= a^2+ab+ab+b^2 \text{ (Inductive)}$$
$$= a^2+2ab+b^2$$

$$(2x+3y)^2 = (2x)^2 + 2.2x.3y + (3y)^2 \text{ (Deductive)}$$

Principle of mathematical Induction

Statement: Let $P(n)$: $n \in N$ be a statement such that

(i) $P(1)$ is true

(ii) $P(k+1)$ is true whenever $P(k)$ is true

Then, $P(n)$ is true for all $n \in N$.

Procedure

(i) Suppose that $P(n)$ be the given problem/statement.

(ii) Show that $P(n)$ is true for $n=1$ i.e. $P(1)$ is true.

(iii) Assume that $P(n)$ is true for $n=k$ i.e. $P(k)$ is true

(iv) Show that $P(k+1)$ is true whenever $P(k)$ is true.

Conclusion: Hence by principle of mathematical induction $P(n)$ is true for all $n \in N$.

Exercise 6.2

1. a) If $P(n)$ is the statement "n^3+n is divisible by 2." Prove that $P(1), P(2), P(3)$ and $P(4)$ are true.

Solution

$P(n) = n^3+n$ is divisible by 2.

$P(1) = 1^3+1 = 1+1 = 2$

$P(2) = 2^3+2 = 8+2 = 10$

$P(3) = 3^3+3 = 27+3 = 30$

$P(4) = 4^3+4 = 64+4 = 68$

Hence, we see that each of $P(1), P(2),$ ~~&~~ $P(3)$ and $P(4)$ is divisible by 2. Hence, $P(1), P(2), P(3),$ and $P(4)$ are true.

b) If $P(n)$ is ~~a~~ the statement "n^2+n is even". Prove that $P(1), P(2), P(3)$ and $P(4)$ are true.

Solution

$P(n) = n^2+n$ is even

$P(1) = 1^2+1 = 1+1 = 2$

$P(2) = 2^2+2 = 4+2 = 6$

$P(3) = 3^2+3 = 9+3 = 12$

$P(4) = 4^2+4 = 16+4 = 20$

Here, we can see that each of $P(1), P(2), P(3)$ and $P(4)$ is even. Here, $P(1), P(2), P(3)$ and $P(4)$ are true.

c) If $P(n)$ is the statement "$n^3 \geq 2^n$" show that $P(1)$ is false and $P(2), P(3)$ are true.

Solution.

$P(n) = n^3 \geq 2^n$

$P(1) = 1^3 > 2^1 = 1 > 2$ which is false

$P(2) = 2^3 > 2^2 = 8 > 4$ which is true

$P(3) = 3^3 > 2^3 = 27 > 8$ which is true

Hence, $P(1)$ is false and $P(2), P(3)$ are true.

d) Let $P(n)$ denote the statement " $\frac{n(n+1)}{6}$ is a natural number". Show that $P(2)$ and $P(3)$ are true but $P(4)$ is not true.

Solution

$P(n) = \frac{n(n+1)}{6}$ is a natural number

$P(2) = \frac{2(2+1)}{6} = \frac{2 \times 3}{6} = 1$

$P(3) = \frac{3(3+1)}{6} = \frac{3 \times 4}{6} = 2$

$P(4) = \frac{4(4+1)}{6} = \frac{20}{6} = \frac{10}{3}$

Here, we see that $P(4)$ is not a natural number where $P(2)$ and $P(3)$ are a natural number. Hence, $P(2)$ and $P(3)$ are true, $P(4)$ is not true.

2. Prove by the method of induction that :

a. $2+5+8+\dots+(3n-1) = \frac{n(3n+1)}{2}$

Solution

Let $P(n) = 2+5+8+\dots+(3n-1) = \frac{n(3n+1)}{2}$

When $n=1$

LHS $= 2$

RHS $= \frac{1.(3.1+1)}{2}$

$= \frac{4}{2}$

$= 2$

$\therefore$ LHS = RHS

thus, P(1) is true

Assume that P(n) is true for n = K

i.e. $P(K) = 2+5+8+\dots+(3K-1) = \frac{K(3K+1)}{2}$

We will show that P(K+1) is true whenever P(K) is true.

i.e. $P(K+1) = 2+5+8+\dots+(3K-1) + \cancel{3(K+1}\ \cancel{3(K}\ (3(K+1)-1) = \frac{(K+1)(3(K+1)+1)}{2}$

Now,

$2+5+8+\dots+(3K-1)+(3(K+1)-1)$

$= \frac{K(3K+1)}{2} + (3K+3-1)$

$= \frac{3K^2+K}{2} + (3K+2)$

$= \frac{3K^2+K+6K+4}{2}$

$= \frac{3K^2+7K+4}{2}$

$= \frac{3K^2+3K+4K+4}{2}$

$= \frac{3K(K+1)+4(K+1)}{2}$

$= \frac{(K+1)(3K+4)}{2}$

this shows that ~~P(K)~~ P(K+1) is true whenever P(K) is true. Hence by principle of mathematical induction P(n) is true for all $n \in N$.

b. $1^2+3^2+5^2+\dots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

Solution

Let, $P(n) = 1^2+3^2+5^2+\dots+(2n-1)^2$

$= \frac{n(2n-1)(2n+1)}{3}$

when $n=1$

LHS $= 1$

RHS $= \frac{1\times(2\times1-1)(2\times1+1)}{3}$

$= \frac{3}{3}$

$= 1$

$\therefore$ LHS = RHS

Thus, $P(1)$ is true,

Assume that $P(n)$ is true for $n=k$.

i.e. $P(k) = 1^2+3^2+5^2+\dots+(2k-1)^2$

$= \frac{k(2k-1)(2k+1)}{3}$

We will show that $P(k+1)$ is true whenever $P(k)$ is true.

i.e. $P(k+1) = 1^2+3^2+5^2+\dots+(2k-1)^2+(2(k+1)-1)^2 =$

$\frac{(k+1)(2(k+1)-1)(2(k+1)+1)}{3}$

Now,

$1^2+3^2+5^2+\dots+(2k-1)^2+(2(k+1)-1)^2$

$= \frac{k(2k-1)(2k+1)}{3} + (2k+2-1)^2$

$= \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2$

$= \frac{k(2k-1)(\cancel{3k}\ 2k+1) + 3(2k+1)^2}{3}$

$= \frac{(2k+1)(k(2k-1)+3(2k+1))}{3}$

$= \dfrac{(2k+1)(2k^2-k+6k+3)}{3}$

$= \dfrac{(2k+1)(2k^2+5k+3)}{3}$

$= \dfrac{(2k+1)(2k^2+3k+2k+3)}{3}$

$= \dfrac{(2k+1)(k(2k+3)+1(2k+3))}{3}$

$= \dfrac{(2k+1)(k+1)(2k+3)}{3}$

$= \dfrac{(k+1)(2(k+1)-1)(2(k+1)+1)}{3}$

This shows that $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by principle of mathematical induction, $P(n)$ is true for all $n \in N$.

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c. $4+8+12+\dots+4n = 2n(n+1)$

Solution

Let, $P(n) = 4+8+12+\dots+4n = 2n(n+1)$

When $n=1$,

$LHS = 4$

$RHS = 2\times1(1+1)$

$= 2\times2$

$= 4$

$\therefore LHS = RHS$

Thus, $P(1)$ is true

Assume that $P(n)$ is true for $n=k$.

i.e. $P(k) = 4+8+12+\dots+4k = 2k(k+1)$

We will show that $P(k+1)$ is true whenever $P(k)$ is true.

i.e. $P(k+1) = 4+8+12+\dots+4k+4(k+1) = 2(k+1)((k+1)+1)$

Now,

$4+8+12+\dots+4k+4(k+1)$

$= 2k(k+1)+4(k+1)$

$= 2k^2 + 2k + 4k + 4$

$= \cancel{2k^2 + 6k + 4}$

$= \cancel{2k^2 + 2 +} \; 2k(k+1) + 4(k+1)$

$= (k+1)(2k+4)$

$= (k+1)\,2(k+2)$

$= 2(k+1)(k+2)$

$= 2(k+1)((k+1)+1)$

This, shows that P(k+1) is true whenever P(k) is true.

Hence by principle of mathematical induction, P(n) is true for all $n \in N$.

d. $1+4+7+\dots+(3n-2) = \frac{n(3n-1)}{2}$

Solution

Let $P(n) = 1+4+7+\dots+(3n-2) = \frac{n(3n-1)}{2}$

When $n = 1$

LHS = 1

$RHS = \frac{1(3\times1-1)}{2} = \frac{2}{2} = 1$

$\therefore$ LHS = RHS

Thus, P(1) is true.

Assume that, P(n) is true for n = k.

i.e. $P(k) = 1+4+7+\dots+(3k-2) = \frac{k(3k-1)}{2}$

We will show P(k+1) is true whenever P(k) is true.

i.e.

$P(k+1) = 1+4+7+\dots+(3k-2)+(3(k+1)-2) = \frac{(k+1)(3(k+1)-1)}{2}$

Now,

$1+4+7+\dots+(3k-2)+(3(k+1)-2)$

$= \frac{k(3k-1)}{2} + (3k+3-2)$

$= \frac{k(3k-1)}{2} + (3k+1)$

$= \frac{3k^2 - k + 2(3k+1)}{2}$

$= \frac{3k^2 - k + 6k + 2}{2}$

$= \frac{3k^2 + 5k + 2}{2}$

$= \frac{3k^2 + 3k + 2k + 2}{\cancel{3}2}$

$= \frac{3k(k+1) + 2(k+1)}{2}$

$= \frac{(k+1)(3k+2)}{2}$

$= \frac{(k+1)(3(k+1)-1)}{2}$

This shows that P(k+1) is true whenever P(k) is true. Hence by principle of mathematical induction P(n) is true for all $n \in N$.

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e. $1.2 + 2.3 + 3.4 + \ldots$ to n terms $= \frac{n(n+1)(n+2)}{3}$

Solution

nth term of $1.2 + 2.3 + 3.4 \ldots = n(n+1)$

Let $P(n) = 1.2 + 2.3 + 3.4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

When $n = 1$

LHS $= 2$

RHS $= \frac{1(1+1)(1+2)}{3} = 2$

$\therefore$ LHS = RHS

Thus, P(1) is true.

Assume that $P(n)$ is true for $n=k$.

i.e. $P(k) = 1.2+2.3+3.4+\dots+k(k+1)$

$$= \frac{k(k+1)(k+2)}{3}$$

We will show $P(k+1)$ is true whenever $P(k)$ is true.

i.e. $P(k+1) = 1.2+2.3+3.4+\dots+k(k+1)+(k+1)(k+2) = \frac{(k+1)((k+1)+1)((k+1)+2)}{3}$

Now,

$1.2+2.3+3.4+\dots+k(k+1)+(k+1)(k+2)$

$$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$

$$= (k+1)(k+2)\left[\frac{k}{3}+1\right]$$

$$= \frac{(k+1)(k+2)(k+3)}{3}$$

This shows that $P(k+1)$ is true whenever $P(k)$ is true. Hence by principle of mathematical induction $P(n)$ is true for all $n \in N$.

3. Prove by the method of induction that :

a. $$\frac{1}{1.3}+\frac{1}{3.5}+\dots+\frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

Solution

Let $P(n) = \frac{1}{1.3}+\frac{1}{3.5}+\dots+\frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$

When $n=1$

$$L.H.S = \frac{1}{1.3}$$

$$= \frac{1}{3}$$

$$RHS = \frac{1}{2.1+1} = \frac{1}{3}$$

$\therefore$ LHS = RHS

Thus, P(1) is true

Assume that P(n) is true for n=K

i.e. $P(K) = \frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \dots + \frac{1}{(2K-1)(2K+1)} = \frac{K}{2K+1}$

We will show P(K+1) is true whenever P(K) is true.

i.e. $P(K+1) = \frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \dots + \frac{1}{(2K-1)(2K+1)} +$

$$\frac{1}{(2(K+1)-1)(2(K+1)+1)} = \frac{K+1}{2(K+1)+1}$$

Now,

$$\frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \dots + \frac{1}{(2K-1)(2K+1)} + \frac{1}{(2(K+1)-1)(2(K+1)+1)}$$

$$= \frac{K}{2K+1} + \frac{1}{(2K+2-1)(2K+2+1)}$$

$$= \frac{K}{2K+1} + \frac{1}{(2K+1)(2K+3)}$$

$$= \frac{1}{2K+1}\left[K + \frac{1}{2K+3}\right]$$

$$= \frac{1}{2K+1}\left[\frac{2K^2+3K+1}{2K+3}\right]$$

$$= \frac{1}{2K+1}\left[\frac{2K(K+1)+1(K+1)}{(2K+3)}\right]$$

$$= \frac{1}{(2K+1)}\,\frac{(2K+1)(K+1)}{(2K+3)}$$

$$= \frac{K+1}{2K+3}$$

This shows that P(K+1) is true, whenever P(K) is true.
Hence, by principle of mathematical induction P(n) is true for all $n \in N$.

b. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

Solution

Let, $P(n) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

When $n = 1$

$LHS = \frac{1}{2}$

$RHS = 1 - \frac{1}{2^1}$

$= \frac{2-1}{2} = \frac{1}{2}$

$\therefore LHS = RHS$

Thus, P(1) is true

Assume that P(n) is true for n=K.

i.e. $P(K) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^K} = 1 - \frac{1}{2^K}$

We will show P(K+1) is true whenever P(K) is true,

i.e. $P(K+1) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^K} + \frac{1}{2^{K+1}} = 1 - \frac{1}{2^{K+1}}$

Now,

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^K} + \frac{1}{2^{K+1}}$

$= 1 - \frac{1}{2^K} + \frac{1}{2^{K+1}}$

$= 1 - \frac{1}{2^K} + \frac{1}{2^K \cdot 2}$

$= 1 - \frac{1}{2^K}\left[1 - \frac{1}{2}\right]$

$= 1 - \frac{1}{2^K} \cdot \frac{1}{2}$

$= 1 - \frac{1}{2^{k+1}}$

This shows that P(k+1) is true whenever P(k) is true.
Hence by principle of mathematical induction P(n) is true for all $n \in N$.

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c. $2 + 2^2 + \cdots + 2^n = 2(2^n - 1)$

Solution

Let, $P(n) = 2 + 2^2 + \cdots + 2^n = 2(2^n - 1)$

When $n = 1$

LHS = 2

$RHS = 2(2^1 - 1)$

$= 2 \times 1$

$= 2$

$\therefore$ LHS = RHS

Thus, P(1) is true.

Assume that P(n) is true for n=k.

i.e. $P(k) = 2 + 2^2 + \cdots + 2^k = 2(2^k - 1)$

We will show P(k+1) is true whenever P(k) is true.

i.e. $P(k+1) = 2 + 2^2 + \cdots + 2^k + 2^{k+1} = 2(2^{k+1} - 1)$

Now,

$2 + 2^2 + \cdots + 2^k + 2^{k+1}$

$= 2(2^k - 1) + 2^{k+1}$

$= 2^{k+1} - 2 + 2^{k+1}$

$= 2.2^{k+1} - 2$

$= 2(2^{k+1} - 1)$

This shows that P(k+1) is true whenever P(k) is true.
Hence, by principle of mathematical induction P(n) is true for all $n \in N$.

d. $3 + 3^2 + \dots + 3^n = \frac{3(3^n - 1)}{2}$

Solution.

Let $P(n) = 3 + 3^2 + \dots + 3^n = \frac{3(3^n - 1)}{2}$

When $n = 1$

$LHS = 3$

$RHS = \frac{3(3^1 - 1)}{2}$

$= \frac{3.2}{2} = 3$

$\therefore LHS = RHS$

Thus, $P(1)$ is true,

Assume that $P(n)$ is true for $n = K$

i.e. $P(K) = 3 + 3^2 + \dots + 3^K = \frac{3(3^K - 1)}{2}$

We will show $P(K+1)$ is true whenever $P(K)$ is true

i.e. $P(K+1) = 3 + 3^2 + \dots + 3^K + 3^{K+1} = \frac{3(3^{K+1} - 1)}{2}$

Now,

$3 + 3^2 + \dots + 3^K + 3^{K+1}$

$= \frac{3(3^{K+1})}{2} + 3^{K+1}$

$= \frac{3^{K+1} - 3 + 2 \cdot 3^{K+1}}{2}$

$= \frac{3 \cdot 3^{K+1} - 3}{2}$

$= \frac{3(3^{K+1} - 1)}{2}$

This shows that $P(K+1)$ is true whenever $P(K)$ is true.

Hence, by principle of mathematical induction $P(n)$ is true for all $n \in N$.

e. $\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \ldots$ to n terms $= \frac{1}{4}\left(1 - \frac{1}{5^n}\right)$

Solution

Let $P(n) = \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \cdots + \frac{1}{5^n} = \frac{1}{4}\left(1 - \frac{1}{5^k}\right)$

When $n = 1$

$LHS = \frac{1}{5}$

$RHS = \frac{1}{4}\left(1 - \frac{1}{5}\right)$

$= \frac{1}{4} \cdot \frac{5-1}{5}$

$= \frac{1}{4} \cdot \frac{4}{5}$

$= \frac{1}{5}$

$\therefore$ LHS = RHS

Thus, P(1) is true

Assume that P(n) is true for n = k.

i.e. $P(k) = \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \cdots + \frac{1}{5^k} = \frac{1}{4}\left(1 - \frac{1}{5^k}\right)$

Now,

$\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \ldots \frac{1}{5^k} + \frac{1}{5^{k+1}}$

$= \frac{1}{4}\left(1 - \frac{1}{5^k}\right) + \frac{1}{5^{k+1}}$

$= \frac{1}{4} - \frac{1}{4.5^k} + \frac{1}{5^k.5}$

$= \frac{1}{4} - \frac{1}{5^k}\left(\frac{1}{4} + \frac{1}{5}\right)$

$= \frac{1}{4}\left(1 - \frac{1}{5^k.5}\right)$

$= \frac{1}{4}\left(1 - \frac{1}{5^{k+1}}\right)$

4. Prove by the method of induction that:

a) $4^n - 1$ is divisible by 3.

Solution

Let $P(n) = 4^n - 1$ is divisible by 3

When $n = 1$

$4^1 - 1 = 4 - 1 = 3$ which is divisible by 3.

$\therefore P(1)$ is true

Assume that $P(n)$ is true for $n = K$

i.e. $P(K) = 4^K - 1$ is divisible by 3.

We will show that $P(K+1)$ is true whenever $P(K)$ is true.

i.e. $P(K+1) = 4^{K+1} - 1$ is divisible by 3.

Now,

$$4^{K+1} - 1 = 4^K . 4^1 - 1$$
$$= 4^K . 4 - 4 + 4 - 1$$
$$= 4(4^K - 1) + 3$$

which is divisible by 3 as $(4^K - 1)$ and 3 are divisible by 3.

This shows that $P(K+1)$ is true whenever $P(K)$ is true.

Hence, by principle of mathematical induction $P(n)$ is true for all $n \in N$.

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b) $3^{2n} - 1$ is divisible by 8.

Solution

Let $P(n) = 3^{2n} - 1$ is divisible by 8.

When $n = 1$

$3^{2\times1} - 1 = 3^2 - 1 = 8$ which is divisible by 8.

$\therefore P(1)$ is true.

Assume that $P(n)$ is true for $n = K$

i.e. $P(K) = 3^{2K} - 1$ is divisible by 8.

We will show that $P(K+1)$ is true whenever $P(K)$ is true.

i.e. $P(K+1) = 3^{2(K+1)} - 1$ is divisibe by 8.
Now,
$$3^{2(K+1)} - 1 = 3^{2K+2} - 1$$
$$= 3^{2k} \cdot 3^2 - 1$$
$$= 3^{2k} \cdot 9 - 1$$
$$= 3^{2k} \cdot 9 - 9 + 9 - 1$$
$$= 9(3^{2k} - 1) + 8$$
which is divisible by 8 as $(3^{2k} - 1)$ and 8 is divisible by 8.
This show that $P(K+1)$ is true whenever $P(K)$ is true.
Hence by principle of mathematical induction $P(n)$ is true for all $n \in N$.

c) $10^{2n-1} + 1$ is divisible by 11.
Solution
Let $P(n) = 10^{2n-1} + 1$ is divisible by 11.
When $n = 1$
$10^{2\times1-1} + 1 = 10^{2-1} + 1 = 11$ which is divisible by 11.
$\therefore P(1)$ is true
Assume that $P(n)$ is true for $n = k$
i.e. $P(k) = 10^{2k-1} + 1$ is divisible by 11.
We will show that $P(K+1)$ is true whenever $P(k)$ is true.
i.e. $P(K+1) = 10^{2(K+1)-1} + 1$ is divisible by 11.
Now,
$$10^{2(K+1)-1} + 1 = 10^{2K+2-1} + 1$$
$$= 10^{2K-1} \cdot 10^2 + 1$$
$$= 10^{2k-1} \cdot 100 + 100 - 100 + 1$$
$$= 100(10^{2k-1} + 1) - 99$$
which is divisible by 11 as $10^{2k-1} + 1$ and 99 is divisible by 11.

This shows that $P(k+1)$ is true whenever $P(k)$ is true.
Hence, by principle of mathematical induction, $P(n)$ is true for all $n \in N$.

d. $x^n - y^n$ is divisible by $x-y$.

Solution

Let $P(n) = x^n - y^n$ is divisible by $x-y$

When $x=1$

$x^1 - y^1 = x-y$ which is divisible by $x-y$

$\therefore$ P(1) is true

Assume that P(k) is true for $n=k$.

i.e. $P(k) = x^k - y^k$ is divisible by $x-y$

we will show that P(k+1) is true whenever P(k) is true.

i.e. $P(k+1) = x^{k+1} - y^{k+1}$ is divisible by $x-y$

Now,

$$
\begin{aligned}
x^{k+1} - y^{k+1} &= x^k \cdot x - y^k \cdot y \\
&= x^k \cdot x - xy^k + xy^k - y^k \cdot y \\
&= x(x^k - y^k) + y^k(x-y)
\end{aligned}
$$

Which is divisible by $(x-y)$ since $(x^k - y^k)$ and $(x-y)$ are divisible by $(x-y)$.

This shows that P(k+1) is true whenever P(k) is true.

Hence, by principle of mathematical induction P(n) is true for all $n \in N$.

e. $n(n+1)(n+2)$ is a multiple of 6.

Solution

Let $P(n) = n(n+1)(n+2)$ is a multiple of 6.

When $n=1$

$$
\begin{aligned}
n(n+1)(n+2) &= 1(1+1)(1+2) \\
&= 2 \times 3 \\
&= 6 \text{ which is multiple of 6.}
\end{aligned}
$$

$\therefore$ P(1) is true.

Assume that P(k) is true for $n=k$.

i.e. $P(k) = k(k+1)(k+2)$ is divisible by 6.
we will show that $P(k+1)$ is true whenever $P(k)$ is true.
i.e. $P(k+1) = (k+1)((k+1)+1)((k+1)+2)$
Now,

$(k+1)(k+2)(k+3)$
$= k(k+1)(k+2) + 3(k+1)(k+2)$
Which is multiple of 6 as $k(k+1)(k+2)$ and $3(k+1)(k+2)$ are multiple of 6.

[$\because (k+1)$ or $(k+2)$ is even. so, $3(k+1)(k+2)$ is multiple of 6.
$3 \times \text{even} \Rightarrow 6$ multiple of 6]

$\therefore P(k+1)$ is true whenever $P(k)$ is true.

R.K

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