

Set I

Attempt **All** Questions:

Group-A [11x1=11]

Rewrite the correct option in your answer sheet

- Which of the following is a statement?
 a) The fishes are beautiful b) Study mathematics.
 c) x is a capital of country y. d) **Water is essential for health.**
- The value of $\sqrt{-16} \times \sqrt{-25}$ is
 a) **-20** b) -20i c) 20i d) 20
- If $\angle C = 60^\circ$, b = 5 cm and a = 4 cm of ABC, what is the value of c?
 a) 3.58 cm b) 4.58 cm **c) 4.89 cm** d) 4.56
- In a triangle ABC, $B = 120^\circ$, a = 1, c = 1 then the other angles and sides are
 a) $35^\circ, 45^\circ, \sqrt{2}$ b) $10^\circ, 50^\circ, \sqrt{3}$ c) $20^\circ, 40^\circ, 2$ **d) $30^\circ, 30^\circ, \sqrt{3}$**
- The cosine of the angle between the vectors $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{b} = \vec{i} + 3\vec{j} + 2\vec{k}$ is
 a) **$\frac{1}{14}$** b) 14 c) $\sqrt{14}$ d) 196
- The equation of parabola with the vertex at the origin and the directrix $y - 2 = 0$ is.
 a) $x^2 - 8y = 0$ b) $y^2 + 8y = 0$ **c) $x^2 + 8y = 0$** d) $y^2 - 8y = 0$

- A mathematical problem is given to three students Sumit, Sujan and Rakesh whose chance of solving it are $\frac{1}{2}, \frac{1}{3},$ and $\frac{1}{a}$ respectively. The probability that the problem is solved is $\frac{3}{4}$. The possible values of 'a' are.
 a) $\frac{9}{2}$ **b) 4** c) $\frac{1}{4}$ d) $\frac{1}{8}$

- $\lim_{\theta \rightarrow 0} \frac{\sin\theta}{\theta}$ is equal to...
 a) 0 b) ∞ **c) 1** d) $\frac{0}{0}$
- The derivatives of $\frac{4x^2+3}{3x^2-2}$ is
 a) **$\frac{-34x}{(3x^2-2)^2}$** b) $\frac{30x^2}{3x^2-2}$ c) $\frac{-32x}{(3x^2-2)^3}$ d) $\frac{-31x}{(3x-2)^2}$
- By Newton's Raphson, the positive root of $x^3 - 18 = 0$ in (2, 3) is
 a) 2.666 b) 2.621 **c) 2.620** d) 2.622
- The total cost function of a producer is given as $C = 500 + 30Q + \frac{1}{2}Q^2$ what is the marginal cost (MC) when Q = 4 is
 a) Rs.38 **b) Rs.34** (c) Rs.30 (d) Rs.28

Group-B

- Attempt all questions** **[8x5=40]**
- A function $f(x) = x^2$ is given. Answer the following question for the function f(x).
 a. What is the algebraic nature of the function? **Ans: Even function**
 b. Write the name of the locus of the curve. **Ans: parabolic**
 c. Write the vertex of the function. **Ans: (0,0)**
 d. Write any one property for sketching the curve. **Ans: sym. about Y-axis**
 e. Write the domain of the function. **Ans $(-\infty, \infty)$**

Group-C

13. Compare the sum of n terms of the series: $1 + 2a + 3a^2 + 4a^3 + \dots$ and $a + 2a + 3a + 4a \dots$ up to n terms. **Ans: a^{n-2}**

14. a. In any triangle, prove that: $(b + c) \sin\left(\frac{A}{2}\right) = a \sin\left(\frac{A}{2} + B\right)$ **[3]**

b. Express $\vec{r} = (4, 7)$ as the linear combination of $\vec{a} = (5, -4)$ and $\vec{b} = (-2, 5)$ **[2]**
Ans: $\vec{r} = 2\vec{a} + 3\vec{b}$

15. Calculate the appropriate measure of Skewness for the data below.

Class	0-10	10-20	20-30	30-40	40-50	50-60
No of workers	10	12	25	35	40	50

Ans: $S_k(B) = -0.095$

16. Define different types of discontinuity of a function. Also write the condition for increasing, decreasing and concavity of function. **[2+3]**

Ans: For increasing and decreasing $f'(x) > 0$ and $f'(x) < 0$ resp. And $f''(x) > 0$, concave up and $f''(x) < 0$ concave down.

17. Evaluate: $\int \frac{x^2 dx}{\sqrt{a^2 - x^2}}$ **[Ans: $\frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) - \frac{x}{2} \sqrt{a^2 - x^2} + c$]**

18. Define Trapezoidal rule. Evaluate using Trapezoidal rule for $\int_0^1 \frac{dx}{1+x}$, $n=4$. **Ans: 0.667**

19. A decline in the price of good X by Rs. 5 causes an increase in its demand by 20 units to 50 units. The new price of X is 15.

- Calculate elasticity of demand.
- The elasticity of demand is negative, what does it mean?

Attempt all questions [8×3=24]

20. a. The factor of the expression $\omega^3 - 1$ are $\omega - 1$ and $\omega^2 + \omega + 1$. If $\omega^3 - 1 = 0$
 i) Find the possible values of ω and write the real and imaginary roots of ω . **[2]**

Ans: 1, ω , ω^3

ii) Prove that $\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix} = 0$ where n is a positive integer. **[4]**

b. Verify that $|x + y| \leq |x| + |y|$ with $x = 2$ and $y = -3$. **[2]**

21. The single equation of pair of lines is $2x^2 + 3xy + y^2 + 5x + 2y - 3 = 0$

a. Find the equation of pair of straight lines represented by the single equation. **[4]**

Ans: $x + y + 3 = 0$ and $2x + y - 1 = 0$

b. Are the pair of lines represented by the given equation passes through origin? Write with reason. **[1]**

Ans: No, as origin donot satisfy the eqns

c. Find the point of intersection of the pair of lines. **[2]**
Ans: (4,-7)

d. If three vectors \vec{a}, \vec{b} and \vec{c} are mutually perpendicular unit vectors in space then write a relation between them. **[1]**

Ans: dot product is zero

22. a. Distinguish between derivative and anti-derivative of a function. Write their physical meanings and illustrate with example in your context. Find, the differential coefficient of $\log \sin x$ with respect to x. **[1+ 2+2]**

Ans: cotx

b. Find the area bounded by the y – axis, the curve $x^2 = 4(y - 2)$ and the line $y = 11$. **[3]**

Ans: 36Sq. units

Set II

Group-A [11x1=11]

Rewrite the correct option in your answer sheet.

- Which of the following is not a statement:
a) Study mathematics.
b) The earth moves round the Sun.
c) $2+6 = 5$.
d) A quadrilateral has three sides.
- The value of $\frac{1}{i} + \frac{1}{i^2} + \frac{1}{i^3} + \frac{1}{i^4}$ is ...
a) 1 b) -1 c) i d) 0
- If $a = 2$, $b = \sqrt{6}$, $A = 45^\circ$, then C may be ...
a) 75° b) 15°
c) Both a and b d) Cannot determined
- If $c^4 - 2(a^2 + b^2)c^2 + a^4 + a^2b^2 + b^4 = 0$, then $C = \dots$
a) 60° b) 120°
c) Both a and b d) Cannot determined
- For what value of m is the pair of vectors $\vec{i} - 2\vec{j} + 4\vec{k}$ and $2\vec{i} + 7\vec{j} + m\vec{k}$ orthogonal?
a) 1 b) -2 c) 3 d) 4
- The focus of the parabola $y^2 = 16x$ is ...
a) (2, 0) b) (4, 0) c) (8, 0) d) (16, 0)
- What is the probability that there will be 5 Saturdays in the month of April?
a) $\frac{5}{30}$ b) $\frac{3}{7}$ c) $\frac{2}{7}$ d) $\frac{1}{7}$

- A function $y = f(x)$ is continuous at point $x = a$ if ...
a) Limit exists at point $x=a$
b) Functional value exists at $x = a$.
c) Both (a) and (b) holds.
d) Both (a) and (b) holds and equal.
- Anti-derivative of $\operatorname{cosec} x$ is
a) $\ln(\operatorname{cosec} x - \cot x) + c$ b) $\ln\left(\tan \frac{x}{2}\right) + c$
c) Both a and b d) $-\operatorname{cosec} x \cdot \cot x + c$
- The equation $f(x)$ is given as $x^2 - 153 = 0$. Considering the initial approximation at $x=12$ then the value of next approximation correct upto 3 decimal places is given as...
a) 12.375 b) 12.384 c) 12.369 d) 12.537
- Given the supply function $P = 160 + 0.5Q$, then the price elasticity of supply at $P = 240$ is...
a) 1 b) 2 c) 3 d) 6

Group-B

- Attempt all questions [8x5=40]
- Define domain and range of a function. Find domain and range of function $f(x) = \sqrt{21 - 4x - x^2}$ ($f: \mathbb{R} \rightarrow \mathbb{R}$)
Ans: Domain = [-7,3] and Range = [0,5]
 - Prove that A.M, G.M and H.M between any two unequal positive numbers satisfy the following relations:
(i) $G.M^2 = A.M \times H.M$
(ii) $A.M > G.M > H.M$ [2+3]
 - a. In any triangle ABC, prove that $\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$ [3]
b. find the value of λ so that three points with the position vectors $-2\vec{i} + 3\vec{j} + \lambda\vec{k}$, $\vec{i} + 2\vec{j} + 3\vec{k}$ and $7\vec{i} - \vec{k}$ are collinear. [2]
Ans: $\lambda = 5$

Group-C

15. Following table gives the distribution of daily wage in a company

Wages(Rs.)	Below 100	100-200	200-300	300-400	400-500
No. of workers:	8	16	30	20	6

Examine whether the above distribution is symmetrical or not by calculating the skewness using appropriate method.

Ans: Not symmetrical, $S_k(B) = -0.01$

16. List the criteria for the local maxima and minima of a function.

Find the local maxima and minima of the function

$$f(x) = 4x^3 - 6x^2 - 9x + 1.$$

Ans: Max = 7/2 at x = -1/2 and Min = -25/2 at x = 3/2

17. Integrate: $\int \sqrt{\frac{a+x}{a-x}} dx$ **Ans: $asin^{-1}\frac{x}{a} - \sqrt{a^2 - x^2} + c$**

18. Define Simpson's rule, hence evaluate $\int_0^1 \frac{1}{1+x} dx$.

for n = 4 correct to 4 places of decimals.

Ans: 0.6932

19. A customer has an income of Rs.4800 to spend on two goods X and Y whose prices per units are Rs. 240 and Rs.160 respectively.

a. Find the equation of budget line. Also find its slope and the intercepts.

Ans: $3x + 2y = 60$, where slope = -1.5 and x-intercept = 20 and y-intercept = 30 [2]

b. How does the slope change when price per units of good X increases to Rs.320, the other remaining same? [1]

Ans: changed slope = -2

c. What will be change in y-intercept when price per unit of Y is doubled? [1]

Ans: changed y-intercept = 15

d. How does the budget line change if total budget increases by 50%? [1]

Ans: $3x+2y= 90$

Attempt all questions

[8×3=24]

20. a. Define absolute value of a complex number.
 b. Write any four properties of conjugate of complex number
 c. Find the square root of $12 - 5i$
 d. Express $i - \sqrt{3}$ into polar form.

[1 + 1 + 4 + 2]

Ans: $\pm \frac{1}{\sqrt{2}}(5 - i)$ and $2(\cos 120^\circ + i \sin 120^\circ)$

21. a. Prove that the straight lines joining the origin to the point of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c^2$ are right angles if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$ [4]

b. Find the equations of the tangents to the circle $x^2 + y^2 = 25$ drawn through the point (13,0). [4]

Ans: $12y = \pm 5(x - 13)$

22. a. Find from first principle derivative of: $\sin^2 3x$ [4]

Ans: $3 \sin 6x$

b. Find the area between two curves $y^2 = 4ax$ and $x^2 = 4ay$ [4]

Ans: $\frac{16}{3} a^2$ Sq. units

Set III

Group-A [11x1=11]

Rewrite the correct option in your answer sheet.

- Contrapositive of the statement “ If 4 is an even number, then 5 is not an even number.” is ...
a) If 4 is not an even number, then 5 is an even number.
b) If 5 is an even number, then 4 is an even number.
c) If 5 is an even number, then 4 is not an even number.
d) If 5 is not an even number, then 4 is not an even number.
- The imaginary part of $\frac{1}{-5i}$ is ...
a) $-\frac{1}{5}$ b) 0 **c) $\frac{1}{5}$** d) 5
- In ΔABC , $B = 60^\circ$, $b : c = \sqrt{3} : \sqrt{2}$, then A is ...
a) 75° b) 15° c) 60° d) Cannot determined
- If $a^4 + b^4 + c^4 = 2a^2(b^2 + c^2)$, then A = ...
a) 135° b) 45°
c) Both a and b d) Cannot determined
- For what value of m is the pair of vectors $3\vec{i} + \vec{j} - \vec{k}$ and $m\vec{i} - 4\vec{j} + 4\vec{k}$ collinear?
a) 1 **b) -12** c) 12 d) 6
- The equation of the parabola with vertex at (0,0) and focus (0, -3) is ...
a) $x^2 = 12y$ b) $x^2 = -12y$ c) $y^2 = 12x$ d) $y^2 = -12x$
- A can solve 80% of the problems given in a book while B can solve 60%. What is the probability that at least one of them will solve a problem selected at random from the book?
a) $\frac{23}{25}$ b) $\frac{12}{25}$ c) $\frac{6}{25}$ d) $\frac{10}{25}$

- $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$ holds when n is
a) **A rational number** b) A whole number
c) A real number d) an integer
- Anti-derivative of $\ln x$ is
a) $\frac{1}{x} + c$ **b) $x \ln x - x + c$** c) $x \ln x + c$ d) $\ln x - x + c$
- The equation f(x) is given as $x^2 - 4 = 0$. Considering the initial approximation at $x=6$ then the value of next approximation correct upto 2 decimal places is given as
a) **3.33** b) 2.33 c) 1.33 d) 4.33
- A firm has a demand function $P = 200 - 3Q$ and cost function $C = 2Q^2 - 3Q$, then marginal cost is, When output is 12.
a) 252 b) 164 **c) 45** d) 83

Group-B

Attempt all questions

[8x5=40]

- Define one-one and onto function with an example each. Prove that a function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3x - 1$ is one- one and onto function.
- If a, b, c are in G.P., a, x, b are in A.P. and b, y, c are in A.P. prove that
$$\frac{1}{x} + \frac{1}{y} = \frac{2}{b}$$
- If $|x| < 1$, $y = x + x^2 + x^3 + \dots$, to ∞ , prove that $x = \frac{1}{1+y}$ [3+2]
- a. In any triangle ABC, prove that $\tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$. [3]
b. Express $\vec{r} = (8, -5)$ as the linear combination of $\vec{a} = (2, -3)$ and $\vec{b} = (-1, -2)$. [2]

Ans: $\vec{r} = 3\vec{a} - 2\vec{b}$

Group-C

15. Sarupya obtained samples of CFL bulbs from two suppliers. He got the samples tested in his laboratory for the lengths of the life. The results of the test are as given below:

Length of the life(in Hrs)	No. of bulbs	
	Supplier A	Supplier B
400 – 500	8	6
500 – 600	20	24
600 – 700	16	12
700 – 800	6	8

Which supplier's bulb shows greater variability in the length of the life?

Ans: CV(A) < CV(B) so B has greater variability

16. A man of height 1.5m walks away from the lamp post of height 4.5m at the rate of 20 cm/sec. How fast is the shadow lengthening when the man is 42cm away from the post?

Ans: 10cm/sec

17. Integrate: $\int_0^{\frac{\pi}{4}} \tan^3 x \, dx$.

Ans: $\frac{1}{2}(1 - \ln 2)$

18. Approximate $\int_0^2 2^x \, dx$ using Simpson's 1/3 rule with $h = \frac{1}{2}$. for n = 4 correct to 4 places of decimals. Also find the absolute error.

Ans: Approx. = 4.371, Absolute error = 0.043

19. The demand function for a good is $P = 60 - 2Q$. Fixed cost for a good is Rs192 and the variable cost for each additional unit of good is Rs. 20.

- Write down the equation for total revenue and total cost in terms of Q.
- Find the break-even point.
- Determine the profit function in terms of Q.
- Determine the maximum profit.

Ans: (a) $R = 60Q - 2Q^2$ and $C = 192 + 20Q$

(b) $Q = 8, 12$. (c) $\pi = -2Q^2 + 40Q - 192$.

(d) P = Rs. 8 when Q = 10

Attempt all questions

[8×3=24]

20. a. Define conjugate of complex number with example. If w and z are any two complex numbers then prove that $|w + z| \leq |w| + |z|$. **[1+4]**

- b. If $x - iy = \sqrt{\frac{1-i}{1+i}}$, prove that $x^2 + y^2 = 1$. **[3]**

21. a. Determine the equations of the bisectors of the angles between the lines $3x - 2y + 1 = 0$ and $18x + y - 5 = 0$. Identify the bisector of the acute angle. **[4]**

Ans: $11x - 3y = 0$ and $3x + 11y - 10 = 0$, $11x - 3y = 0$ containing origin.

- b. If $y - x = 2$ is the equation of a chord of the circle $x^2 + y^2 + 2x = 0$, find the equation of the circle of which this chord is a diameter. **[4]**

Ans: $x^2 + y^2 + 3x - y + 2 = 0$

22. a. Find by first principle derivative of: $\sqrt{\tan x}$ **[4]**

Ans: $\frac{1}{2\sqrt{\tan x}} \sec^2 x$

- b. Using integration. Find the area of the curve: $x^2 + y^2 = a^2$. **[4]**

Ans: πa^2 Sq. units

*****The End*****