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5c. $\frac{b^2-c^2}{a^2}\sin 2A+\frac{c^2-a^2}{b^2}\sin 2B+\frac{a^2-b^2}{c^2}\sin 2C=0$

$LHS=\frac{b^2-c^2}{a^2}\sin 2A+\frac{c^2-a^2}{b^2}\sin 2B+\frac{a^2-b^2}{c^2}\sin 2C$

$=\frac{b^2-c^2}{\cancel{2}\cancel{4}R^2\sin^2 A}\cancel{2}\sin A\cos A+\frac{c^2-a^2}{\cancel{2}\cancel{4}R^2\sin^2 B}\cancel{2}\sin B\cos B+\frac{a^2-b^2}{\cancel{2}\cancel{4}R^2\sin^2 C}\cancel{2}\sin C\cos C$

$=\frac{b^2-c^2}{2R^2}\frac{\cos A}{\sin A}+\frac{c^2-a^2}{2R^2}\frac{\cos B}{\sin B}+\frac{a^2-b^2}{2R^2}\frac{\cos C}{\sin C}$

$=\frac{1}{2R^2}\left\{\frac{(b^2-c^2)\left(\frac{b^2+c^2-a^2}{2bc}\right)}{\frac{a}{2R}}+\frac{(c^2-a^2)\left(\frac{a^2+c^2-b^2}{2ac}\right)}{\frac{b}{2R}}+\frac{(a^2-b^2)\left(\frac{a^2+b^2-c^2}{2ab}\right)}{\frac{c}{2R}}\right.$

$=\frac{\cancel{2R}}{2R^2}\left\{\frac{(b^2-c^2)(b^2+c^2-a^2)}{2abc}+\frac{(c^2-a^2)(a^2+c^2-b^2)}{2abc}+\right.$

$\left.\frac{(a^2-b^2)(a^2+b^2-c^2)}{2abc}\right\}$

$=\frac{1}{R.2abc}\{b^4+b^2c^2-a^2b^2-b^2c^2-c^4+a^2c^2+a^2c^2+c^4-b^2c^2$

$-a^4-a^2c^2+a^2b^2+a^4+a^2b^2-a^2c^2-a^2b^2-b^4+b^2c^2\}$

$=\frac{1}{2Rabc}\times 0$

$=0$

$=RHS$ (Proved)

d. $a\cos A+b\cos B+c\cos C=4R\sin A.\sin B\sin C$

$LHS=a\cos A+b\cos B+c\cos C$

$=2R\sin A\cos A+2R\sin B\cos B+c\cos C$

$=R(2\sin A\cos A+2\sin B\cos B)+c\cos C$

$=R(\sin 2A+\sin 2B)+c\cos C$

$=R.2\sin\frac{2A+2B}{2}\cos\frac{2A-2B}{2}+c\cos C$

$=2R\sin(A+B)\cos(A-B)+\cancel{c\cos C}\,2R\sin C\cos(180-(A+B)$

$=2R\sin C(\cos(A-B)-\cos(A+B)$

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$$= 2R\sin C\,\{\cos(A-B) - \cos(A+B)\}$$

$$= 2R\sin C\cdot 2\sin\frac{A-\cancel{B}+A+\cancel{B}}{2}\sin\frac{\cancel{A}+B-\cancel{A}+B}{2}$$

$$= 4R\sin C\sin\frac{2A}{2}\sin\frac{2B}{2}$$

$$= 4R\sin A\sin B\sin C$$

= RHS (proved)$_4$

e. $(b+c-a)\left[\cot\frac{B}{2}+\cot\frac{C}{2}\right] = 2a\cot\frac{A}{2}$

$$LHS = (b+c-a)\left[\cot\frac{B}{2}+\cot\frac{C}{2}\right]$$

$$= (b+c-a)\left[\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}+\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}\right]$$

$$= (b+c-a)\left[\frac{\sqrt{s(s-b)^2}+\sqrt{s(s-c)^2}}{\sqrt{(s-a)(s-b)(s-c)}}\right]$$

$$= (b+c-a)\left(\frac{(s-b)\sqrt{s}+(s-c)\sqrt{s}}{\sqrt{(s-a)(s-b)(s-c)}}\right)$$

$$= (b+c-a)\left\{\frac{\sqrt{s}\,(s-b+s-c)}{\sqrt{(s-a)(s-b)(s-c)}}\right\}$$

$$= (b+c-a)\frac{\sqrt{s}\,(a+\cancel{b}+\cancel{c}-\cancel{b}-\cancel{c})}{\sqrt{(s-a)(s-b)(s-c)}}$$

$$= (a+b+c-2a)\frac{\sqrt{s}\cdot a}{\sqrt{(s-a)(s-b)(s-c)}}$$

$$= \frac{2\cancel{(s-a)}^{2}\sqrt{s}\cdot a}{\cancel{\sqrt{s-a}}\sqrt{(s-b)(s-c)}}$$

$$= \frac{2a\sqrt{s(s-a)}}{\sqrt{(s-b)(s-c)}}$$

$$= 2a\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$$

$$= 2a\cot\frac{A}{2}$$

= RHS (proved)$_5$

6. In ΔABC, if $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$ prove that $C=60°$.

Sol; We have,

$$\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$$

or, $$\frac{b+c+a+c}{(a+c)(b+c)}=\frac{3}{a+b+c}$$

or, $$\frac{(a+b+2c)(a+b+c)}{(a+c)(b+c)}=\frac{3}{\cancel{a+b}\ 1}$$

or, $a^2+ab+ac+ab+b^2+bc+2ac+2bc+2c^2=3(a+c)(b+c)$

or, $a^2+b^2+2c^2+2ab+\cancel{3ac}+\cancel{3bc}=3ab+\cancel{3ac}+\cancel{3bc}+3c^2$

or, $a^2+b^2+2c^2+2ab-3ab-3c^2=0$

or, $a^2+b^2-c^2-ab=0$

or, $$\frac{a^2+b^2-c^2}{2ab}=\frac{ab}{2ab}$$

or, $$\cos C=\frac{1}{2}$$

or, $\cos C=\cos 60°$

$\therefore C=60°$

(Proved),

7. In ΔABC, if $(a+b+c)(a-b-c)+3bc=0$, find A.

Sol; We have

$0=(a+b+c)(a-b-c)+3bc$

or, $0=a^2-\cancel{ab}-\cancel{ac}+\cancel{ab}-b^2-bc+\cancel{ac}-bc-c^2+3bc$

or, $0=a^2-b^2-c^2+bc$

or, $a^2-b^2-c^2=-bc$

or, $$\frac{b^2+c^2-a^2}{2bc}=\frac{bc}{2bc}$$

or, $$\cos A=\frac{1}{2}$$

$\therefore A=60°$

8. If $A = 2B$, then prove that either $c = b$ or $a^2 = b(c+b)$

We have,

$$A = 2B$$

By sine law,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\Rightarrow \frac{a}{\sin 2B} = \frac{b}{\sin B} \qquad [\because A = 2B]$$

$$\Rightarrow a \sin B = b \sin 2B$$

$$\Rightarrow a \cancel{\sin B} = b \cdot 2 \cancel{\sin B} \cos B$$

$$\Rightarrow a = 2b\left(\frac{a^2+c^2-b^2}{2ac}\right)$$

$$\Rightarrow a^2 c = b(a^2+c^2-b^2)$$

$$\Rightarrow a^2 c = a^2 b + b(c^2-b^2)$$

$$\Rightarrow a^2(c-b) = b(c^2-b^2)$$

$$\Rightarrow a^2(c-b) - b(c^2-b^2) = 0$$

$$\Rightarrow a^2(c-b) - b(c-b)(c+b) = 0$$

$$\Rightarrow (c-b)\{a^2 - b(c+b)\} = 0$$

$$\Rightarrow c-b = 0 \quad \underline{\text{or}} \quad a^2 - b(c+b) = 0$$

$$\text{i.e. } c = b \quad \underline{\text{or}} \quad a^2 = b(c+b)$$

(Proved)

9. If $a^4+b^4+c^4-2c^2(a^2+b^2)+a^2b^2=0$, show that $\angle C = 60°$ or $120°$

Given,

$$a^4+b^4+c^4-2c^2(a^2+b^2)+a^2b^2=0$$

$$\Rightarrow (-1)a^4+b^4+c^4-2a^2c^2-2b^2c^2+a^2b^2=0\ (-1)$$

$$\Rightarrow 2a^2c^2+2b^2c^2+2a^2b^2-a^4-b^4-c^4-a^2b^2-2a^2b^2=0$$

$$\Rightarrow 16\times\left(\frac{1}{4}\sqrt{2a^2c^2+2b^2c^2+2a^2b^2-a^4-b^4-c^4}\right)^2 = 3a^2b^2$$

$$\left(\because \frac{1}{4}\sqrt{2a^2c^2+2b^2c^2+2a^2b^2-a^4-b^4-c^4} = \Delta\right)$$

$$\Rightarrow 16\Delta^2 = 3a^2b^2$$

$$\Rightarrow \Delta^2 = \frac{3a^2b^2}{16}$$

$$\Rightarrow \left(\frac{1}{2}ab\sin C\right)^2 = \frac{3a^2b^2}{16}$$

$$\Rightarrow \frac{1}{4}a^2b^2\sin^2 C = \frac{3a^2b^2}{\cancel{16}\,4}$$

$$\Rightarrow \sin^2 C = \frac{3}{4}$$

$$\Rightarrow \sin C = \pm\frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin C = \frac{\sqrt{3}}{2} \quad \underline{\text{or}} \quad \sin C = -\frac{\sqrt{3}}{2}$$

$$= \sin 60^\circ \qquad\qquad = \sin 120^\circ$$

$$\therefore C = 60^\circ \quad \text{or} \quad C = 120^\circ$$

10. In ΔABC, if $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$, prove that a^2, b^2, c^2 are in AP.

Sol: To prove: a^2, b^2, c^2 are in AP

$$\Rightarrow b^2 = \frac{a^2+c^2}{2}$$

Given,

$$\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$$

$$\Rightarrow \sin(B+C)\sin(B-C) = \sin(A+B)\sin(A-B)$$

$$\Rightarrow \sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B$$

$$\Rightarrow \left(\frac{b}{2R}\right)^2 - \left(\frac{c}{2R}\right)^2 = \left(\frac{a}{2R}\right)^2 - \left(\frac{b}{2R}\right)^2$$

$$\Rightarrow \frac{1}{4R^2}(b^2-c^2) = \frac{1}{4R^2}(a^2-b^2)$$

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$$\Rightarrow b^2 - c^2 = a^2 - b^2$$
$$\Rightarrow 2b^2 = a^2 + c^2$$
$$\Rightarrow b^2 = \frac{a^2+c^2}{2}$$

$\therefore a^2, b^2, c^2$ are in A.P. (Proved)

11. If $\frac{\sin(A-B)}{\sin(A+B)} = \frac{a^2-b^2}{a^2+b^2}$, prove that the ΔABC is either isosceles or right angled.

Sol: To prove: ΔBC is either isosceles or right angled

Given,

$$\frac{\sin(A-B)}{\sin(A+B)} = \frac{a^2-b^2}{a^2+b^2}$$

$$\Rightarrow \frac{\sin(A-B)}{\sin(A+B)} \times \frac{\sin(A+B)}{\sin(A+B)} = \frac{a^2-b^2}{a^2+b^2}$$

$$\Rightarrow \frac{\sin(A-B)\sin(A+B)}{\sin C \cdot \sin C} = \frac{a^2-b^2}{a^2+b^2}$$

$$\Rightarrow \frac{\sin^2 A - \sin^2 B}{\sin^2 C} = \frac{a^2-b^2}{a^2+b^2}$$

$$\Rightarrow \frac{\left(\frac{a}{2R}\right)^2 - \left(\frac{b}{2R}\right)^2}{\left(\frac{c}{2R}\right)^2} = \frac{a^2-b^2}{a^2+b^2}$$

$$\Rightarrow \frac{\cancel{\frac{1}{4R^2}}(a^2-b^2)}{\cancel{\frac{1}{4R^2}}(c^2)} = \frac{(a^2-b^2)}{(a^2+b^2)}$$

$$\Rightarrow c^2 = a^2 + b^2 \quad \text{--- (1)}$$

This shows that ΔABC is right angled Δ.

12. In ΔABC, if $(\cos A + 2\cos C) : (\cos A + 2\cos B) = \sin B : \sin C$, prove that the triangle is either isosceles or right angle.

Sol:

We have,

$$\frac{\cancel{f}\cos A + 2\cos C}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$$

$\Rightarrow \sin C\,(\cos A + 2\cos C) = \sin B\,(\cos A + 2\cos B)$

$\Rightarrow \sin C\cos A + 2\sin C\cos C = \sin B\cos A + 2\sin B\cos B$

$\Rightarrow \sin C\cos A - \sin B\cos A = 2\sin B\cos B - 2\sin C\cos C$

$\Rightarrow \cos A\,(\sin C - \sin B) = \sin 2B - \sin 2C$

$\Rightarrow \cos A\left(2\cos\frac{C+B}{2}\sin\frac{C-B}{2}\right) = 2\cos\frac{2B+2C}{2}\sin\frac{2B-2C}{2}$ $[\sin C - \sin]$

$\Rightarrow \cancel{2}\cos A\cos\frac{B+C}{2}\sin\frac{C-B}{2} = \cancel{2}\cos(B+C)\sin(B-C)$

$\Rightarrow \cos A\cos\frac{B+C}{2}\sin\frac{C-B}{2} = \cos(180^\circ - A)\sin(B-C)$ $\left[\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]$

$\Rightarrow \cos A\cos\frac{B+C}{2}\sin\frac{C-B}{2} = -\cos A\left(2\sin\left(\frac{B-C}{2}\right)\cos\left(\frac{B-C}{2}\right)\right)$

$\Rightarrow \cos A\cos\frac{B+C}{2}\sin\frac{C-B}{2} + 2\cos A\sin\frac{B-C}{2}\cos\frac{B-C}{2} = 0$

$\Rightarrow \cos A\sin\frac{C-B}{2}\left(\cos\frac{B+C}{2} - 2\cos\frac{B-C}{2}\right) = 0$ $\left[\sin\frac{B-C}{2} = -\sin\frac{C-B}{2}\right]$

$\Rightarrow \cos A = 0$ or $\sin\frac{C-B}{2} = 0$ or $\cos\frac{B+C}{2} - 2\cos\frac{B-C}{2} = 0$

$\Rightarrow \cos A = \cos 90$ or $\sin\frac{C-B}{2} = \sin 0$

$\therefore A = 90^\circ$ $\qquad \Rightarrow \frac{C-B}{2} = 0$

$\Rightarrow C - B = 0$

$\Rightarrow C = B$

This shows that ΔABC is either right angled or isosceles.

13. If the cosines of two of the angles of a Δ are proportional to the opposite sides, prove that the triangle is isosceles.

Sol; By given condition,

$$\frac{a}{\cos A} = \frac{b}{\cos B}$$

$\Rightarrow a\cos B = b\cos A$

$\Rightarrow \cancel{2R}\sin A\cos B = \cancel{2R}\sin B\cos A$

$\Rightarrow \sin A\cos B = \sin B\cos A$

$\Rightarrow \sin A\cos B - \cos A\sin B = 0$

$\Rightarrow \sin(A-B) = 0$

$\Rightarrow \sin(A-B) = \sin 0$

$\Rightarrow A - B = 0$

$\Rightarrow A = B$

$\therefore \Delta ABC$ in isosceles triangle triangle.

14. If $b-a=mc$, prove that $\cot\frac{(B-A)}{2} = \frac{1+m\cos B}{m\sin B}$

Sol.

Here,

$b-a=mc$

$\Rightarrow m = \frac{b-a}{c}$

$$RHS = \frac{1+m\cos B}{m\sin B}$$

$$= \frac{1+\left(\frac{b-a}{c}\right)\cos B}{\left(\frac{b-a}{c}\right)\sin B}$$

$$\Rightarrow \frac{\frac{c+(b-a)\cos B}{c}}{\frac{(b-a)\sin B}{c}}$$

$$= \frac{c+(b-a)\cos B}{(b-a)\sin B}$$

$$= \frac{a\cos B + b\cos A +}{}$$

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15.) In ΔABC, if $a=3, b=4$, and $c=5$, prove that $\sin 2A = \frac{24}{25}$.

Sol: Here,

$$\sin 2A = \frac{24}{25}$$

$$LHS = \sin 2A$$
$$= 2\sin A\cos A$$
$$= 2\sqrt{1+\cos^2 A}\cdot\frac{b^2+c^2-a^2}{2bc}$$
$$= 2\sqrt{1-\left(\frac{b^2+c^2-a^2}{2bc}\right)^2}\cdot\frac{b^2+c^2-a^2}{2bc}$$
$$= 2\sqrt{1-\left(\frac{4^2+5^2-3^2}{2.4.5}\right)^2}\cdot\frac{4^2+5^2-3^2}{2.4.5}$$
$$= 2\sqrt{1-\left(\frac{16+25-9}{40}\right)^2}\cdot\frac{32}{40}$$
$$= 2\sqrt{1-\left(\frac{4}{5}\right)^2}\cdot\frac{4}{5}$$
$$= 2\sqrt{\frac{25-16}{25}}\cdot\frac{4}{5}$$
$$= 2\cdot\frac{3}{5}\cdot\frac{4}{5}$$
$$= \frac{24}{25}$$
$$= RHS \text{ (Proved)}$$

16) In any ΔABC, if $\frac{\sin A + \cos A}{\cos B} = \sqrt{2}$, show that $\angle C = 135°$

Sol: Given,

$$\frac{\sin A + \cos A}{\cos B} = \sqrt{2}$$

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Squaring both sides

$\Rightarrow \dfrac{\sin^2 A + 2\sin A\cos A + \cos^2 A}{\cos^2 B} = 2$

$\Rightarrow 1 + 2\sin A\cos A = 2\cos^2 B$

$\Rightarrow \sin 2A = 2\cos^2 B - 1$

$\Rightarrow \sin 2A = \cos 2B$

$\Rightarrow \sin 2A = \sin(90^\circ - 2B)$

$\Rightarrow 2A = 90^\circ - 2B$

$\Rightarrow 2A + 2B = 90^\circ$

$\Rightarrow A + B = \dfrac{\cancel{90}^{45}}{\cancel{2}}$

$\Rightarrow A + B = 45^\circ$

$\Rightarrow 180 - C = 45^\circ$

$\Rightarrow C = 135^\circ$

$\therefore \angle C = 135^\circ$

(~~Pr~~ Showed)

17. In $\triangle ABC$, if $a = 13, b = 14, c = 15$, find $\Delta, s, \sin\frac{A}{2}, \cos\frac{A}{2}$

Sol: Here,

$a = 13, b = 14, c = 15$

$\therefore s = \dfrac{a+b+c}{2}$

$= \dfrac{13+14+15}{2}$

$= 21$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-13)(21-14)(21-15)}$

$= \sqrt{21 \times 8 \times 7 \times 6}$

$= \sqrt{7056}$

$= 84$

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$= \sqrt{\frac{(21-14)(21-15)}{14\times15}}$$

$$= \sqrt{\frac{17\times8}{14\times15}}$$

$$= \frac{1}{\sqrt{5}}$$

$$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

$$= \sqrt{\frac{21(21-13)}{14\times15}}$$

$$= \sqrt{\frac{21\times8}{14\times15}}$$

$$= \frac{2}{\sqrt{5}}$$

14. If $b-a=mc$, prove that $\cot\frac{B-A}{2} = \frac{1+m\cos B}{m\sin B}$

Sol:

Given,

$$b-a = mc$$

$$\Rightarrow m = \frac{b-a}{c}$$

$$RHS = \frac{1+m\cos B}{m\sin B}$$

$$= \frac{1+\frac{b-a}{c}\cos B}{\frac{b-a}{c}\sin B}$$

$$= \frac{\frac{c+(b-a)\cos B}{c}}{\frac{(b-a)\sin B}{c}}$$

$$= \frac{c+(b-a)\cos B}{(b-a)\sin B}$$

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$$= \frac{2R\sin C + (2R\sin B - 2R\sin A)\cos B}{(2R\sin B - 2R\sin A)\sin B}$$

$$= \frac{\sin C + (\sin B - \sin A)\cos B}{(\sin B - \sin A)\sin B}$$

$$= \frac{\sin(A+B) + (\sin B - \sin A)\cos B}{\sin B(\sin B - \sin A)}$$

$$= \frac{\cancel{\sin A\cos B} + \cos A\sin B + \sin B\cos B - \cancel{\sin A\cos B}}{\sin B(\sin B - \sin A)}$$

$$= \frac{\cos A\sin B + \sin B\cos B}{\sin B(\sin B - \sin A)}$$

$$= \frac{\cancel{\sin B}(\cos A + \cos B)}{\cancel{\sin B}(\sin B - \sin A)}$$

$$= \frac{\cos A + \cos B}{\sin B - \sin A}$$

$$= \frac{\cancel{2}\left(\cos\frac{A+B}{2}\right)\cos\frac{A-B}{2}}{\cancel{2}\left(\cos\frac{A+B}{2}\right)\sin\frac{B-A}{2}}$$

$$= \frac{\cos\left(-\frac{(B-A)}{2}\right)}{\sin\left(\frac{B-A}{2}\right)}$$

$$= \frac{\cos\frac{B-A}{2}}{\sin\frac{B-A}{2}}$$

$$= \cot\frac{B-A}{2}$$

= RHS (proved),

Exercise 8.2

1. Solve the triangle $A=60°, B=45°, c=6\sqrt{2}$

Sol:

We have,

$$A=60°, B=45°, C=6\sqrt{2}$$

Now,

$$A+B+C=180°$$

$$\Rightarrow C=180°-(60°+45°)$$

$$\therefore C=75°$$

By using Sine Law,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$\Rightarrow \frac{a}{\sin 60°}=\frac{b}{\sin 45°}=\frac{6\sqrt{2}}{\sin 75°}$$

$$\Rightarrow \frac{2a}{\sqrt{3}}=\frac{2b}{\sqrt{2}}=\frac{24\sqrt{2}}{\sqrt{6}+\sqrt{2}}$$

$$\Rightarrow \frac{2a}{\sqrt{3}}=\frac{2b}{\sqrt{2}}=\frac{24\sqrt{2}}{\sqrt{2}(\sqrt{3}+1)}$$

$$\Rightarrow \frac{2a}{\sqrt{3}}=\sqrt{2}b=\frac{24}{\sqrt{3}+1}$$

Taking 1st and 3rd ratios,

$$\frac{2a}{\sqrt{3}}=\frac{24}{\sqrt{3}+1}$$

$$\Rightarrow a=\frac{\cancel{24}^{12}\sqrt{3}}{\cancel{2}(\sqrt{3}+1)}$$

$$\Rightarrow a=\frac{12\sqrt{3}}{\sqrt{3}+1}$$

$$\Rightarrow a=\frac{12\sqrt{3}}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$\Rightarrow a = \frac{36 - 12\sqrt{3}}{3-1}$$

$$\Rightarrow a = \frac{\cancel{12}^{6}(3 - \sqrt{3})}{\cancel{2}}$$

$$\therefore a = 6(3-\sqrt{3})$$

Taking 2^{nd} & 3^{rd} ratios,

$$\sqrt{2}\, b = \frac{24}{\sqrt{3}+1}$$

$$\Rightarrow b = \frac{24}{\sqrt{6}+\sqrt{2}}$$

$$\Rightarrow b = \frac{24}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$$

$$\Rightarrow b = \frac{24\sqrt{6} - 24\sqrt{2}}{6-2}$$

$$\Rightarrow b = \frac{\cancel{24}^{6}(\sqrt{6}-\sqrt{2})}{\cancel{4}}$$

$$\therefore b = 6\sqrt{2}(\sqrt{3}-1)$$

2. Solve $\triangle ABC$, if $a = \sqrt{3}+1$ and $b = \sqrt{3}-1$, $C = 60°$.

Sol: Here,

$a = \sqrt{3}+1$

$b = \sqrt{3}-1$

$C = 60°$

Using cosine law,

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$= (\sqrt{3}+1)^2 + (\sqrt{3}-1)^2 - 2(\sqrt{3}+1)(\sqrt{3}-1)\cos 60°$$

$$= 3 + \cancel{2\sqrt{3}} + 1 + 3 - \cancel{2\sqrt{3}} + 1 - 2(3-1)\cos 60°$$

$$= 8 - \cancel{4}^{2} \cdot \frac{1}{\cancel{2}}$$

$$= 6$$

$$\therefore c = \sqrt{6}$$

Now,

Using sine law,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\Rightarrow \frac{\sin A}{\sqrt{3}+1} = \frac{\sin B}{\sqrt{3}-1} = \frac{\sin 60^\circ}{\sqrt{6}}$$

$$\Rightarrow \frac{\sin A}{\sqrt{3}+1} = \frac{\sin B}{\sqrt{3}-1} = \frac{\sqrt{3}}{2\sqrt{3}\cdot\sqrt{2}}$$

$$\Rightarrow \frac{\sin A}{\sqrt{3}+1} = \frac{\sin B}{\sqrt{3}-1} = \frac{1}{2\sqrt{2}}$$

Taking 1^{st} & 3^{rd} ratios,

$$\frac{\sin A}{\sqrt{3}+1} = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \sin A = \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\Rightarrow \sin A = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\Rightarrow \sin A = \sin 105^\circ$$

$$\therefore A = 105^\circ$$

In ΔABC;

$$A+B+C = 180^\circ$$

$$\Rightarrow B = 180^\circ - (105+60^\circ)$$

$$\therefore B = 15^\circ$$

4. If $a=2$, $b=\sqrt{6}$ and $c=\sqrt{3}-1$, solve the triangle.

Sol: We have,

$a=2$, $b=\sqrt{6}$ & $c=\sqrt{3}-1$

By cosine law,

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\Rightarrow \cos A = \frac{6+3-2\sqrt{3}+1-4}{2\sqrt{6}\cdot(\sqrt{3}-1)}$$

$$\Rightarrow \cos A = \frac{6-2\sqrt{3}}{2\sqrt{6}(\sqrt{3}-1)}$$

$$= \frac{\cancel{2}\sqrt{3}\,\cancel{(\sqrt{3}-1)}}{\cancel{2}\sqrt{6}\,\cancel{(\sqrt{3}-1)}}$$

$$= \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

$$\Rightarrow \cos A = \cos 45^\circ$$

$$\therefore \quad A = 45^\circ$$

Also,

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{4+3-2\sqrt{3}+1-6}{2\cdot 2(\sqrt{3}-1)}$$

$$= \frac{2-2\sqrt{3}}{4(\sqrt{3}-1)}$$

$$= \frac{\cancel{2}\,\cancel{(1-\sqrt{3})}}{-\cancel{4}_{2}\,\cancel{(1-\sqrt{3})}}$$

$$= -\frac{1}{2}$$

$$\Rightarrow \cos B = \cos 120^\circ$$

$$\therefore \quad B = 120^\circ$$

Also, $A+B+C = 180^\circ \Rightarrow C = 180-45-120$.

$\therefore C = 15^\circ$

5. If three angles of a triangle are in the ratio 2:3:7 find a:b:c.

Sol: Here,

$$A:B:C = 2:3:7$$

i.e. $$\frac{A}{2} = \frac{B}{3} = \frac{C}{7} = k \text{ (say)}$$

$$\Rightarrow \frac{A}{2} = k, \ \frac{B}{3} = k, \ \frac{C}{7} = k$$

$$\Rightarrow A = 2k, \ B = 3k; \ C = 7k$$

Now,

$$A+B+C = 180^\circ$$

$$\Rightarrow 2k+3k+7k = 180^\circ$$

$$\Rightarrow 12k = 180^\circ$$

$$\Rightarrow k = \frac{180}{12}$$

$$\therefore k = 15^\circ$$

$$\therefore A = 2k = 30^\circ$$

$$B = 3k = 45^\circ$$

$$C = 7k = 105^\circ$$

By using sine law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{a}{\sin 30^\circ} = \frac{b}{\sin 45^\circ} = \frac{c}{\sin 105^\circ}$$

$$\Rightarrow \frac{a}{\frac{1}{2}\times 4} = \frac{b}{\frac{\sqrt{2}}{2}\times 4} = \frac{c}{\frac{\sqrt{6}+\sqrt{2}}{4}\times 4}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{2\sqrt{2}} = \frac{c}{\sqrt{6}+\sqrt{2}}$$

$$\Rightarrow \frac{a}{\sqrt{2}\cdot\sqrt{2}} = \frac{b}{2\sqrt{2}} = \frac{c}{\sqrt{2}(\sqrt{3}+1)}$$

$$\Rightarrow \frac{a}{\sqrt{2}} = \frac{b}{2} = \frac{c}{\sqrt{3}+1}$$

$$\therefore a:b:c = \sqrt{2}:2:\sqrt{3}+1$$

6. In ΔABC, $B = 60^\circ$; $b:c = \sqrt{3}:\sqrt{2}$. Show that $A = 75^\circ$.

Sol; Here,

$$B = 60^\circ$$

$$b:c = \sqrt{3}:\sqrt{2}$$

$$\Rightarrow \frac{b}{c} = \frac{\sqrt{3}}{\sqrt{2}}$$

Using sine law,

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{\sqrt{3}}{\sin 60^\circ} = \frac{\sqrt{2}}{\sin C}$$

$$\Rightarrow \sin C = \frac{\frac{\sqrt{3}}{2} \times \sqrt{2}}{\sqrt{3}}$$

$$= \frac{\cancel{\sqrt{3}}}{\sqrt{2}\cdot\cancel{\sqrt{3}}}$$

$$= \frac{1}{\sqrt{2}}$$

$$= \cancel{\frac{\sqrt{2}}{2}}$$

$$\Rightarrow \sin C = \sin 45^\circ$$

$$\therefore C = 45^\circ$$

Now, $A + B + C = 180^\circ \Rightarrow A = 180 - (45 + 60) =$

$$\therefore A = 75^\circ$$

(Proved)

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7. If $a:b:c = 4:5:6$ in $\triangle ABC$, prove that $C = 2A$.

Sol: Here,

$$a:b:c = 4:5:6$$

$$\Rightarrow \frac{a}{4} = \frac{b}{5} = \frac{c}{6} = k \text{ (say)}$$

$$\Rightarrow a = 4k,\ b = 5k,\ c = 6k$$

Using cosine law,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{25k^2 + 36k^2 - 16k^2}{2 \cdot 5k \cdot 6k}$$

$$= \frac{k^2(25+36-16)}{k^2(2\cdot 5\cdot 6)}$$

$$= \frac{\cancel{45}\,3}{\cancel{60}\,4}$$

$$\therefore \cos A = \frac{3}{4}$$

Now,

$$\cos 2A = 2\cos^2 A - 1$$

$$= 2\left(\frac{3}{4}\right)^2 - 1$$

$$= \frac{\cancel{2} \times 9}{\cancel{16}\,8} - 1$$

$$= \frac{9-8}{8}$$

$$\therefore \cos 2A = \frac{1}{8}$$

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$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{16k^2+25k^2-36k^2}{2\cdot 4k\cdot 5k}$$

$$= \frac{5k^2}{40k^2}$$

$$= \frac{1}{8}$$

$\Rightarrow \cos C = \cos 2A$

$\therefore C = 2A$

(proved)

8. If $\cos A = \frac{4}{5}$, $\cos B = \frac{3}{5}$, find $a:b:c$.

Sol;

we have,

$$\cos A = \frac{4}{5} \text{ and } \cos B = \frac{3}{5}$$

we know,

$$\sin A = \sqrt{1-\cos^2 A}$$

$$= \sqrt{1-\left(\frac{4}{5}\right)^2}$$

$$= \sqrt{\frac{25-16}{25}}$$

$$= \frac{3}{5}$$

and $\sin B = \sqrt{1-\cos^2 B}$

$$= \sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$= \frac{4}{5}$$

we know,

$$\sin C = \sin(A+B)$$
$$= \sin A \cos B + \cos A \sin B$$
$$= \frac{3}{5} \cdot \frac{3}{5} + \frac{4}{5} \cdot \frac{4}{5}$$
$$= \frac{9}{25} + \frac{16}{25}$$
$$= 1$$

By using sine law,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\therefore a : b : c = 3 : 4 : 5$$

(Proved),

10. If $a = 2$, $b = \sqrt{6}$ and $c = \sqrt{3}+1$, find the greatest and least angle of the triangle ABC.

Sol: Here,

Given, $a = 2$, $b = \sqrt{6}$, $c = \sqrt{3}+1$

$$\cos A = \frac{6+3+1+2\sqrt{3}-4}{2\sqrt{6}(\sqrt{3}+1)}$$
$$= \frac{2\sqrt{3}(\sqrt{3}+1)}{2\sqrt{6}(\sqrt{3}+1)}$$
$$= \frac{1}{\sqrt{2}}$$
$$= \cos 45^\circ$$
$$\therefore A = 45^\circ$$

$$\cos B = \frac{4+3+1+2\sqrt{3}-6}{2 \cdot 2(\sqrt{3}+1)}$$
$$= \frac{2+2\sqrt{3}}{4(\sqrt{3}+1)}$$

$= \frac{1}{2}$

$= \cos 60^\circ$

$\therefore B = 60^\circ$

Now,

$A + B + C = 180^\circ$

$\Rightarrow C = 180^\circ - 45^\circ - 60^\circ$

$\therefore C = 75^\circ$

Hence, $A = 45^\circ$ is the least angle & $C = 75^\circ$ is the greatest $\angle$ of $\triangle ABC$.

11 If $a = 2, b = \sqrt{3} + 1, C = 60^\circ$, solve the triangle.

Sol: Given,

$a = 2, b = \sqrt{3} + 1, C = 60^\circ$

Here,

$$\cos 60^\circ = \frac{4 + 3 + 1 + 2\sqrt{3} - c^2}{2 \cdot 2(\sqrt{3} + 1)}$$

$$\Rightarrow \frac{1}{2} = \frac{8 + 2\sqrt{3} - c^2}{4(\sqrt{3} + 1)}$$

$\Rightarrow 4\sqrt{3} + 4 = 16 + 4\sqrt{3} - 2c^2$

$\Rightarrow 2c^2 = 12$

$\therefore c = \sqrt{6}$

And,

$$\cos B = \frac{4 + 6 - (3 + 1 + 2\sqrt{3})}{2 \times 2 \times \sqrt{6}}$$

$$= \frac{10 - 4 - 2\sqrt{3}}{4\sqrt{6}}$$

$$= \frac{2\sqrt{3}(\sqrt{3} - 1)}{4\sqrt{6}}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$= \cos 75^\circ$

$\therefore B = 75^\circ$

$A + B + C = 180^\circ$
$\Rightarrow A = 180^\circ - 75^\circ - 60^\circ$
$\therefore A = 45^\circ$

Hence,
$A = 45^\circ, B = 75^\circ, c = \sqrt{6}$

12. Find the ambiguous solution

a. $a = 2, b = \sqrt{3} + 1, A = 45^\circ$

Sol: We have,
$a = 2, b = \sqrt{3} + 1, A = 45^\circ$

By sine law,

$$\frac{2}{\sin 45^\circ} = \frac{\sqrt{3}+1}{\sin B}$$

$$\Rightarrow \sin B = \frac{(\sqrt{3}+1) \times \frac{1}{\sqrt{2}}}{2}$$

$$\Rightarrow \sin B = \frac{\sqrt{3}+1}{\sqrt{2}}$$

$\therefore B = 75^\circ$ or 105°

When $B = 75^\circ, C = 180^\circ - 120^\circ = 60^\circ$
When $B = 105^\circ, C = 180^\circ - 105^\circ = 75^\circ$
$\therefore C = 30^\circ$ or 75°

Again, by sine law,

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

When $C = 30^\circ$

$$\Rightarrow \frac{a}{1/\sqrt{2}} = \frac{c}{1/2}$$

$$\Rightarrow 2\sqrt{2} \times \frac{1}{2} = c$$

$$\therefore c = \sqrt{2}$$

when $c = 75^\circ$

$\Rightarrow \frac{2}{\sin 45^\circ} = \frac{c}{\sin 75^\circ}$

$\Rightarrow \frac{2}{1/\sqrt{2}} = \frac{c}{\frac{\sqrt{6}+\sqrt{2}}{4}}$

$\Rightarrow \frac{2}{1/\sqrt{2}} = \frac{4c}{\sqrt{6}+\sqrt{2}}$

$\Rightarrow 2\sqrt{2} = \frac{4c}{\sqrt{6}+\sqrt{2}}$

$\Rightarrow 2\sqrt{16} + 4 = 4c$

$\Rightarrow 2 \times 4 + 4 = 4c$

$\Rightarrow 12 = 4c$

$\therefore c = 3$

b. $a = 3,\ b = 8\sqrt{3},\ A = 30^\circ$

Sol: Here,

$a = 3,\ b = 3\sqrt{3},\ A = 30^\circ$

$\cos 30^\circ = \frac{27 + c^2 - 9}{2 \cdot 3\sqrt{3} \cdot c}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{18 + c^2}{6\sqrt{3} \cdot c}$

$\Rightarrow 18c = 36 + 2c^2$

$\Rightarrow c^2 - 9c + 18 = 0$

$\Rightarrow c^2 - 6c - 3c + 18 = 0$

$\Rightarrow c(c-6) - 3(c-6) = 0$

$\Rightarrow c = 6, 3$

If $c = 6$, $\cos B = \frac{9 + 6^2 - 27}{2 \times 3 \times 6} = \frac{1}{2} = \cos 60^\circ \Rightarrow B = 60^\circ$

If $c = 3$,

$\cos B = \frac{9+9-27}{2\times 3\times 3} = \frac{-1}{2} = \cos 120^\circ$

$\therefore B = 120^\circ$

~~If c = 3.~~

When $B = 60^\circ, A = 30^\circ, C = 180^\circ - 90^\circ = 90^\circ$

When $B = 120^\circ, A = 30^\circ, C = 180^\circ - 150^\circ = 30$

Required solution are

$B = 60^\circ, C = 90^\circ, c = 6$

$B = 120^\circ, C = 30^\circ, c = 3$

9. Solve the triangle ABC in the following cases.

a) $a = 2, b = \sqrt{2}, c = \sqrt{3}+1$

Sol: Here,

$a = 2, b = \sqrt{2}, c = \sqrt{3}+1$

By Sine law,

$a : b : c = \sin A : \sin B : \sin C$

$\Rightarrow 2 : \sqrt{2} : \sqrt{3}+1 = \sin A : \sin B : \sin C$

$\Rightarrow \sin A : \sin B : \sin C = \frac{2\cdot 1}{2\sqrt{2}} : \sqrt{2} \times \frac{1}{2\sqrt{2}} : \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\Rightarrow \sin A : \sin B : \sin C = \sin 45^\circ \cdot \sin 30^\circ : \sin 105^\circ$

$\therefore \angle A = 45^\circ, \angle B = 30^\circ, \angle C = 105^\circ$

b) $A = 45^\circ, B = 60^\circ, C = 45^\circ$

Sol; Here,

$A = 45^\circ, B = 60^\circ, C = 45^\circ$

By Sine law,

$a : b : c = \sin 75^\circ : \sin 60^\circ : \sin 45^\circ$

$= \frac{\sqrt{3}+1}{2\sqrt{2}} : \frac{\sqrt{3}}{2} : \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}} \times 2\sqrt{2} : \frac{\sqrt{3}}{1} \times 2\sqrt{2} : \frac{1}{\sqrt{2}} \times 2\sqrt{2}$

$= \sqrt{3}+1 : \sqrt{6} : 2$

$\therefore a = \sqrt{3}+1,\ b = \sqrt{6},\ c = 2$

c) $a = 1;\ b = \sqrt{3},\ C = 30^\circ$

Sol: Here,

$a = 1, b = \sqrt{3}, C = 30^\circ$

By sine law,

$\cos 30^\circ = \frac{1+3-c^2}{2 \times 1 \times \sqrt{3}}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{4-c^2}{2\sqrt{2}}$

$\Rightarrow 3 = 4 - c^2$

$\Rightarrow c = \pm 1$ (+ve only)

And,

$\cos B = \frac{1+1-3}{2 \times 1 \times 1} = \frac{-1}{2} = \cos 120^\circ$

$\therefore B = 120^\circ$

Now,

$A + B + C = 180^\circ$

$\Rightarrow A = 180^\circ - 120^\circ - 30^\circ$

$\therefore A = 30^\circ$

d) $a = \sqrt{57}, A = 60^\circ, \Delta = 2\sqrt{3}$

Sol: Here,

$a = \sqrt{57}, A = 60^\circ, \Delta = 2\sqrt{3}$

We know that,

$\Delta = \frac{1}{2} bc \sin 60^\circ$

$\Rightarrow 2\sqrt{3} = \frac{1}{2} bc \times \frac{\sqrt{3}}{2}$

$\Rightarrow bc = 8, \; b = 8/c$ ——— (1)

Also,

$\cos 60^\circ = \frac{b^2 + c^2 - 57}{16}$

$\Rightarrow \frac{1}{2} = \frac{b^2 + c^2 - 57}{16}$

$\Rightarrow 8 = b^2 + c^2 - 57$

$\Rightarrow \frac{64}{c^2} + c^2 = 64$ $\quad [b = 8/c]$

$\Rightarrow \cancel{c^4 - 64c^2}$

$\Rightarrow c^4 - 65c^2 + 64 = 0$

$\Rightarrow c^4 - 64c^2 - c^2 + 64 = 0$

$\Rightarrow (c^2 - 64)(c^2 - 1) = 0$

$\therefore c = \pm 1, \pm 8$ (+ve only)

when $c = 1, b = 8$

when $c = 8, b = 1$

By sine law,

when $a = \sqrt{57}, c = 1, b = 8$

$\frac{\sqrt{57}}{\sin 60^\circ} = \frac{1}{\sin C}$

$\Rightarrow \sin C = \frac{\sqrt{3}}{2\sqrt{57}}$

$\Rightarrow C = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{57}}\right)$

Also,

$$\frac{\sqrt{57}}{\sin 60^\circ} = \frac{8}{\sin B}$$

$$\Rightarrow \sin B = \frac{8\sqrt{3}}{2\sqrt{57}} = \frac{4\sqrt{3}}{\sqrt{57}}$$

$$\therefore B = \sin^{-1}\left(\frac{4\sqrt{3}}{\sqrt{57}}\right)$$

When $a = \sqrt{57}$; $b = 1$, $c = 8$

$$\frac{\sqrt{51}}{\sin 60^\circ} = \frac{1}{\sin B}$$

$$\Rightarrow \sin B = \frac{\sqrt{3}}{2\sqrt{57}}$$

$$\therefore B = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{57}}\right)$$

And,

$$\frac{\sqrt{57}}{\sin 60^\circ} = \frac{8}{\sin C}$$

$$\therefore C = \sin^{-1}\left(\frac{4\sqrt{3}}{\sqrt{57}}\right)$$

Required solution are:-

$b = 8$, $c = 1$, $C = \sin^{-1}\left(\sqrt{3}/2\sqrt{57}\right)$, $B = \sin^{-1}\left(\frac{4\sqrt{3}}{57}\right)$

or, $b = 1$, $c = 8$, $B = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{57}}\right)$, $C = \sin^{-1}\left(\frac{4\sqrt{3}}{\sqrt{57}}\right)$